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Quantifiers

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Quantifiers are terms of generality. They are also among our prime examples of operators which take scope. The distinction between singular and general terms, as well as the ways that general terms enter into scope relations, are certainly fundamental to our understanding of language. Yet when we turn to natural language, we encounter a huge and apparently messy collection of general terms; not just *every* and *some*, but *most*, *few*, *between five and ten*, and many others. Natural language sentences also display a complex range of scope phenomena; unlike first-order logic, which clearly and simply demarcates scope in its notation.

In spite of all this complexity, the study of quantification in natural language has made remarkable progress. Starting with a seminal trio of papers from the early 1980s, Barwise and Cooper (1981), Higginbotham and May (1981), and Keenan and Stavi (1986), quantification in natural language has been investigated extensively by philosophers, logicians, and linguists. The result has been an elegant and far-reaching theory. This chapter will present a survey of some of the important components of this theory. Section (I) will present the core of the theory of generalized quantifiers. This theory explores the range of expressions of generality in natural language, and studies some of their logical properties. Section (II) will turn to issues of how quantifiers enter into scope relations. Here there is less unanimity than in the theory of generalized quantifiers. Two basic approaches, representative of the main theories in the literature, will be sketched and compared. Finally, Section (III) will turn briefly to the general question of what a quantifier is.

I Generality in Natural Language

The first of our topics is the notion of quantifiers as expressions of generality. We have already observed that natural languages present us with a wide range of such expressions. We thus confront a number of questions, both foundational and descriptive: what are the semantics of expressions of generality, what sorts of basic semantic properties do they have, and what expressions of generality appear in natural language?

One of the accomplishments of research over the last 25 years is to give interesting answers to these questions. Though many problems remain open, a great deal about the basic semantic properties of natural-language quantifiers is known. This is encapsulated in what is often known as *generalized quantifier theory*. This section will be devoted to the core of this theory. It should be noted at the outset that generalized quantifier theory is a large and well-developed topic, and there is too much in it to cover in any exhaustive way. There are, fortunately, two very good more specialized surveys to which interested readers may turn for more details and more references: Westerståhl (1989) and Keenan and Westerståhl (1997).

I.1 Generalizing Quantifiers

Before examining the variety of natural language quantification, we should return to the basic question of what quantifiers are.

The basic idea about what quantifiers are comes from Frege (1879, 1891, 1893). Frege observed that the familiar quantifiers $\forall$ and $\exists$ can be thought of, in Frege's terms, as *second level concepts*. We do not need Frege's particular idea of a concept, though. Let us suppose, for instance, that the semantic value of a predicate F is its extension: the set of objects that fall under it. Then the universal quantifier takes such an extension as input, and outputs *true* if that extension is the entire universe, and *false* if it is not. It is thus a function whose inputs are sets, and outputs are truth values. Likewise, the existential quantifier takes a set as input, and outputs *true* if that set is non-empty, and *false* if it is empty.

The standard quantifiers of first-order logic can thus be thought of as functions from sets to truth values, or equivalently, as *sets of sets*. This captures Frege's idea that quantifiers are second-level concepts.[1] This is a thesis about the semantic values of quantifiers:

(1) The semantic values of quantifiers are sets of sets.

This thesis, though it will be refined in some ways as we progress, is the core of the theory of quantifiers we will develop.

Once we see quantifiers as sets of sets, we can quickly observe that being non-empty and being the entire universe are merely two among many. So, for instance, relative to a fixed universe M, we can define:

(2) a. $(\mathbf{Q}_R)_M = \{X \subseteq M \mid |X| > |M \setminus X|\}$

b. $(\mathbf{Q}_\alpha)_M = \{X \subseteq M \mid |X| \geq \aleph_\alpha\}$

($|X|$ is the cardinality of a set X. In many cases, where we have some set which is to be thought of as the semantic value of an expression, I will put the set in **bold**; so $(\mathbf{Q}_R)_M$ interprets Q_R relative to a universe M. I shall use 'semantic value' and 'denotation' interchangeably.)

These are often called *generalized quantifiers* or *Mostowski quantifiers*, in honor of their first extensive study by Mostowski (1957). Mostowski quantifiers can be added to the usual first-order logic. $Q_\alpha x F(x)$ says that the extension of F has cardinality $\geq \aleph_\alpha$. $(\mathbf{Q}_R)_M$ is the Rescher quantifier (Rescher, 1962). For a finite universe M, $(Q_R)xF(x)$ says that the extension of F is more than half the size of M.

One fairly technical distinction needs to be made before we close this subsection. We defined Mostowski quantifiers for a fixed universe M. These are what are usually called *local* generalized quantifiers. *Global* generalized quantifiers are simply functions from sets M to local generalized quantifiers on M. So, for instance, for each M, $(\mathbf{Q}_R)_M$ is the local Rescher quantifier on M. $\mathbf{Q}_R$, the global Rescher quantifier, is the function which takes M to $(\mathbf{Q}_R)_M$.

[1]I am here assuming that semantic values are sets, and that they are *extensional*. Much of what follows is independent of these assumptions, though there are a number of places at which it will be crucial that predicate semantic values have *cardinalities*.

I.2 Generalized Quantifiers in Natural Language

Though the kind of generalization of ∀ and ∃ given by Mostowski quantifiers is a major step, it is not enough to accurately explain natural language quantifiers. For instance, in a way the Rescher quantifier Q_R expresses *most*, but not the way natural language does. Consider:

(3) a. Most students attended the party.

b. Most birds fly.

c. Most people have ten fingers.

These do not compare the size of a predicate extension with the size of the entire universe. Rather, they compare the size of one subset of the universe with another. The first, for instance, says that the set of students who came to the party is larger than the set of students who did not come to the party.

Once we see this sort of *binary* structure, we see that it is quite widespread in natural language. We see, for instance:

(4) a. Few students attended the party.

b. Most students attended the party.

c. Both students attended the party.

d. Enough students attended the party.

