BLIND CHANNEL IDENTIFIABILITY/EQUALIZABILITY OF SINGLE INPUT MULTIPLE OUTPUT NONLINEAR CHANNELS FROM SECOND ORDER STATISTICS

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ABSTRACT

We explore the utility of second-order statistics for blind identification/equalization of nonlinear channels. Under standard assumptions it is shown that the channel cannot be identified to within a scaling factor from the output second order statistics, but that the ambiguity is at a level that permits equalization. We show that these results cover cases that the prior literature does not address.

1. INTRODUCTION

Blind channel identification and equalization have received considerable attention over the last decade. In particular, methods based on second-order statistics have raised great interest since they can perform their tasks with relatively short data records [10]. With a few exceptions [1] - [5], almost all the available literature is devoted to the linear channel case. However, it is of interest to address the issue of *nonlinear* channels since real world systems such as satellite communication channels, high density magnetic recordings, voiceband data transmission systems, physiological signal models, etc. present non-negligible nonlinear effects.

In this work we consider the problem of blind identification/equalization of nonlinear single-input multiple-output (SIMO) channels. The model we assume is as follows:

$$x_n = \sum_{j=0}^{M_0} h_{0j} d_{n-j} + \sum_{i=1}^{D} \sum_{j=0}^{M_i} h_{ij} z_{n-j}^{(i)} + \eta_n, \qquad (1)$$

where $\{d_n\}$ is the scalar, stationary input, $z_n^{(i)} = f_i(d_n, d_{n-1}, \cdots)$ are known scalar-valued nonlinear causal functions of $\{d_n\}$, h_{ij} are $L \times 1$ coefficient vectors, and η_n, x_n are $L \times 1$ signal vectors representing an additive disturbance and the observed signal, respectively. The number of subchannels is L. The noise $\{\eta_n\}$ and the signal $\{d_n\}$ are assumed to be independent. This SIMO model may result by oversampling the continuous output of a single sensor, and/or by taking sampled outputs of a sensor array, when the continuous-time channel presents nonlinearities. It accommodates, polynomial approximations of nonlinear channels [8].

The paper most influencing our current work is [1]. It considers the direct linear equalization of FIR Volterra channels using an approach similar to that in [11]. The method is attractive because of its simplicity. Two of its notable features are as follows. First it assumes that an associated channel matrix is square and nonsingular, and argues that this can always be achieved, if neccessary, by decreasing the number of channels and increasing the equalizer length. A longer equalizer increases the computational complexity. Further if some channels are to be dropped there is no way to ensure *a priori* that the surviving channels satisfy the corresponding full-rank condition, even if the original set of channels did. Thus the selection of the channels to drop is a difficult issue. Thus, the first issue motivating this paper is whether this squareness assumption can be dropped.

Secondly, under the assumption that the length of the linear kernel exceeds the length of every other kernels, [1] shows that one can equalize the channel input. This raises the issue of whether one can estimate the channel itself, and if not then, what is the precise level of channel information that can be gained from the second order output statistics. In resolving this issue we assume the knowledge of the input statistics and adopt an approach similar to that in [9].

Third, in the event the linear kernel has the same length as another kernel, [1] has to resort to higher order methods to equalize the channel. Using the theory developed in this paper we show that this restriction on the length of the kernel is not necessary for second order methods to apply.

Fourth, when a kernel other than the linear one has the largest memory, then the techniques of [1] only resolve this largest kernel. In our case, we show that under the right conditions even the linear kernel is resolvable despite the violation of this particular length condition.

Observe that in principle the model (1) could be seen within a multiuser framework by treating the $\{z_n^{(i)}\}_{i=1}^D$ as inputs corresponding to different users. Subspace based techniques exist for equalization within such a framework, [6]. However, *these techniques only resolve the inputs to within a mixing matrix.* In the current context, as $z_n^{(i)}$ are functions of $\{d_n\}$, it will mean that only a memoryless nonlinear function of the input can be obtained. One way of viewing the results of this paper is that they show that under right conditions the structure of the mixing matrix (referred to as the *Ambiguity Matrix* in this paper) permits obtaining d_n directly.

