BLIND EQUALIZATION OF NONLINEAR DIGITAL SATELLITE LINKS WITH PSK MODULATION

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ABSTRACT

Satellite communication systems suffer from nonlinear intersymbol interference due to the limited bandwidth of the filters and the need to operate the onboard amplifier near saturation. We consider the blind equalization problem for this class of systems in a multichannel setting. By exploiting the bandpass nature of the channel and the statistical properties of PSK constellations, a simple necessary and sufficient condition for blind equalizability from the second-order statistics of the received signal is derived. An algorithm for the extraction of the equalizers is also given.

1. INTRODUCTION

In satellite repeaters operation of the high power amplifier near saturation is often necessary for efficiency reasons [4]. The subsequent nonlinear effect, together with the restricted bandwidth of the receive and transmit filters, result in an overall channel presenting nonlinear intersymbol interference (ISI). Among the techniques suggested to cope with this effect one finds adaptive nonlinear predistortion [7] and nonlinear equalizers [1] or ISI cancellers [5]. Recently blind equalization techniques for nonlinear channels, both deterministic [6] and second-order statistics (SOS) based [8], have been proposed. These methods hinge on the fact that if several subchannels are available, under some conditions a linear FIR equalizer is capable of completely removing both linear and nonlinear ISI. Moreover this equalizer can be computed blindly, therefore disposing of the need for training signals which reduce data rate efficiency.

We investigate SOS-based blind equalization of nonlinear satellite links using phase-shift keying (PSK) modulation, a popular format for these systems [2]. As shown in [3], the baseband equivalent nonlinear channel can be accurately represented by a truncated Volterra series of the form

$$y(n) = \sum_{i=1}^{q} \sum_{j=0}^{l_i} h_{ij} s_i(n-j) + z(n), \qquad (1)$$

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where $y(n)$ is the sampled received signal, $z(n)$ is the additive noise, $s_1(n) \triangleq a(n)$ are the transmitted PSK symbols and the remaining terms $s_i(n)$, $i > 1$ have the form

$$s_i(n) = \prod_{k=1}^{m_i} a^*(n - t_{ik}) \prod_{k=m_i+1}^{2m_i+1} a(n - t_{ik}). \qquad (2)$$

The h_{ij} are the coefficients of the i-th baseband equivalent Volterra kernel. It is assumed that p subchannels are available so that h_{ij}, $y(n)$ and $z(n)$ are $p \times 1$ vectors. This multichannel configuration can be obtained by oversampling the received signal and/or by using multiple antennas. Observe from (2) that only odd-order kernels appear in (1), and that these have one more unconjugated input than conjugated inputs. The absence of even-order distortions is due to the fact that they generate spectral components which lie outside the channel bandwidth (centered at the carrier frequency) and therefore are rejected by the bandpass filter following the amplifier [3]. The structure of this class of channels allows us to derive necessary and sufficient conditions under which the equalizers can be obtained blindly from the SOS of the received signal $y(n)$. A general algorithm is also presented, which is different from the one in [8] in that the linear kernel is not required to be the longest.

In our notation, $(\cdot)^T$, $(\cdot)^H$ denote transpose and conjugate transpose respectively; I_m, J_m denote respectively the $m \times m$ identity matrix and the shift matrix with ones in the first subdiagonal and zeros elsewhere; e_k denotes the k-th unit vector; and $\oplus$ stands for block diagonal concatenation, e.g. $A \oplus B = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$.

Section 2 introduces the equalization problem. Blind equalizability conditions and algorithms are given in section 3 and 4 respectively. Simulations are in section 5.

2. PROBLEM FORMULATION

By defining the $pm \times 1$ vectors

$$Y^T(n) \triangleq [\; y(n)^T \quad y(n-1)^T \quad \cdots \quad y(n-m+1)^T \;],$$

$$Z^T(n) \triangleq [\ z(n)^T \quad z(n-1)^T \quad \cdots \quad z(n-m+1)^T\ ],$$

one can write the channel input-output relation as

$$Y(n) = \mathcal{F}S(n) + Z(n), \tag{3}$$

where $S(n) \triangleq [\ S_1^T(n) \quad S_2^T(n)\ ]^T$ is the signal vector. $S_1(n)$ and $S_2(n)$ represent the linear and nonlinear contribution to ISI, being given by

$$S_1^T(n) \triangleq [\ a(n) \quad a(n-1) \quad \cdots \quad a(n-l_1-m+1)\ ], \tag{4}$$

$$S_2^T(n) = [\ s_2(n) \quad \cdots \quad s_2(n-l_2-m+1) \quad | \quad \cdots \quad | \\ s_q(n) \quad \cdots \quad s_q(n-l_q-m+1)\ ]. \tag{5}$$