We also see such constructions as:

(5) a. Between five and ten students attended the party.

b. At least ten students attended the party.

c. All but five students attended the party.

d. More male than female students attended the party.

e. John's mother attended the party.

f. More of John's than Mary's friends attended the party.

In fact, the English analogs of ∀ and ∃ have the same structure:

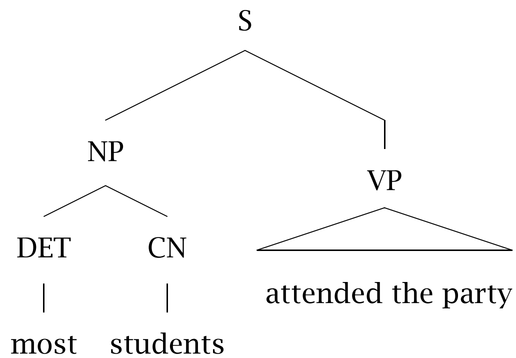
(6) a. Every student attended the party.

b. Some student attended the party.

(These examples are modeled on the much more extensive list in Keenan and Stavi (1986).)

The binary pattern in natural-language expressions of generality is no accident. It reflects a fundamental feature of the syntax of natural languages. Simplifying somewhat, we can observe that sentences break down into combinations of noun phrases (NPs) and verb phrases (VPs). Noun phrases also break down, into combinations of *determiners* (DETs) and common nouns (CNs) (or more complex construction with adjectival modifiers like *small brown dog*). Quantifier expressions of the sorts we see in (4–6) occupy the determiner positions in subject noun phrases. The basic structure we see in all those examples follows the pattern:

(7)



We saw a moment ago that to capture the meaning of *most* in (3), we need to see the CN position as *semantically significant*.[2]

To do this, we need a modest extension of the idea of a Mostowskian generalized quantifier. We may retain the basic idea that quantifiers are sets of sets; only now, we need to think of them as relations between sets. They take two inputs, the semantic value of a CN expression like *student* and a VP expression like *attended the party*, and output a truth value. Each of these syntactic constituents gets interpreted as having a set as its semantic value, so quantifier expressions like *most* should be interpreted as functions from pairs of sets to truth values, i.e. as binary relations between sets. This is our next thesis:

[2]There are a number of syntactic issues I am putting aside here. See any current syntax text, or the handbook discussions of Bernstein (2001) and Longobardi (2001). For some interesting cross-linguistic work, see Matthewson (2001) and the papers in Bach *et al.* (1995).

(8) The semantic values of many quantifier expressions (determiners) in natural languages are functions which take as input two sets, and output truth values, i.e. they are relations between sets.

This is often called the *relational theory of determiner denotations*.

It is common to classify quantifiers by their inputs. A Mostowski quantifier, which takes one set input, is classified as type $\langle 1 \rangle$. The quantifiers we have just looked at are classified as type $\langle 1, 1 \rangle$, taking two set (unary relation) inputs. We can now offer a definition:

(9) a. A (local) type $\langle 1, 1 \rangle$ quantifier on M is a function $\mathbf{Q}_M(X, Y)$ mapping sets $X, Y \subseteq M$ to truth values.

b. A (global) type $\langle 1, 1 \rangle$ quantifier is a function from universes M to local quantifiers $\mathbf{Q}_M$.

This allows us to give definitions for a range of quantifiers. For instance, for each M and $X, Y \subseteq M$:

(10) a. $\mathbf{every}_M(X, Y) \longleftrightarrow X \subseteq Y$

b. $\mathbf{most}_M(X, Y) \longleftrightarrow |X \cap Y| > |X \setminus Y|$

c. $\mathbf{neither}_M(X, Y) \longleftrightarrow |X| = 2 \wedge X \cap Y = \emptyset$

d. $\mathbf{at\ least\ 10}_M(X, Y) \longleftrightarrow |X \cap Y| \geq 10$

Similar definitions can be given for other quantifiers, including those in (4) and (5).

We thus see that natural-language *determiners* can be interpreted as type $\langle 1, 1 \rangle$ quantifiers. Full NPs can be understood as these quantifiers with one argument fixed, which are then type $\langle 1 \rangle$ quantifiers.[3]

I.3 Restricted Quantifiers

Type $\langle 1, 1 \rangle$ quantifiers appear to be *restricted* quantifiers. Whereas $\forall$ and $\exists$, and other type $\langle 1 \rangle$ quantifiers, range over the entire universe, a quantifier like **most** ranges over its first input, corresponding to the CN position in a noun phrase.

[3] Terminology varies on whether determiners or full NPs are called 'quantifiers'; for instance, Barwise and Cooper (1981) reserve the term 'quantifier' for NP denotations, i.e. type $\langle 1 \rangle$ quantifiers.

It does turn out that natural language quantifiers display important features of restricted quantification, but the reason is more complex, and more interesting, than the mere presence of an extra argument. It is entirely possible to define type $\langle 1,1\rangle$ quantifiers which is not restricted. For instance:

(11) $\textbf{more}_M{}^{\langle 1,1\rangle}(X,Y) \longleftrightarrow |X| > |Y|$

This does not behave as if its domain is restricted to X, in cases where Y and X do not overlap.

The core feature which makes natural language quantifiers behave like restricted quantifiers is exhibited by the following pattern:

(12) a. i. Every student attended the party.
ii. Every student is a student who attended the party.
b. i. Few students attended the party.
ii. Few students are students who attended the party.
c. i. Most students attended the party.
ii. Most students are students who attended the party.

In each of these, (i) and (ii) are equivalent. The corresponding feature for $\textbf{more}_M{}^{\langle 1,1\rangle}$ would be $|X| > |Y| \longleftrightarrow |X| > |X \cap Y|$, which is easily falsified.

The pattern we see in (12) but not in (11) is called conservativity:[4]

(13) (CONSERV) $\mathbf{Q}_M(X,Y)$ is *conservative* iff for all $X, Y \subseteq M$, $\mathbf{Q}_M(X,Y) \longleftrightarrow \mathbf{Q}_M(X, X\cap Y)$.

