

# Rule Based Fuzzy Cognitive Maps - Expressing Time in Qualitative System Dynamics<sup>♦</sup>

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*Abstract* - Time is essential in the study of System Dynamics. When we are trying to represent and analyze the dynamics of complex quantitative systems, the problem of expressing the "effect" of time flow is naturally solved since the mathematical equations that describe the relations between the entities of the system are a function of time. However, if we are dealing with real world qualitative systems that are impossible or difficult to model using mathematical equations, then the use of natural language becomes the best tool to represent the system and expressing time influence becomes a real issue that has not been addressed before. This paper introduces a coherent procedure to implicitly represent time in Rule Based Fuzzy Cognitive Maps which are a previously introduced methodology and tool to represent and simulate the dynamics of qualitative systems.

**Keywords:** *Fuzzy Rulebased Cognitive Maps, System Dynamics*

## I. INTRODUCTION

Decision makers, whether they are social scientists, politicians or economists, usually face serious difficulties when approaching significant, real-world dynamic systems. Such systems are composed of a number of dynamic concepts or actors, which are interrelated in complex ways usually including feedback links that propagate influences in complicated chains. Axelrod work on Cognitive Maps (CMs) [1] introduced a way to represent real-world qualitative dynamic systems, and several methods and tools have been developed to analyze the structure of causal maps [2][3]. However, complete, efficient and practical mechanisms to analyze and predict the evolution of data in CMs are necessary but not yet available for several reasons [4]. System Dynamics tools like those developed by J.W.Forrester [5] could be a solution, but since in Cognitive Maps numerical data may be uncertain or hard to come by, and the formulation of a mathematical model may be difficult, costly or even impossible, then efforts to introduce knowledge on these systems should rely on natural language arguments in the absence of formal models. Fuzzy Cognitive Maps (FCM), as introduced by Kosko [6], are an alternative approach to system dynamics. However, FCM are Causal Maps (a subset of Cognitive Maps that only allow causal relations) [7] and in most applications, a FCM is indeed a man-trained Neural Network that is not Fuzzy in a traditional sense and does not explore usual Fuzzy capabilities [7]. They do not share the properties of other fuzzy systems and the causal maps end up being quantitative matrixes without any qualitative

knowledge [7]. To avoid the above-mentioned limitations of existing approaches, Rule Based Fuzzy Cognitive Maps (RB-FCM) were introduced in [7][8][9][10] and are being developed as a tool that models and simulates real world qualitative system dynamics.

One very important issue that has apparently been ignored (or avoided) when approaching Qualitative System Dynamics is time. Time is obviously essential in the study of System Dynamics (SD). When we are dealing with quantitative SD like [5] or [12], timing issues are naturally solved, since Time is explicitly expressed in the mathematical equations used to describe the relations between concepts. However, in qualitative SD, where natural language is more adequate to express the relations between concepts, time must be implicitly expressed in those relations (in the case of RBFCM, time must be somehow included in the fuzzy rule bases that define the relations). Without imposing this implicit timing knowledge it is impossible to guarantee a minimally acceptable simulation.

Oddly, almost none of the published studies (for instance [13][14][15]) involving application and simulation of systems using FCM (which despite its limitations still is the most widespread procedure to analyze the dynamics of qualitative systems), show any considerations regarding timing issues. Considering that all relations involved in FCM are causal relations, and that the effect of a causal relation always involves a change (represented by an amount of variation in a given period of time) [9][10][13][15], one might make considerations about the validity and utility of these models. This paper addresses the problem of Time in qualitative system dynamics and introduces a coherent procedure to implicitly represent time in RB-FCM.