In our notation, $(\cdot)^T$, $(\cdot)^H$ denote transpose and Hermitian transpose respectively; $\otimes$ denotes the Kronecker product. J_m is the $m \times m$ shift matrix with ones in the first subdiagonal and zeros elsewhere.

By collecting N successive observations into $X_n = [\ x_n^T \ \cdots \ x_{n-N+1}^T\ ]^T$, one can write

$$X_n = \mathcal{F} S_n + V_n, \qquad (2)$$

where $V_n = [\ v_n^T \ \cdots \ v_{n-N+1}^T\ ]^T$,

$$S_n = [\ d_n \ \cdots \ d_{n-M_0-N+1} \ | \ z_n^{(1)} \ \cdots \ z_{n-M_1-N+1}^{(1)} \ | \ \cdots \ | \ z_n^{(D)} \ \cdots \ z_{n-M_D-N+1}^{(D)}\ ]^T, \qquad (3)$$

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and the channel matrix $\mathcal{F} = [\ \mathcal{F}_0 \quad \mathcal{F}_1 \quad \cdots \quad \mathcal{F}_D\ ]$, with

$$\mathcal{F}_i = \begin{bmatrix} h_{i0} & \cdots & h_{iM_i} & & \\ & \ddots & & \ddots & \\ & & h_{i0} & \cdots & h_{iM_i} \end{bmatrix} \qquad LN \times (N+M_i).$$

Therefore the covariance sequence of X_n can be written as

$$C_x(k) = \text{cov}(X_n, X_{n-k}) = \mathcal{F} C_s(k) \mathcal{F}^H + C_v(k), \qquad (4)$$

where $C_s(k) = \text{cov}(S_n, S_{n-k})$, $C_v(k) = \text{cov}(V_n, V_{n-k})$. Thus the identification and equalization questions can be phrased as: To what extent are the channel matrix $\mathcal{F}$ and the input sequence $\{d_n\}$ determined by $\{C_x(k)\}$? *Note the use of the covariance as opposed to the autocorrelation of X_n. This responds to the nonlinearity induced non-zero mean nature of X_n.*

Section 2 presents the limitations of second-order methods in the form of an ambiguity matrix. Section 3 studies the case of white input symbols. Identifiability conditions are given in section 4. Section 5 gives an algorithm for equalizer design. As an example, the particular case of a linear-quadratic-cubic channel is considered in section 6.

2. THE AMBIGUITY MATRIX

We adopt the following standard assumptions:
A1: The channel matrix $\mathcal{F}$ has full column rank.
A2: $\{\eta_n\}$ is zero-mean, white, with covariance $\sigma_\eta^2 I_L$.
A3: The covariance matrix $C_s(0)$ is positive definite.

We begin by noting the first evident limitation of second order statistics. Consider the case where in (1), $D = 1$ and $z_n^{(1)} = d_n^2$ and that d_n is a zero mean symmetrically distributed iid sequence. Then one can show that for suitable, known constants a_i and all real θ_i,

$$\begin{aligned} C_x(k) &= a_0 \mathcal{F}_0 J^k_{N+M_0} \mathcal{F}_0^H + a_1 \mathcal{F}_1 J^k_{N+M_1} \mathcal{F}_1^H + C_v(k) \\ &= a_0 \left[e^{j\theta_0} \mathcal{F}_0\right] J^k_{N+M_0} \left[\mathcal{F}_0^H e^{-j\theta_0}\right] \\ &\quad + a_1 \left[e^{j\theta_1} \mathcal{F}_1\right] J^k_{N+M_1} \left[\mathcal{F}_1^H e^{-j\theta_1}\right] + C_v(k). \end{aligned}$$

Thus in general $\mathcal{F} = [\ \mathcal{F}_0 \quad \mathcal{F}_1 \quad \cdots \quad \mathcal{F}_D\ ]$ that satisfies (4) cannot be resolved to within a single scaling constant. The question addressed in this paper reduces to the following: Under what conditions can enough information be obtained from the second statistics of the output, to permit blind equalization.