Conservativity expresses the idea that the interpretation of a sentence with a quantified noun phrase only looks as far as the CN, so the CN restricts the domain of quantification. However, attention to global vs. local notions of quantification suggests that we should strengthen the condition for restricted quantification a little bit. For a restricted global quantifier, we do not want any aspect of the universe M beyond its intersection with X to affect $\mathbf{Q}_M(X,Y)$. Hence, we want:

(14) (UNIV) For each M and $X, Y \subseteq M$, $\mathbf{Q}_M(X,Y) \longleftrightarrow \mathbf{Q}_X(X, X\cap Y)$.

[4] This same property was called the 'lives on' property by Barwise and Cooper (1981) and 'intersectivity' by Higginbotham and May (1981). I believe the terminology 'conservativity' is due to Keenan and Stavi (1986).

('UNIV' for 'universe-restricting'. Note the subscript on the right-hand side is X.)

The difference between CONSERV and UNIV is relatively small, but not entirely trivial. It was observed by van Benthem (1983, 1986) that UNIV is equivalent to CONSERV together with the property EXT (for 'extension'):

(15) (EXT) For each $X, Y \subseteq M \subseteq M'$, $\mathbf{Q}_M(X, Y) \longleftrightarrow \mathbf{Q}_{M'}(X, Y)$.

As observed by Westerståhl (1985b, 1989) EXT, expresses the idea that quantifiers do not change their meanings on different domains.

One of the striking facts about natural languages, observed in Barwise and Cooper (1981) and Keenan and Stavi (1986), is that all natural-language determiner denotations satisfy CONS. Once EXT was isolated as a distinct condition, it was observed that determiner denotations satisfy EXT as well. These are proposed *linguistic universals*: non-trivial empirical restrictions on possible natural languages. We have seen that logic can give us type $\langle 1, 1\rangle$ quantifiers which violate these constrains. (11) violates conservativity. A quantifier violating EXT is given by Westerståhl (1985b):

(16) $\mathbf{many}^*{}_M(X, Y) \longleftrightarrow |X \cap Y| > 1/3 \cdot |M|$

The proposed linguistic universals says that all natural language determiner denotations satisfy CONS and EXT.

There are a number of potential counter-examples to these universals. Some of them remain controversial, but the consensus in the literature is that the universals hold. Let me give a couple of examples. Why is $\mathbf{more}_M{}^{\langle 1,1\rangle}$ not a counter-example? Because the determiner *more* appears to be a *two-place* determiner, figuring in constructions like:

(17) More students than professors attended the party.

More than is conservative. (Quantifiers taking more than two arguments have been investigated by Keenan and Moss (1984) and Beghelli (1994) (see Keenan and Westerståhl (1997) for additional discussion).

Another much-discussed case is *only*. It may appear to be an easy example of the failure of conservativity. Consider:

(18) Only dogs bark.

A natural reading of this sentence makes it true if and only if the set of barking things is included in the set of dogs. This suggests a highly simplified semantics for *only*:

(19) $\mathbf{only}_M(X, Y) \longleftrightarrow Y \subseteq X$

This is simplified in many ways, but it makes the failure of conservativity vivid. $Y \subseteq X \longleftrightarrow (Y \cap X) \subseteq X$ only holds when $Y \subseteq X$. Hence, any false sentence suffices to show that conservativity fails.

Even so, there is good reason to think that *only* is not a determiner. It appears outside of noun phrases, in:

(20) John only talked to Susan.

It also appears in places we do not see determiners in English noun phrases:

(21) a. Only the Provost/John talked to Susan.

b. Only between five and ten students came to the party.

We have good reason to think that *only* is not a counter-example to conservativity because it is not a determiner.[5]

Finally, what about possible failures of EXT, such as **many***? It is often observed that *many* is context-dependent, in that what counts as many is heavily influenced by context. Depending on how this sort of context-dependence is handled, it may be possible to interpret *many* in accord with EXT and CONS. This remains a controversial issue. See Westerståhl (1985b) for extensive discussion.[6]

Conservativity has proved an extremely important property. The space of conservative quantifiers is much more orderly than the full range of type $\langle 1, 1 \rangle$ quantifiers. This is brought

[5]For more on *only*, see Rooth (1985, 1996). Related to expressions like *only* are adverbs of quantification, such as *always* and *never*. For discussion of these, see Lewis (1975) and von Fintel (1994).

[6]The context-dependence proposed for determiners like *many* is in the meaning of the determiner, not in the restriction of its domain. For discussions of how context restricts the domains of quantifiers, see Westerståhl (1985a), von Fintel (1994), Stanley and Szabó (2000) with comments by Bach (2000) and Neale (2000), and Cappelen and Lepore (2002). I am skipping over the issue, related to paradoxes, of whether all quantifiers, including such apparently unrestricted ones as *everything*, wind up with some non-trivial contextual domain restriction. This is discussed in Williamson (2004) and Glanzberg (2005).

out most vividly in the *conservativity theorem* due initially to Keenan and Stavi (1986), further investigated by van Benthem (1983, 1986) and Keenan (1993). The key insight is that the class of conservative quantifiers can be build up *inductively*, from a base stock of quantifiers and some closure conditions. Let M be a fixed *finite* universe and let $CONS_M$ be the collection of conservative quantifiers on M. We will build up a class of quantifiers $D - GEN_M$ on M as follows. $D - GEN_M$ contains *every* and *some* (as type $\langle 1, 1 \rangle$ quantifiers). We also assume that each set of members of M is definable by a predicate, and that $D - GEN_M$ is closed under *Boolean combination* and *predicate restrictions*. The latter assumes that if $\mathbf{Q}_M(X, Y)$ is in $D - GEN_M$, so is $\mathbf{Q}_M(X \cap C, Y)$ for $C \subseteq M$. This amounts to closure under (intersective) adjectival restriction in an NP.