## II. RULE BASED FUZZY COGNITIVE MAPS

RB-FCM provide a representation of the dynamics of complex real-world qualitative systems with feedback and allow the simulation of the occurrence of events and their influence in the system. They are fuzzy directed graphs with feedback, which are composed of fuzzy nodes (**Concepts**), and fuzzy links (**Relations**). RB-FCM are true cognitive maps (CM) since are not limited to the representation of causal relations. Unlike FCM, Concepts are fuzzy variables defined by fuzzy linguistic variables, and fuzzy rule bases are used to

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express Relations. RB-FCM are essentially fuzzy rule based systems where we added fuzzy mechanisms to deal with feedback, and different kinds of relations in order to cope with the complexity and diversity of the qualitative systems we are trying to model.

RB-FCM are iterative: the current value of each concept is computed with its inputs previous values. The evolution of the system through time might reach equilibrium and converge to a single state or a cycle of states under certain conditions [10].

Introduction or removal of concepts and/or relations, or the change of state in one or more concepts affect the modeled systems in ways that are usually difficult or impossible to predict due to the complex feedback links. RB-FCM are a tool to predict the evolution through time caused by those changes. RB-FCM provide 2 kinds of Concepts (**Levels** and **Variations**), several kinds of Relations (Causal, Inference, Common, Similarity, Hill-defined, Crisp, Level-Variation, and other common relations), and mechanisms that support invariant and time-variant probabilities, the possibility of simulating real-time alternatives and the definition of subsystems including decision support systems to simulate the process of decision making by “actors” within the system [9][10][11].

With the introduction of the mechanisms presented in this paper, which allow the modeling of delays and inhibition of certain relations when they have no influence on a given instant, RB-FCM can deal with timing issues and become a tool to represent and analyse the dynamics of qualitative systems.

RB-FCMSyntax is a language that was developed to describe real-world qualitative systems in RB-FCM[11]. It allows the definition of the concepts, relations, linguistic variables, membership functions, etc. that compose the system. In order to simplify the description of the systems, macro mechanisms allow the definition of simple monotonic and symmetric causal relations (which are the only relations present in FCM) with a single instruction.

A few simple examples:

**Concept** Inflation

Variation;

{Decrease,Maintain,Increase};

**End Concept;**

Define a Variation concept named Inflation, and the name of its linguistic variables.

**MBF** Good triang 0.6,0.8,0.9; Defines the triangular mbf Good

**RB** idRB

/\*fuzzy rules\*/

**END RB;** Define a fuzzy rulebase

**FCR** idRB CCorp\_Results, CcorpValue; Defines a Fuzzy Causal Relation between 2 concepts. The rules are defined in the Rulebase idRB

**FCR+** Interest\_rate, Inflation; A macro (set of rules) that defines a positive causal relation between the Interest Rate

and Inflation (If the Interest Rate increases then Inflation will increase, If the Interest Rate increases much then Inflation will increase much, If the Interest Rate decrease then Inflation will decrease, etc.)

### III. EXPRESSING TIME IN RB-FCM

The procedures presented in this section were developed in order to maintain coherence in the process of modeling the dynamics of a qualitative system. To allow the representation of time flow, delays, and the inhibition of certain relations when they have no influence on a given instant, changes were made to the engine of RB-FCM, and the RB-FCMSyntax was extended to control those timing mechanisms.

#### A. Implicit time

As it was mentioned above, time in RB-FCM must inevitably be represented implicitly in every relation; therefore, the responsibility of maintaining temporal coherence in the process of modeling a system relies heavily in the modeler. He or she must not only be responsible to find the nature and characteristics of the relation, but must also ensure that the magnitude of the relation is adequate to the time interval that it represents.

For example, let us consider two concepts that can be included in a map representing an imaginary corporation named CorpC: CorpC\_Results and CorpC\_Value, the former represents the annual presentation of the results, and the latter the stock market value of the company. Several other concepts affect the results, but there is an obvious strong positive causal relation between those two concepts (good results lead to an increase of the value, very good results cause a really large increase). The modeler of the system not only would have to identify the causal relation, but he would also have to quantify the meaning of the increase based on the time that each iteration is supposed to simulate, since if each iteration represents one single hour, then the variation of the stock market value of CorpC must be much smaller than if each iteration represents one day...