$\mathcal{F} = [\ \mathcal{F}_1 \quad \mathcal{F}_2 \quad \cdots \quad \mathcal{F}_q\ ]$ is the channel matrix, with

$$\mathcal{F}_i = \begin{bmatrix} h_{i0} & \cdots & h_{il_i} & & \\ & \ddots & & \ddots & \\ & & h_{i0} & \cdots & h_{il_i} \end{bmatrix} \qquad pm \times (m+l_i).$$

The covariance sequence of the process $Y(n)$ is

$$C_y(k) \triangleq \text{cov}[Y(n), Y(n-k)] = \mathcal{F}C_s(k)\mathcal{F}^H + C_z(k),$$

where $C_s(k) \triangleq \text{cov}[S(n), S(n-k)]$ and $C_z(k) \triangleq \text{cov}[Z(n), Z(n-k)]$. Our assumptions are:

A1: The channel matrix $\mathcal{F}$ is tall and has full column rank.
A2: $z(n)$ is zero-mean, white, with covariance $\sigma_z^2 I_p$.
A3: The covariance matrix $C_s(0)$ is positive definite.
A4: The symbols $a(n)$ are iid and drawn from an M-ary PSK constellation: $a(n) \in \{\exp(j2\pi k/M), \quad k = 0, 1, \ldots, M-1\}$.

Assumption **A1** is a 'coprimeness' requirement on the subchannels; it ensures the existence of vectors g_d such that $g_d^H \mathcal{F} = e_{d+1}^H$ for $0 \leq d \leq m+l_1-1$. Hence in the noiseless case $g_d^H Y(n) = a(n-d)$, so that g_d provides the coefficients of a zero-forcing (ZF) FIR equalizer of degree $m-1$ with associated delay d. We are interested in finding these equalizers from $C_y(0)$ and $C_y(1)$.

Under **A2**, one has $C_z(k) = \sigma_z^2 J_{pm}^{pk}$. Then, because of **A1**, σ_z^2 can be estimated as the smallest eigenvalue of $C_y(0)$, so that the effect of noise can be removed from $C_y(k)$. Henceforth we shall assume that $C_y(k) = \mathcal{F}C_s(k)\mathcal{F}^H$.

Assumption **A3** allows one to write

$$C_s(0) = QQ^H \qquad \text{with } Q \text{ invertible.} \tag{6}$$

By using this square root Q, one can define the *normalized* channel and source covariance matrices respectively as

$$F = \mathcal{F}Q, \qquad \bar{C}_s(k) = Q^{-1}C_s(k)Q^{-H}. \tag{7}$$

With these, the matrices $C_y(k)$ become

$$C_y(k) = F\bar{C}_s(k)F^H, \qquad \text{with } \bar{C}_s(0) = I. \tag{8}$$

As shown in [8], $\bar{C}_s(1)$ plays a pivotal role in our problem, which can be stated as follows. Suppose that one is able to obtain a channel matrix estimate, say $\hat{\mathcal{F}}$, satisfying $\hat{\mathcal{F}}C_s(k)\hat{\mathcal{F}}^H = C_y(k)$ for $k = 0, 1$, and from it one computes the equalizer vectors $\hat{g}_d$ such that $\hat{g}_d^H \hat{\mathcal{F}} = e_{d+1}^H$. Can we ensure that these equalizers still provide good performance with the *actual* channel matrix $\mathcal{F}$?