The conservativity theorem tells us that for each M:

(22) $CONS_M = D - GEN_M$

This tells us that the domain of possible natural language determiners is far more orderly than it might have appeared: we can build up all the conservative determiner denotations by the inductive process which builds $D - GEN_M$. The proof of the conservativity theorem also tells us that for a given finite universe M, we can build a natural language determiner (possibly quite long) which expresses each conservative quantifier on M. This is the *Finite Effability Theorem* of Keenan and Stavi (1986):

(23) For a finite M, each element of CON_M is expressed by a determiner of English.

Thus, the conservativity property makes for a much more tractable space of determiner denotations, built up in a systematic way which is closely tied to constructions we can carry out in natural language.

I.4 Logicality

We began this section with the idea that quantifiers are expressions of generality. Though we have seen a wide range of determiner denotations which fall within $CONS$ and EXT, we have yet to give any statement of what makes them general. One way to articulate the notion of generality is that it requires the truth of a sentence to be independent of exactly which individuals are involved in interpreting a given quantifier. This can be captured formally by the constraint of *permutation invariance*:

(24) (PERM) Let π be a permutation of M (i.e. a bijection from M to itself). Then $\mathbf{Q}_M(X, Y) \longleftrightarrow \mathbf{Q}_M(\pi[X], \pi[Y])$.

PERM is a local condition. But a global variant, *isomorphism invariance*, is quite natural:

(25) (ISOM) For any M and M', if $\iota\colon M \to M'$ is a bijection, then $\mathbf{Q}_M(X, Y) \longleftrightarrow \mathbf{Q}_M{}'(\iota[X], \iota[Y])$.

ISOM is commonly assumed in the mathematical literature, and is built into the definitions of quantifiers in Mostowski (1957) and Lindström (1966).[7]

Westerståhl (1985b, 1989) observed that if we assume EXT, the domain of quantification ceases to matter, and ISOM and PERM are equivalent. Following van Benthem (1983, 1986), one sometimes sees quantifiers satisfying CONS, EXT, and ISOM called *logical quantifiers*. One of the key features of these logical quantifiers is that they are entirely determined by two cardinal numbers:[8]

(26) For any M and M', and $X, Y \subseteq M$ and $X', Y' \subseteq M'$, if $|X \setminus Y| = |X' \setminus Y'|$ and $|X \cap Y| = |X' \cap Y'|$, then $\mathbf{Q}_M(X, Y) \longleftrightarrow \mathbf{Q}_{M'}(X', Y')$.

There has been a great deal of mathematical work investigating these quantifiers; much of it elegantly discussed in van Benthem (1986) and Westerståhl (1989).

ISOM (or PERM) does appear to capture the idea that quantifiers are *general*, and so not about any objects in particular. It is a further question whether this makes them genuinely *logical constants*, as the label 'logical quantifier' suggests. The idea that some sort of permutation-invariance is a key feature of logical notions has been proposed by Mautner (1946) and Tarski (1986). A vigorous defense of the logicality of ISOM quantifiers is given in Sher (1991).

[7]The condition is called 'ISOM', as ι induces an isomorphism between the structures $\mathfrak{M} = \langle M, X, Y \rangle$ and $\mathfrak{M}' = \langle M', \iota[X], \iota[Y] \rangle$. In essence, as Lindström (1966) observed, a type $\langle 1, 1 \rangle$ generalized quantifier is a class of structures of the form $\langle M, X, Y \rangle$; if it satisfies ISOM, we have a class of structures closed under isomorphism.

[8]I am not sure who should be credited with this result. As far as I know, it appeared in this form first in van Benthem (1984, 1986), but related ideas appear in Mostowski (1957) and Higginbotham and May (1981).

I.5 Quantifiers and Noun Phrases

We have seen that, with a few controversial potential exceptions, natural language determiner denotations satisfy CONS and EXT. But there are some clear cases treated by generalized quantifier theory which do not satisfy ISOM, and so are not logical quantifiers. Possessive constructions like *John's* in (5), for instance, often violate ISOM.

Another example of a quantifier violating ISOM is provided by proper names. Suppose *John* denotes an individual **j**. We can build a type $\langle 1 \rangle$ generalized quantifier to interpret the NP *John* following Montague (1973). Let $\mathbf{John}_M = \{X \subseteq M \mid \mathbf{j} \in X\}$. This is a quantifier violating ISOM.

There are two ways to respond to these exceptions. One is to give up on ISOM as a feature of quantifiers in natural language. This leaves the generalization that determiners denote type $\langle 1, 1 \rangle$ quantifiers satisfying CONS and EXT, but not necessarily ISOM. These determiners build type $\langle 1 \rangle$ quantifiers satisfying CONS and EXT when combined with a CN denotation, so we might even make the further generalization that all NPs denote type $\langle 1 \rangle$ generalized quantifiers, once we have given up on ISOM.

Another response is to attempt to explain away the apparent violations of ISOM. In the type $\langle 1 \rangle$ case, we can easily observe that though it is possible to treat *John* as a generalize quantifier, it can also be treated as simply denoting an individual. There are good reasons to take this simpler route (cf. Partee, 1986). Thus, an apparently non-ISOM quantifier in natural language may not be a quantifier at all. Likewise, in the type $\langle 1, 1 \rangle$ case, we might find analyses of possessive constructions which do not treat them as syntactically on par with other determiners, or do not treat them as determiners at all. (See Barker (1995) for an extensive discussion of the syntax and semantics of possessives.) This might allow for the stronger hypothesis, that quantifiers in natural language are the denotations of determiners (or a syntactically natural subclass of determiners, perhaps), and they are logical generalized quantifiers satisfying CONS, EXT, and ISOM.