#### B. Base Time (B-Time)

Due to the iterative nature of the RB-FCM simulation process, each system iteration must represent the flow of a given amount of time. In RB-FCM this period is called Base Time (B-Time). It represents the “resolution” of the simulation, i.e., the highest level of temporal detail that a simulation can provide in the modeled system.

B-Time must always be implicit while defining each rule in each rule base, especially in the case of causal relations, since Variations are always involved in those relations and there is an intrinsic relation between B-Time and Variations: the linguistic variable indicating the highest possible amount of change in each Variation (*Increase\_Very\_Much*, *Decrease\_Very\_Much*, *Gets\_Incredibly\_Better*, etc.), represents the largest amount of change that the physical entity modeled by the Variation is expected to suffer during the period of time that B-Time models. The following example allows a better understanding of this issue.

Example: If on a given system, B-Time=1 day, then the meaning of the linguistic variable *Increase\_Very\_Much* of concept *Oil\_Price*, is completely different of the meaning of the same linguistic variable on the same system with B-Time=1 year. 3 USD can be a huge variation in 1 day, but is a small variation when we consider a 1 year period.

Therefore, while modeling a system, B-Time must be defined before the Concept's linguistic variables and the Relations between Concepts.

### C. B-Time selection

The choice of B-Time is highly dependent on the Real-world system being modeled. It could be one hour, one day, two days, one week, one year, one century or any other time period. It depends on the desired or advisable level of detail, complexity and intended long-term analysis of the system. Shorter B-Times usually need more detailed and complex rule bases and imply a much more careful approach to the precision and validity of the rules. Longer B-Times should provide more valid long-term simulations, but short-term detail, precision and validity will possibly be sacrificed.

The intrinsic nature of the system is also an important factor for B-Time selection. Chaotic or pseudo-chaotic systems obviously need shorter B-Times (1 hour, 1 day), since small deviations in the first steps of the simulation will cause huge errors in later steps. On the other hand, stable systems that tend to converge to a single state or single cycle of states usually handle very well long B-Times (1 month, 1 year), which should be fully utilized to allow the implementation of less detailed rule bases and provide faster simulations.

Some examples of possible B-Time values for sample systems:

System	B-Time	Justification
A qualitative model of Inflation	1 month	Economic indicator published every month
Stock Market	1 day	Rather chaotic, daily indicators
City Council Politics	1 month	Depends on City size, population involvement...
World Politics	1 year	Huge social dimension prevents lesser resolutions, except for localized events

### D. Other possible time intervals

Since some relations only make sense at intervals larger than B-Time, in RB-FCM it is possible to include time intervals other than B-Time. Each concept and relation in RB-FCM can be associated with a time interval that indicates at which iterations should the involved relations resolve. To allow different time intervals, relations are inhibited when indicated.

For example, some economic indicators are published continuously 24 hours a day (Oil Price, Gold Price), most are published every month (Productivity, Inflation) but others have a rather large period of divulgation like once every 3-month (Stocks) or once every year (GDP per capita, External Debt, etc.). The effect of these indicators in some of the other

economic or social concepts is felt only when they are published. If we are using the period of 1 month as B-Time, then it makes sense to inhibit those relations except when the indicators are actualized in the model.

It is important to notice that relations, not concepts, are inhibited: The same concept that is published once every year can have a continuous effect in other concept (i.e. the relation is active in every iteration), but may affect a third concept only when it is published.

Not all economic indicators are published synchronously. Even if they have the same period, they can be published in different times (a 3-month indicator can be published every January, April, July,..., and other every February, May, etc.) – the same can happen with several kinds of concepts and relations, not only economic ones. Therefore RB-FCM not only allows the modeling of different time periods (relation inhibition), but also allows asynchronous timing of different relations.