Observe that assumption **A1** ensures the existence of a vector g such that $g^T \mathcal{F} = e_1^T = [\ 1 \quad 0 \quad \cdots \quad 0\ ]$. Suppose that the estimated channel $\tilde{\mathcal{F}}$ equals the "true" channel, to within an ambiguity matrix $\tilde{P}$, i.e.

$$\tilde{\mathcal{F}} = \mathcal{F} \tilde{P}. \qquad (5)$$

Then provided $\tilde{P}$ has the structure

$$\tilde{P} = \begin{bmatrix} \alpha I_{N+M_0} & 0 \\ 0 & \Gamma \end{bmatrix}, \qquad \alpha \neq 0, \qquad (6)$$

$g^T \tilde{\mathcal{F}} = \alpha e_1^T = [\ \alpha \quad 0 \quad \cdots \quad 0\ ]$, regardless of Γ. Thus resolution to within the ambiguity structure of (6) suffices for equalizability. We now ask, when is this resolution possible?

Under **A2**, $C_\eta(k) = \sigma_\eta^2 J^{kL}_{NL}$, so that $C_x(0) = \mathcal{F} C_s(0) \mathcal{F}^H + \sigma_\eta^2 I_{NL}$. Now $\mathcal{F} C_s(0) \mathcal{F}^H$ is singular (as long as N is chosen so as to have more rows than columns in $\mathcal{F}$), and thus σ_η^2 can be estimated as the smallest eigenvalue of $C_x(0)$. The effect of the noise can therefore be removed from $C_x(k)$, so henceforth we shall consider that this has already been done, i.e. $C_x(k) = \mathcal{F} C_s(k) \mathcal{F}^H$.

Assumption **A3**, the 'persistent excitation' condition, on the vector S_n allows us to write $C_s(0) = QQ^H$ where Q is nonsingular (not necessarily unique). Then we can introduce the *normalized* channel and covariance matrices:

$$F = \mathcal{F} Q, \qquad \bar{C}_s(k) = Q^{-1} C_s(k) Q^{-H}. \qquad (7)$$

(Note that by **A1** and **A3**, F has full column rank). Then

$$C_x(0) = FF^H, \qquad C_x(k) = F \bar{C}_s(k) F^H. \qquad (8)$$

Our goal is to all $\tilde{F}$ of the same size as F such that

$$C_x(k) = \tilde{F} \bar{C}_s(k) \tilde{F}^H \qquad \text{for all } k \leq K, \qquad (9)$$

for some K (usually one takes $K = 1$). For $k = 0$, (9) gives $FF^H = \tilde{F}\tilde{F}^H$, which in turn implies

$$\tilde{F} = FP \qquad \text{for some unitary matrix } P. \qquad (10)$$

Substituting (10) in (9) and using assumption **A1**,

$$P \bar{C}_s(k) = \bar{C}_s(k) P, \qquad (11)$$

i.e. P must commute with $\bar{C}_s(k)$. Observe that $P = e^{j\theta} I$ always satisfies (11); hence identifiability from SOS is at best modulo a constant $e^{j\theta}$. This is the case when linear channels are considered [9]. However, for nonlinear channels, P need not be as simple as $e^{j\theta} I$, see above. Observe that in (5)

$$\tilde{P} = QPQ^{-1}, \qquad (12)$$

and thus *for equalizability it suffices to force QPQ^{-1} to have the structure in (6).*

Determining the general form of all unitary matrices P satisfying (11) requires solving a linear system of equations with quadratic constraints. *Fortunately these can be replaced by an equivalent set of linear constraints, as the following result shows.* First recall that a matrix P is unitary iff $P = e^{jW}$ for some Hermitian W [7].

Lemma 1 *Let W be square Hermitian, and $P = e^{jW}$. Then (11) holds iff $\bar{C}_s(k)$ and W commute.*

3. THE WHITE INPUT CASE

Define the vectors of the linear and nonlinear parts as $S_n^{(1)} = [\ d_n \quad \cdots \quad d_{n-M_0-N+1}\ ]^T$ (size $k_a = N + M_0$) and $S_n^{(2)}$ (size k_b) respectively; then $S_n = [\ (S_n^{(1)})^T \quad (S_n^{(2)})^T\ ]^T$. In the sequel the following assumptions are made:
A4: $\{d_n\}$ is stationary zero-mean i.i.d. with $E[|d_n|^2] = \sigma^2$.
A5: $\text{cov}(S_n^{(2)}, d_{n-N-M_0}) = 0$.