The latter hypothesis predicts important differences between quantified noun phrases, build up out of determiners denoting ISOM quantifiers, and other noun phrases like proper names or possessive constructions. And in fact, we do see such differences. One way in which quantified noun phrases behave differently from other noun phrases is brought out by what are called *weak crossover* cases. Compare:

(27) a. * His_i mother loves every boy_i.

b. His_i mother loves Mary's Brother_i.

c. His_i mother loves John_i.

(The subscripts here are to indicate that the desired reading has *his* bound by or coreferring with the subsequent expression it is co-indexed with.)

A number of authors have noted that we get unacceptability in weak crossover environments with ISOM quantified noun phrases, but not with non-ISOM or non-quantified ones. This is often cited as evidence that there is a genuine subdivision between quantificational noun phrases, built from ISOM determiners, and other noun phrases; just as the strong hypothesis predicts (cf. Higginbotham and May, 1981; Lasnik and Stowell, 1991; Larson and Segal, 1995). (Readers of the logic literature should be aware that regardless of their status in natural language, most logicians take generalized quantifiers to satisfy ISOM by definition.)

I.6 Glimpses Beyond

We have now seen the beginnings of generalized quantifier theory, but only the beginnings. The surveys of Westerståhl (1989) and Keenan and Westerståhl (1997) discuss a number of extensions of the theory, and applications of generalized quantifier theory in linguistics.

Among the results they discuss is one that shows that the quantifier **most** defined in (10) cannot be defined by any combination of (ISOM) $\langle 1 \rangle$ quantifiers. This shows that we really do need at least type $\langle 1, 1 \rangle$ quantifiers (cf. Väänänen, 1997). They also investigate the delicate issue of whether we need to go beyond $\langle 1, 1 \rangle$. We saw that *more* should be interpreted as taking *three* arguments. Whether we will also need to consider what are called *polyadic* quantifiers, which take *relations* rather than sets as inputs, remains an active area of research (cf. Higginbotham and May, 1981; van Benthem, 1989; May, 1989; Keenan, 1992; Westerståhl, 1994; Hella *et al.*, 1996; Moltmann, 1996)

II Quantification and Scope

The relational theory of determiners which we examined, if all too briefly, in Section (I) explains some of the important properties of the semantic values of determiners. But it does not do very much to explain how determiners interact with the rest of semantics. As an example of where quantifiers fit into semantic theory, I shall present some ideas about how quantifiers take scope in natural language.

Perhaps more so than the theory of generalized quantifiers, this area remains controversial. There are a number of good textbook presentations of the basic material, including Larson and Segal (1995) and Heim and Kratzer (1998) (I follow the latter quite closely here). But there is also some significant disagreements in the literature. To illustrate this disagreement, I shall discuss two representative examples of approaches to quantifier scope. I shall need some machinery to do so, which is built up in Sections (II.1–II.4). The actual discussion of scope will be in Section (II.5).

II.1 Quantifiers and Semantic Types

The theory of generalized quantifiers as relations between sets pays no attention to the order in which a quantifier's arguments are 'processed'. For studying the properties of determiners, this has proved a useful idealization, but it is not what we want from a compositional semantic theory in general.

A glance at the sentence structure in (7) tells us that the compositional semantics of determiners should first have the determiner's value combine with the value of the CN, resulting in an NP semantic value. It is the NP value which combines with the VP value to determine the value of the sentence.

It is easy enough to re-formulate our account of determiner denotations to keep track of the order of arguments. The official definition (9) made a type $\langle 1, 1 \rangle$ quantifier $\mathbf{Q}_M$ a function on two arguments X and Y. We can think of this as a sequence of two functions: $\mathbf{Q}_M(X)$ is the function from subsets of M to truth values, which is true iff $\mathbf{Q}_M(X, Y)$ is true. Thus, a binary relation on subsets of M like $\mathbf{Q}_M$ can be thought of as a function from subsets of M to functions from subsets of M to truth values.[9] $\mathbf{Q}_M(X)$ is a denotation for an NP, and is itself a type $\langle 1 \rangle$ quantifier.

It will be useful to have some notation to keep track of the inputs and outputs of functions. One way to do this is to use *type theory*. Type theory is a highly general theory of functions. (In order to try to avoid confusion between types in the sense of quantifier types and this type theory, I shall sometimes call the latter *semantic type theory*.)

Semantic type theory starts with two basic types: t is the type (set) of truth values, which we may take to have two elements $\top$ and $\bot$; e is the type (set) of individuals, which we may take to be some fixed universe M. The theory then builds up functions out of these. The type (e, t) is the type of functions from individuals to truth values, i.e. it is a notation for $\wp(M)$, the set of subsets of M. We will continue with our assumption that the values of VPs and CNs are sets of individuals, i.e. of type (e, t). We will also continue with the extensional perspective, which gives sentences semantic values of type t (This is of course, an idealization.) We will also assume that non-quantified NPs are of type e. The analysis of determiners which takes their order into account makes them functions whose inputs are sets of individuals, and whose outputs are functions from sets of individuals to truth values. Hence, determiners should denote functions of type $((e, t), ((e, t), t))$. As we are working with a fixed universe M, giving type e, all quantifiers are now local. Generally, for any two types a and b, (a, b) is the type of functions from a to b.[10]

II.2 Quantifiers in Object Position

Our semantic analysis thus starts with the idea that determiners are of type $((e, t), ((e, t), t))$, CNs are of type (e, t), and VPs are of type (e, t). This makes quantified NPs of type $((e, t), t)$. As a result, in sentences like (7), the VP value is the argument of the NP value, but we still have an appropriate match-up of functions and arguments to produce a type t value for the

[9]This is what is sometimes called 'Currying' a binary relation, in honor of the logician Haskell B. Curry.