A different timing situation appears when B-Time is adequate for most of the system, but the effect of some relations is too small to provide a discernible effect for the selected B-Time. These situations can be prevented maintaining B-Time, modeling the critical relations as if its period is a B-time multiple (allowing a more detailed description of the effects of the relation), and inhibiting those relations during the simulation for the specified multiple of B-Time iterations.

### E. Delays

Delays are the most obvious timing issue, and the only one that has been addressed on FCM on some occasions. Sometimes the effect of a relation is not immediate (or at least can't be felt during B-Time), and the delay must somehow be modeled on the RB-FCM. It is important to note that this is a different issue from the one mentioned on the previous section. We are not trying to model an event that produces a small discernible effect during a single iteration, but an event which as a real delay. A common example of this kind of timing events is the effect of oil-price variations on the cost of fuel or electricity: due to the fact that most countries have long term reserves, and the process of oil transport and refining is slow, then the people only feels those effects after a few months.

On RB-FCM, delays on the effect of a relation are modeled recurring to a FIFO buffer with size  $Bufsize=delay/B-Time$  on the consequent of such relation. The value of the consequent Concept is computed with the first element of the buffer, creating an effective delay of  $Bufsize$  iterations. Obviously the precision of the delay depends on B-Time.

### F. RB-FCM Timing Syntax

To implement the above mentioned timing control mechanisms, each rule always resolves in each iteration (B-Time) unless the iterations for when it should resolve are

expressly defined in the rule<sup>2</sup>. The following features were introduced to the language:

**B-Time**=... A purely indicative definition of B-Time

Each rule can have up to 3 Timing annotations: one that indicates that the rule only resolves at a certain multiple of B-Time (for example, if B-Time is 1 week, then a 4 indicates that the rule is only relevant once every 4 weeks - 1month); a second one represented between [ ] that indicates when the relation will resolve for the first time; and a third one, between {} that indicates a possible delay.

A few examples of timing definition using RB\_FC syntax:

**FCR+** Interest\_rate, Inflation

The rules in this macro resolve every iteration (B-Time), i.e., every time the Interest Rate changes, the Inflation will react accordingly.

**12FCR++** CorpC\_Results, CorpC\_Value

A macro that defines a strong positive causal relation between the presentation of the results of corporation C and the stock market value of its actions. These rules resolve only every 12 B-Time iterations (assuming that B-Time is 1 month, this would indicate the presentation of results once every year).

**12[5]{3}FCR++** CorpC\_Results, CorpC\_Value

The same set of rules as above, but in this case the first presentation of results would be done 5 months after the beginning of the simulation, and its effect would be felt on the 8<sup>th</sup> month, due to a 3 month delay in the effect of the relation.

#### IV. CONCLUSIONS

This paper introduced some simple considerations, guidelines and mechanisms to introduce the effect of time in the representation of the dynamics of qualitative systems. Even if all of the presented above might seem redundant and should be common sense to anyone involved in the design or modeling of the dynamics of qualitative systems, it is important to remember that these issues have been ignored until now. Published models do not show any considerations regarding timing issues. One never knows approximately how long lasts each iteration, after how long the system converges or if time was considered when selecting the weight for each relation. One might argue that the models involved are too imprecise to indicate any measure of time with confidence, but in that case, are the models of any use? Being FCM causal maps (usually with lots of feedback), all relations involved are highly time-dependent, and therefore all relations must be time-coherent or the penalty would be an invalid and useless model.

A different issue is the possibility of representing relations that are not always active during a simulation and Delays. These mechanisms, albeit simple, allow a direct modeling of common real world situations that are not addressed in FCM

or other tools, and can be easily adapted and implemented in them.

Finally one can mention a limitation of the adopted approach for dealing with timing issues: the fact that all time intervals in a system must be B-Time multiples.

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<sup>2</sup> Note that the rule might still not have any effect if the conditions for it to resolve are not met due to constraints of the system itself