A sufficient, though *not necessary* condition for assumption **A5** to hold under **A4** is to have the memory of the linear part of the channel be strictly greater than that of the nonlinear part.

Let $A_{ij} = \text{cov}(S_n^{(i)}, S_n^{(j)})$, $B_{ij} = \text{cov}(S_n^{(i)}, S_{n-1}^{(j)})$; then

$$C_s(0) = \begin{bmatrix} A_{11} & A_{12} \\ A_{12}^H & A_{22} \end{bmatrix}, \qquad C_s(1) = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}.$$

Under assumption **A4**, one has

$$A_{11} = \sigma^2 I_{k_a}, \qquad B_{12} = J_{k_a} A_{12}. \qquad (13)$$

Define the Schur complement $A_0 = A_{22} - \frac{1}{\sigma^2} A_{12}^H A_{12}$, which is positive definite. Then Q can be chosen as:

$$Q = \begin{bmatrix} I_{k_a} & 0 \\ \frac{1}{\sigma^2} A_{12}^H & I_{k_b} \end{bmatrix} \begin{bmatrix} \sigma I_{k_a} & 0 \\ 0 & A_0^{1/2} \end{bmatrix}, \quad (14)$$

where $A_0^{1/2}$ is a square root of A_0.

Assumption **A5** allows one to write

$$B_{21} = A_{12}^H J_{k_a}. \quad (15)$$

Using (13) and (15), and with

$$B_0 = B_{22} - \frac{1}{\sigma^2} A_{12}^H J_{k_a} A_{12}, \quad (16)$$

one obtains the following factorization for $C_s(1)$:

$$C_s(1) = \begin{bmatrix} I_{k_a} & 0 \\ \frac{1}{\sigma^2} A_{12}^H & I_{k_b} \end{bmatrix} \begin{bmatrix} \sigma^2 J_{k_a} & 0 \\ 0 & B_0 \end{bmatrix} \begin{bmatrix} I_{k_a} & \frac{1}{\sigma^2} A_{12} \\ 0 & I_{k_b} \end{bmatrix}. \quad (17)$$

Therefore, with Q as in (14), the matrix $\bar{C}_s(1) = Q^{-1} C_s(1) Q^{-H}$ turns out to be block diagonal:

$$\bar{C}_s(1) = \begin{bmatrix} J_{k_a} & 0 \\ 0 & A_0^{-1/2} B_0 A_0^{-H/2} \end{bmatrix}. \quad (18)$$

By lemma 1, in order to find the structure of the ambiguity matrix it suffices to consider Hermitian matrices commuting with $\bar{C}_s(1)$. Let W be such a matrix, partitioned as

$$W = \begin{bmatrix} W_{11} & W_{12} \\ W_{12}^H & W_{22} \end{bmatrix}, \quad (19)$$

with W_{11} of size $k_a \times k_a$. If we let $Z = A_0^{-1/2} B_0 A_0^{-H/2}$, then $W \bar{C}_s(1) = \bar{C}_s(1) W$ gives

$$\begin{aligned} W_{11} J_{k_a} &= J_{k_a} W_{11}, & (20) \\ W_{22} Z &= Z W_{22}. & (21) \\ W_{12} Z &= J_{k_a} W_{12}, & (22) \\ Z W_{12}^H &= W_{12}^H J_{k_a}, & (23) \end{aligned}$$

Since W_{11} is Hermitian, eq. (20) immediately implies

$$W_{11} = \theta I_{k_a} \qquad \text{for some real } \theta, \quad (24)$$

while eq. (21) shows that W_{22} can be any Hermitian matrix commuting with $Z = A_0^{-1/2} B_0 A_0^{-H/2}$. Choose $A_0^{1/2}$ to be the unique Hermitian square root of A_0; then (21) yields

$$\bar{W}_{22} B_0 = B_0 \bar{W}_{22}, \quad (25)$$

where $\bar{W}_{22} = A_0^{1/2} W_{22} A_0^{-1/2}$ is $k_b \times k_b$ Hermitian: one must find all the Hermitian matrices commuting with B_0.