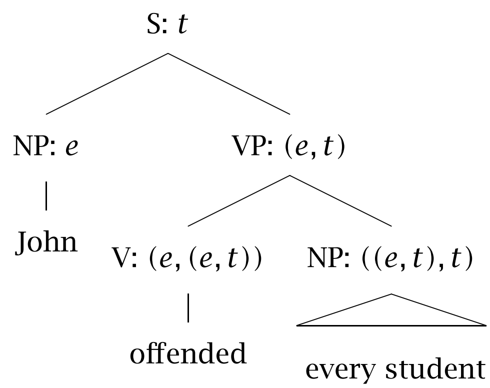
[10]I am writing semantic types with round brackets, such as (a, b). Much of the literature writes semantic types with angle brackets, but these are already being used for quantifier types.

sentence.

If we look at little more widely, however, we run into problems of composition. Transitive verbs with quantifiers in object position provide one sort of problem. A transitive verb will be of type $(e,(e,t))$, taking two type e arguments. But consider an example like:

(28) a. John offended every student.

b.



The entries for the VP simply do not match. *Offended* is of type $(e,(e,t))$. But the quantified NP *every student* is of type $((e,t),t)$. Neither can be the argument for the other. If, as the basic type-theoretic perspective supposes, semantic composition is composition of function and argument, we have no way to combine them.

The theory of generalized quantifiers, as a theory of determiner denotations, does not help us to solve this problem.[11] Instead, some more apparatus is needed, either in the semantics or in the syntax. There are two basic approaches to solving this problem. One involves significant claims about *logical form.* The other makes some corresponding claims about *semantic types.*

II.3 Logical Form and Variable Binding

One approach to the problem of quantifiers in object position, perhaps the dominant one, is to posit underlying logical forms for sentences which are in some ways closer to the ones used in the standard formalisms of logic.

The goal is to replace the quantified NP *every student* with a variable of type e in the VP. This variable would function as the argument of the type $(e,(e,t))$ verb, and also be *bound* by

[11] There is one drastic generalized quantifier theory option we might take, which would be to appeal to polyadic quantifiers of the sort hinted at in Section (I.6), following Keenan (1992).

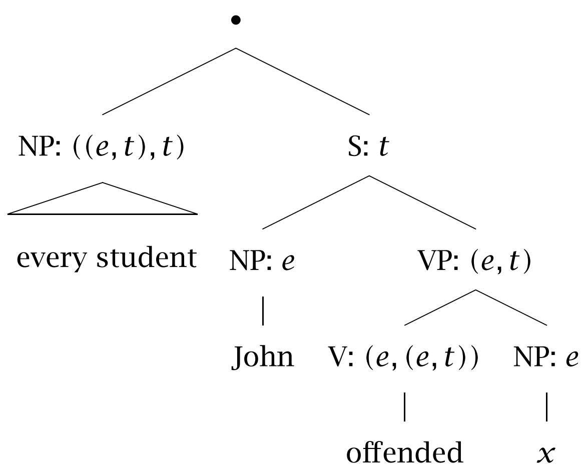
the quantifier. We thus want a structure that looks something like:

(29) Every student$_x$ (John offended x).

To get this, we need to look a little further at how variable work in type theory.

If we simply replace *every student* in (28) by a variable of type e, we do not get what we want. We would have:

(30)

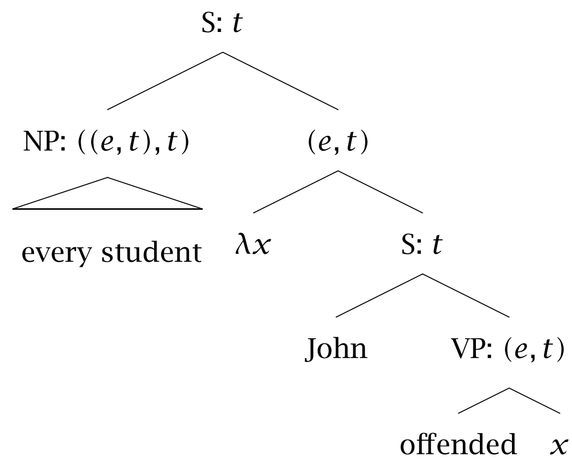


Again the types do not match, and the structure cannot be interpreted. The S node is of type t, and cannot combine with an NP of type $((e,t),t)$.

To get the structure we had in mind in (29), we need to cash out the idea that *every student*$_x$ really *binds* x in the VP. In the type-theoretic setting, this is done by λ-abstraction. If σ is an element of type t, and x is a variable of type e, then $\lambda x.\sigma$ is a function from type e to type t, i.e. an element of type (e,t). To get something that works like (29), we need to add λ-abstraction.

Using λ-abstraction, we can resolve the mismatch between types we see in (28) along the following lines:

(31)



Adding the variable in VP produces an element *John offended* x of type t. λ-abstraction then yields the desired element of type (e, t), which can combine directly with the quantified noun phrase.[12]

The use of λ-abstraction also explains what we intuitively represented by the subscript x on *every student*$_x$ in (29). We wanted to make clear that the quantified NP *every student* binds the x position. This is explicitly done by the λ-node in (31).

The role of λ-abstraction also highlights a point about generalized quantifier theory. Generalized quantifier theory as discussed in Section (I) is not a theory of variable binding. Describing relations between sets does not explain how they can figure into variable binding. On the approach I am sketching here, variable binding is done by λ-abstraction, which produces semantic values of appropriate type to be inputs into generalized quantifiers.

(31) represents a very rough proposal for the *logical form* of (28); the fully worked out version is that of Heim and Kratzer (1998). This is a significant proposal. The claim is not merely that a formalism like (29) makes the logical dependencies of a sentence clear. Rather, it is that the semantic interpretation of a sentence of natural language is derived from a structure like (31). Thus, logical form is posited as a genuine level of linguistic representation. This is a substantial empirical claim.[13]

[12]Technically, we should say that we add syntactic elements which are interpreted as variables and λs. See Heim and Kratzer (1998) and Büring (2004) for more discussion of the syntax and semantics of these particular structures.