3. EQUALIZABILITY CONDITIONS

The channel structure (2) leads to the following result:

Lemma 1 *Consider the generating terms $s_i(n)$ of the baseband equivalent Volterra channel, given by (2). Under assumption* **A4**, *each $s_i(n)$ is a white process, and if $i \neq j$ then* $\text{cov}[s_i(n), s_j(k)] = 0$ *for all n, k.*

As a consequence, with $\sigma_i^2 \triangleq E[|s_i(n)|^2]$ the source covariance matrices are block diagonal:

$$C_s(k) = \sigma_1^2 J_{m+l_1}^k \oplus \sigma_2^2 J_{m+l_2}^k \oplus \cdots \oplus \sigma_q^2 J_{m+l_q}^k.$$

Thus with the obvious choice $Q = \sigma_1 I_{m+l_1} \oplus \cdots \oplus \sigma_q I_{m+l_q}$, the corresponding normalized quantities become simply

$$\bar{C}_s(k) = J_{m+l_1}^k \oplus J_{m+l_2}^k \oplus \cdots \oplus J_{m+l_q}^k. \tag{9}$$

With this, we can invoke Lemma 2 from [8] to obtain:

Theorem 1 *Under assumptions* **A1**-**A4**, *let $\hat{\mathcal{F}}$ be any matrix such that*

$$\hat{\mathcal{F}}C_s(k)\hat{\mathcal{F}}^H = C_y(k) \tag{10}$$

for $k = 0$, 1, and let the vectors $\hat{g}_d$ satisfy

$$\hat{g}_d^H \hat{\mathcal{F}} = e_{d+1}^H \quad \text{for } 0 \leq d \leq m+l_1-1. \tag{11}$$

If the kernel lengths satisfy $l_i \neq l_1$ for $i \neq 1$, then one has $\hat{g}_d^H \mathcal{F} = c \cdot e_{d+1}^H$ for some c with $|c| = 1$.

In addition, if $l_i = l_1$ for some $i \neq 1$, then there exist matrices $\hat{\mathcal{F}}$ satisfying (10) for all k such that the $\hat{g}_d$ in (11) do not completely remove all ISI.

To see this last point, consider the case in which $l_2 = l_1$. Then any matrix of the form $\hat{\mathcal{F}} = \mathcal{F}(P \oplus I_{m+l_3} \oplus \cdots \oplus I_{m+l_q})$ with P of the form

$$P = \begin{bmatrix} \cos\alpha \cdot I_{m+l_1} & \frac{\sigma_1}{\sigma_2} e^{j\beta} \sin\alpha \cdot I_{m+l_1} \\ \frac{\sigma_2}{\sigma_1} e^{j\gamma} \sin\alpha \cdot I_{m+l_1} & -e^{j(\beta+\gamma)} \cos\alpha \cdot I_{m+l_1} \end{bmatrix} \tag{12}$$

satisfies (10) for all k. Therefore the ambiguity represented by the parameters α, β, γ in (12) cannot be resolved using SOS. And if $\hat{g}_d$ satisfies (11), then these do not totally remove ISI since in the noiseless case,

$$\hat{g}_d^H Y(n) = \cos\alpha \cdot a(n-d) + \frac{\sigma_2}{\sigma_1} e^{-j\beta} \sin\alpha \cdot s_2(n-d).$$

4. A BLIND EQUALIZATION ALGORITHM

Assuming the channel satisfies the 'length disparity condition' of Lemma 2, the equalizers can be computed as follows. To begin, perform an SVD of $C_y(0)$:

$$C_y(0) = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma^2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} U_1^H \\ U_2^H \end{bmatrix},$$

where Σ is $r \times r$ diagonal and U_1 has r columns, r being the number of columns of $\mathcal{F}$: $r = (m + l_1) + \cdots + (m + l_q)$. Since from (8) $C_y(0) = FF^H$ and F has full column rank, one has $F = U_1 \Sigma V$ for some $r \times r$ unitary V. Let V_1 comprise the first $m + l_1$ columns of V. It can be readily verified that the ZF equalizers are then given by

$$\mathcal{G}_{\rm ZF} \triangleq \begin{bmatrix} \hat{g}_0 & \cdots & \hat{g}_{m+l_1-1} \end{bmatrix} = \sigma_1 U_1 \Sigma^{-1} V_1. \quad (13)$$

In order to obtain V_1 we shall make use of the matrix

$$R \triangleq \Sigma^{-1} U_1^H C_y(1) U_1 \Sigma^{-1} = V \bar{C}_s(1) V^H. \quad (14)$$