[13]Following May (1977, 1985), many linguists think of logical form as the result of *movement processes* which move quantifiers from their *in situ* positions to positions more or less like the ones in (31). A survey of ideas

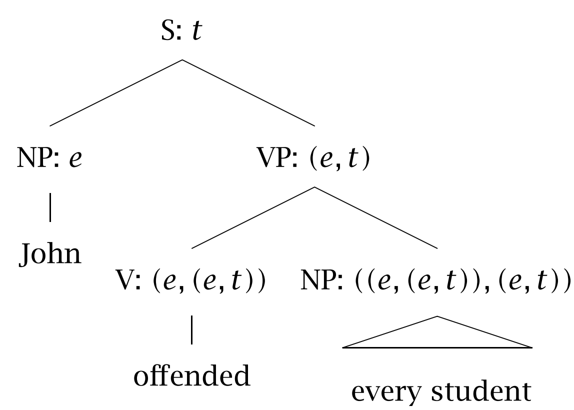
It should be noted that once we have forms looking like (29), it is possible to treat binding in a more Tarskian way, without relying on the apparatus of λ-abstraction and types. One example is the more Davidsonian treatment of Larson and Segal (1995). There are some general methodological questions about the use of higher types in semantics, but the basic idea of treating quantifiers in object position by way of a substantial level of logical form is not particularly sensitive to them.[14]

II.4 Type Shifting

The other leading idea for how to resolve the problem of quantifiers in object position makes much more heavy use of semantic type theory. Rather than positing a distinct level of *logical form*, it posits more complex modes of *composition* in the semantics.

Suppose we change the type of a quantified NP from $((e,t),t)$ to $((e,(e,t)),(e,t))$. Then we can interpret (28) directly:

(32)



The values of the V and NP compose by the NP value taking the V value as an argument.

How can we change something's type? In this case, the transformation from $((e,t),t)$ to $((e,(e,t)),(e,t))$ is in fact much more natural than it might seem. It is an instance of what is known as the *Geach Rule* (cf. Geach, 1972):

(33) $(b,c) \Longrightarrow ((a,b),(a,c))$

about logical form in syntactic theory is given in Huang (1995).

[14] Lepore (1983) and Pietroski (2002) offer critiques of type-based semantics from a broadly Davidsonian viewpoint. Another view of logical form and its role in semantics, more explicitly Davidsonian than the one I am sketching here, is presented in Higginbotham (1985).

This can be thought of as introducing an additional mode of composition, over and above function application. It is essentially *function composition*:

(34) a. i. $(a, b) + (b, c) \Longrightarrow (a, c)$

ii. $\alpha_{(a,b)} + \beta_{(b,c)} \Longrightarrow (\beta \circ \alpha)_{(a,c)}$

b. i. $(e, (e, t)) + ((e, t), t) \Longrightarrow (e, t)$

ii. $\gamma_{(e,(e,t))} + \delta_{((e,t),t)} \Longrightarrow (\delta \circ \gamma)_{(e,t)}$

(34) displays the scheme of function composition, according to which we apply one function α followed by another β, and shows what this does for types. (34b) shows the specific case of (34a) in which we are interested.

(33) is a schema for adding function composition (34) by adding a type-shifting operator. It can be spelled out by:

(35) $Geach_a(\beta_{(b,c)}) = (\lambda X_{(a,b)} \lambda y_a[\beta_{(b,c)}(X_{(a,b)}(y_a))])_{((a,b),(a,c))}$

For $\mathbf{Q}_{((e,t),t)}$ of type $((e, t), t)$, $Geach_e(\mathbf{Q}_{((e,t),t)}) = \lambda v_{(e,(e,t))} \lambda x_e[\mathbf{Q}_{((e,t),t)}(v_{(e,(e,t))}(x_e)))]$ So, for instance $(Geach_e(\textbf{every student}))(\textbf{offended}) = \textbf{every student} \circ \textbf{offended}$. This is now of the right type to combine with **John**.

The operator *Geach* carries out λ-abstraction. Thus again in this framework, the essential function of having a quantifier interact with the right position in a VP in the right way is done by λ-abstraction. This is a beginnings of a theory of binding which does not invoke logical forms different from the surface forms of sentences. For more development along these lines, see Hendriks (1993), Jacobson (1999), Steedman (2000), and Barker (forthcoming), as well as the earlier Cooper (1983).[15]

The basic idea of the type-shifting approach exemplified here is to think of expressions as *polymorphic*. They inhabit multiple types at once. We think of expressions as entered into the lexicon with their minimal type, which can then be *shifted* by type-shifting rules, like the

[15] Much of this literature works in the framework of categorial grammar, and attempts to develop 'variable-free' accounts of binding phenomena. The background mathematics for this work is combinatory logic, which is a close cousin of the λ-calculus I have employed here. See Hindley and Seldin (1986) for extensive comparisons.

Geach rule. This makes expressions in a way ambiguous. (See Partee and Rooth (1983), Partee (1986), and the extensive discussion in van Benthem (1991).)

Whereas the logical form approach made relatively minor use of type theory, the type-shifting approach leans very heavily on it. Type-shifting approaches do not posit additional levels of linguistic representation, over and above the more or less overt surface structure of the sentence, but they do make use of some powerful mathematics. It is a significant question, both empirical and methodological, which approach is right.

II.5 Scope Relations

The problem of quantifiers in object position barely hints at the complexity of the semantics of quantification. To give a slightly richer example, I will finally turn to some aspects of quantifier scope relations.