Because $\bar{C}_s(1)$ is as in (9), one finds that

$$RV_1 = V_1 J_{m+l_1}, \qquad R^H V_1 = V_1 J_{m+l_1}^H, \quad (15)$$

showing how V_1 can be estimated from either its first or its last column. To estimate these, we extract the information about V_1 in R through several of its powers. By relabeling the channel kernels if neccessary, we can write $\bar{C}_s(1) = J_{m+l_1} \oplus Y \oplus \hat{Y}$, where $\hat{Y}$ comprises the J blocks of size less that $m + l_1$ and Y comprises those of size greater that $m + l_1$. Partitioning $V = \begin{bmatrix} V_1 & V_2 & V_3 \end{bmatrix}$ where V_2, V_3 have the same number of columns as Y, $\hat{Y}$ respectively,

$$R = \underbrace{V_1 J_{m+l_1} V_1^H}_{\bar{R}} + \underbrace{V_2 Y V_2^H}_{\Gamma} + \underbrace{V_3 \hat{Y} V_3^H}_{\hat{\Gamma}}. \quad (16)$$

Moreover for all k, $R^k = \bar{R}^k + \Gamma^k + \hat{\Gamma}^k$. Now if we denote $\mathcal{J}_{rs} \triangleq J_s \oplus \overset{(r \text{ times})}{\cdots} \oplus J_s$, then for some u and $s_1 > s_2 > \cdots > s_u > m + l_1$, one can write without loss of generality

$$Y = \mathcal{J}_{r_1 s_1} \oplus \cdots \oplus \mathcal{J}_{r_u s_u}, \quad (17)$$

Partitioning accordingly $V_2 = \begin{bmatrix} V_{21} & \cdots & V_{2u} \end{bmatrix}$ one has $\Gamma = V_2 Y V_2^H = \sum_{i=1}^{u} \Gamma_i$ where $\Gamma_i \triangleq V_{2i} \mathcal{J}_{r_i s_i} V_{2i}^H$; and

$$\Gamma^k = \sum_{i=1}^{u} \Gamma_i^k \quad (18)$$

holds for all k. Let $R^{\#}$ denote the pseudoinverse of the matrix R. One can check that $R^{\#} = V \bar{C}_s^H(1) V^H$ so that

$$R^{\#} V_{2i} = V_{2i} \mathcal{J}_{r_i s_i}^H, \qquad (R^{\#})^H V_{2i} = V_{2i} \mathcal{J}_{r_i s_i}. \quad (19)$$

Using (19), it can be shown that

$$\Gamma_i = \sum_{k=1}^{s_i - 1} (R^{\#})^{s_i - k - 1} \Gamma_i^{s_i - 1} (R^{\#})^{k-1}. \quad (20)$$

Consider now the following iteration. Set $R_1 = R$, and for $i = 1, 2, \ldots, u$, do:

$$\begin{aligned} \Gamma_i &= \sum_{k=1}^{s_i - 1} (R^{\#})^{s_i - k - 1} R_i^{s_i - 1} (R^{\#})^{k-1}, \\ R_{i+1} &= R_i - \Gamma_i. \end{aligned}$$

By combining (16)-(20), one finds that at the end of the iteration, $R_{u+1} = R - \Gamma = \bar{R} + \hat{\Gamma}$. Therefore

$$R_{u+1}^{m+l_1-1} = V_1 J_{m+l_1}^{m+l_1-1} V_1^H = (V_1 e_{m+l_1})(V_1 e_1)^H, \quad (21)$$

since $\hat{\Gamma}^{m+l_1-1} = 0$. Thus an SVD of the matrix $R_{u+1}^{m+l_1-1}$ provides the first and last columns of V_1 up to a constant c with $|c| = 1$. The remaining columns can be found via (15) and then the equalizers are computed as per (13).

5. SIMULATION RESULTS

We considered a baseband equivalent Volterra channel with $q = 3$ kernels with lengths $l_1 = 2$, $l_2 = 1$, $l_3 = 3$. The modulation is 8-PSK with unit radius and the nonlinear terms are $s_2(n) = a^*(n)a(n-1)a(n-2)$, $s_3(n) = a(n)a^*(n-1)a(n-2)$. The number of subchannels was $p = 4$. The coefficients are given in table 1. The linear-to-nonlinear distortion ratio for this channel is 8.35 dB.

For illustration purposes the phase ambiguity inherent to the algorithm was removed before computing the error rates, which were averaged based on 100 independent runs. The noise is white Gaussian with variance σ_z^2. The SNR is defined as SNR= $10 \log \frac{\sigma_1^2}{\sigma_z^2} \sum_{j=0}^{l_1} ||h_{1j}||^2$.