One important feature of quantifiers in natural language is that they can generate scope ambiguities. Recall, as every student of first-order logic learns, *Everyone likes someone* has two first-order representations:

(36) Everyone likes someone.

a. $\forall x \exists y L(x, y)$

b. $\exists y \forall x L(x, y)$

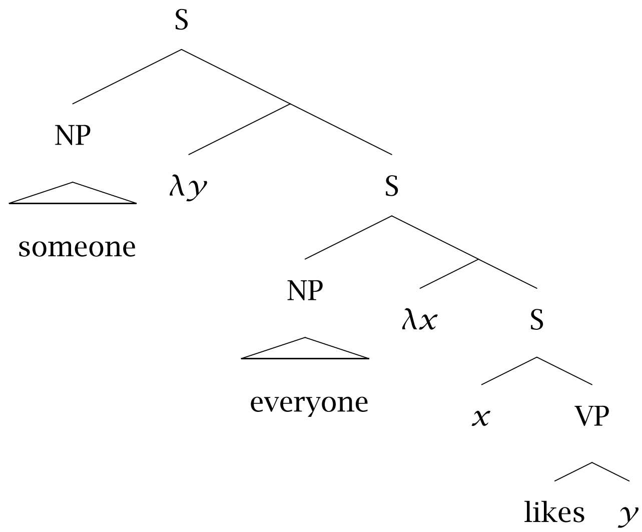
The second is usually called the *inverse scope reading*, as it inverts the surface order of the quantifiers. Another, more complicated inverse scope example is that of inverse linking (May, 1977):

(37) Someone from every city despises it.

May observed that in this sort of case, the inverse scope reading is the only natural one (or perhaps the only one available).

The logical form approach has no fundamental problem with the existence of inverse scope readings. Basically, the logical form approach treats quantifier scope much the way it is treated in first-order logic, modified to employ generalized quantifiers and the account of binding outlined in Section (II.3). Direct and inverse scope readings are simply the result of different mappings of a sentence to logical forms, corresponding to different orders in which the quantifiers are 'moved' from their *in situ* positions to positions further to the left and higher in the tree. For instance, the inverse scope reading of (36) is given by:

(38)



If we adopt the logical form theory, quantifier scoping is taken care of by the same apparatus which handled quantifiers in object position.[16]

This is an elegant result, and part of a battery of arguments often marshaled to show the existence of a level of logical form (cf. May, 1985). Scope ambiguity is explained by holding that in fact sentences like (36) have *two* distinct logical forms—two distinct linguistic structures. At logical form, scope ambiguity is structural ambiguity.

Type-shifting approaches have to do more work to handle inverse scope. The Geach rule described in Section (II.4) is not sufficient. One approach to scope via type shifting is to introduce two type-shifting operators which raise the types of the arguments of a transitive verb from e to $((e,t),t)$, allowing the verb to combine with two quantifiers. The *order* in which these operators are applied determines the scope relations between the quantifiers, much as

[16]The syntax of scope is a rich area of linguistics. The basics can be found in many syntax books. For a recent survey, see Szabolcsi (2001).

Though many logical form theories take the syntax of logical form to determine scope, May (1985, 1989) considers a theory in which it does not completely do so.

the order in which the quantifiers are moved does on the logical form approach. Hendriks (1993) shows that these operators can be derived from a single type-shifting principle, but I will leave the rather technical details to him.[17]

Both approaches thus *can* handle inverse scope (though I have suppressed more detail in the type-shifting approach). Which one is right is a substantial question, both methodological and empirical. We face general questions about the apparatus of type shifting and linguistic levels like logical form. We also face empirical issues about which theories can explain the full range of data related to scope and binding. Perhaps the preponderance of current research (at least, research close to syntax) takes place in some version of the logical form approach, but see Jacobson (2002) for a spirited defense of the type-shifting approach.

Though both approaches can handle basic scope inversion cases like (36), the phenomena related to scope in natural language are in fact quite complex. I shall close this section by mentioning a few of the many issues that a full theory of quantifier scope must face.

Though in many cases quantifiers can enter into arbitrary scope relations, there are some well-know situations where they cannot. For instance, quantifiers cannot scope out of relative clauses. Consider (Rodman, 1976):

(39) Guinevere has a bone that is in every corner of the house.

This cannot be given the (more plausible) interpretation in which *every corner of the house* has wide scope. This fact is often cited as further evidence in support for logical form theories, which seek to explain it by general syntactic principles, but see Hendriks (1993) for a discussion in type-shifting terms.

Different languages display different scope interactions. Aoun and Li (1993) note sentences which are ambiguous in English but not in Chinese, including the simple:

(40) Every man loves a woman.

(The example is credited to Huang.) It is also known that not all quantifiers exhibit the same

[17]There are systems which produce inverse scope readings with type-shifting operations more closely related to the Geach rule, like the elegant Lambek calculus with permutation of van Benthem (1991). Unfortunately, this system over-generates scope ambiguities, predicting one in *John loves Paris*, as Hendriks (1993) shows. A more refined theory along van Benthem's lines is given a textbook presentation in Carpenter (1997).

scope potentials, even in one language. Beghelli and Stowell (1997) and Szabolcsi (1997) note that inverse scope readings do not appear to be available in:

(41) a. Three referees read few abstract.

b. Every man read more than three books.

Aoun and Li (1993) and Beghelli and Stowell (1997) and Szabolcsi (1997) use this data to support their own developments of the logical form approach (cf. Takahashi, 2003). There are also much-discussed difficult issues about the scope of *the* and *a*. See Heim (1991) and van Eijck and Kamp (1997) for surveys.

III What is a Quantifier?

Can we now say what quantifiers are? Perhaps. Generalized quantifier theory, and the relational theory of determiner denotations which goes with it, offer an answer. The strong hypothesis we considered in Section (I.5) holds that natural language quantifiers are logical generalized quantifiers, satisfying the constraints CONS, EXT, and ISOM. These are expressed by determiners, which combine with CNs to build quantified noun phrases. A somewhat weaker hypothesis holds that natural language quantifiers need not be ISOM, but must be CONS and EXT. Section (I.5) offered some reasons to prefer the stronger hypothesis.

In a way, this tells us what quantifiers *are* in remarkably specific terms. But the moral of Section (II) is that it does not tell us all that much about how quantifiers *work*. The examples there show us that to understand *quantification* in natural language is to understand more than what quantifiers are; it is also understand significant aspects of semantics, and the ways semantics interact with syntax. Being a quantifier is a property with significant semantic and grammatical implications.

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