We chose an equalizer length $m = 7$. Two different approaches were considered. In the first one, no denoising of the estimated covariance matrices $\hat{C}_y(k)$ was performed and the ZF equalizers were obtained by the algorithm of section 4. In the second approach, the noise variance was estimated as the smallest eigenvalue of $\hat{C}_y(0)$. Then the covariance matrices were denoised prior to computation of the ZF equalizers, and then the MMSE equalizers were obtained using the following relation from [9]:

$$\mathcal{G}_{\rm MMSE} = \left(I - \hat{\sigma}_z^2 \hat{C}_y^{-1}(0) \right) \mathcal{G}_{\rm ZF} \quad (22)$$

where $\hat{C}_y(0)$ represents the *undenoised* estimate of the channel output covariance matrix. Figures 1 and 2 show the symbol error rate of the equalizers with delays 0 and 8 respectively as a function of SNR, when 1000 received samples were used for covariance estimation. The performance

channel	h_{10}	h_{11}	h_{12}	h_{20}	h_{21}	h_{30}	h_{31}	h_{32}	h_{33}
1	0.4+0.6j	-0.5+0.2j	1.0+0.5j	0.1+0.0j	0.2-0.2j	-0.1+0.0j	-0.1-0.1j	0.1+0.2j	-0.1+0.2j
2	0.5+0.4j	0.6+0.8j	-1.0+0.8j	0.3+0.2j	-0.1+0.1j	0.1-0.1j	0.3+0.1j	0.0+0.0j	0.1+0.5j
3	-0.5+1.0j	0.4-0.9j	0.6+0.3j	-0.1+0.1j	0.1+0.1j	0.3+0.1j	-0.1+0.1j	0.2-0.2j	0.1-0.4j
4	-0.2+0.5j	0.8-0.3j	0.1-0.7j	0.1+0.1j	-0.1+0.0j	-0.1+0.1j	-0.2+0.1j	-0.1+0.2j	0.2+0.1j

Table 1: Coefficients of the baseband equivalent Volterra channel used in the simulations

of the equalizers with intermediate delays lies somewhere between these two.

Quite surprisingly, the MMSE equalizers perform worse than the ZF ones for low SNR. This is due to the fact that in noisy environments accurate estimation of the noise variance requires a larger sample size.

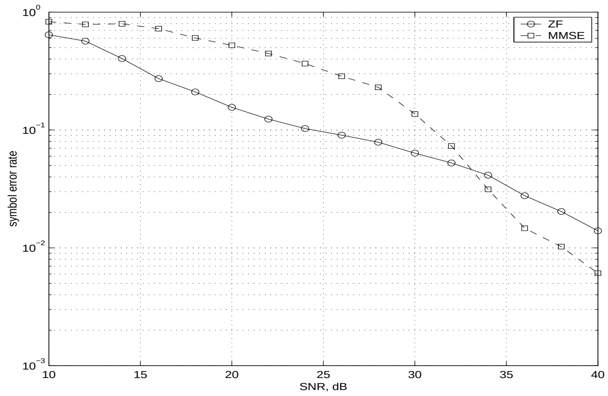

Figure 1: Symbol error rate vs. SNR for the delay 0 equalizers. 1000 samples were used for covariance estimation

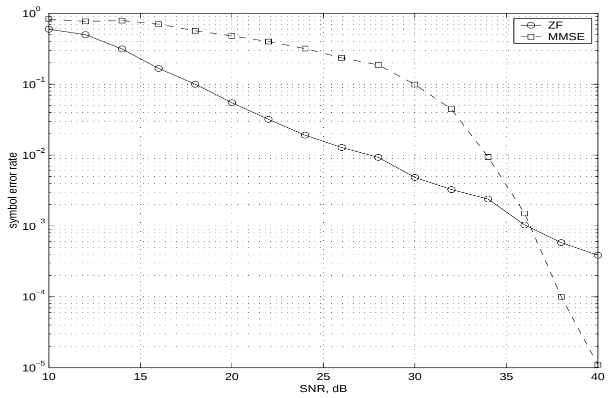

Figure 2: Symbol error rate vs. SNR for the delay 8 equalizers. 1000 samples were used for covariance estimation

6. CONCLUSIONS

We have presented necessary and sufficient conditions under which a multichannel baseband equivalent Volterra model of a nonlinear satellite link with PSK modulation can be equalized from the SOS of the received signal alone: namely, that the lengths of the nonlinear kernels all be different from that of the linear part. An algorithm was provided for the computation of the equalizers.

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