4. IDENTIFIABILITY CONDITIONS

Lemma 2 specifies conditions on $C_s(0)$ and $C_s(1)$ under which $W_{12} = 0$. Under these conditions, from (14), and Lemma 1, QPQ^{-1} has the structure in (6), and blind equalizability obtains. To understand this Lemma, observe from the development in Section 3, that $C_s^{-1}(0) C_s(1)$ is similar to the matrix

$$\begin{bmatrix} J_{k_a} & 0 \\ 0 & A_0^{-1} B_0 \end{bmatrix}. \quad (26)$$

Thus, the Jordan decomposition of the matrix $C_s^{-1}(0) C_s(1)$ has a Jordan block of dimension k_a.

Lemma 2 *Under assumptions* **A1-A5**, *suppose that among the blocks associated with the eigenvalue $\lambda = 0$ in the Jordan decomposition of $C_s^{-1}(0) C_s(1)$ there is exactly one with size equal to $k_a = N + M_0$. Then eqs. (22)-(23) imply $W_{12} = 0$.*

Observe that $C_s^{-1}(0) C_s(1)$ is available from the statistics of the symbol sequence $\{d_n\}$, and therefore the condition in lemma 2 can be checked *a priori*. In view of (26), the requirement in Lemma 2 translates to the requirement that there is *no block associated with the eigenvalue $\lambda = 0$ in the Jordan decomposition of $A_0^{-1} B_0$ with size equal to* $k_a = N + M_0$. Note only Jordan blocks associated with the zero eigenvalue are of interest. Non-zero eigenvalues do not affect the outcome.

5. ALGORITHM

The following algorithm estimates the channel matrix from $C_x(0)$, $C_x(1)$, under **A1-A5** and a somewhat stronger version of lemma 2: it is assumed that all Jordan blocks of $A_0^{-1} B_0$ associated to $\lambda = 0$ have size *strictly less than* k_a.

1. Perform an SVD of $C_x(0)$:

$$C_x(0) = \begin{bmatrix} U_a & U_b \end{bmatrix} \begin{bmatrix} \Sigma^2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} U_a^H \\ U_b^H \end{bmatrix}.$$

2. Let $G = \Sigma^{-1} U_a^H C_x(1) U_a \Sigma^{-1}$.
3. Let p be the number of nonzero eigenvalues of $\bar{C}_s(1)$, and let these be the roots of $\alpha_0 \lambda^p + \alpha_1 \lambda^{p-1} + \cdots + \alpha_p$, with $\alpha_0 \alpha_p \neq 0$.
4. Let r be the number of zero singular values of $\bar{C}_s(1)$. Let $W = [\ w_1 \ \cdots \ w_r\ ]$ be a set of left singular vectors of G associated to its r smallest singular values.
5. If $p = 0$ (i.e. all eigenvalues of $\bar{C}_s(1)$ are zero), let $A = G^{k_a - 1} W$. Otherwise let

$$A = \left(G^{k_a - 1} + \frac{1}{\alpha_p} \sum_{i=0}^{p-1} \alpha_{p-i-1} G^{k_a + i} \right) W$$

6. Let $\hat{v}_{k_a}$ be the column of A with largest norm, divided by its norm.
7. Let $\hat{v}_i = G^H \hat{v}_{i+1}$, $i = k_a - 1, \ldots, 1$. Let $\hat{V}_a = [\ \hat{v}_1 \ \cdots \ \hat{v}_{k_a}\ ]$.
8. Perform an SVD of $I - \hat{V}_a \hat{V}_a^H$:

$$I - \hat{V}_a \hat{V}_a^H = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Gamma & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} U_1^H \\ U_2^H \end{bmatrix}.$$

   Let $\hat{V}_b = U_1$ and $\hat{V} = [\ \hat{V}_a \ \ \hat{V}_b\ ]$.
9. Let $\hat{\mathcal{F}} = U_a \Sigma \hat{V} Q^{-1}$.

Then the rows 1 to k_a of $\hat{\mathcal{F}}^\dagger$ ($(.)^\dagger$ the pseudoinverse) constitute zero-forcing equalizers (delays 0 to $k_a - 1$) for the actual channel $\mathcal{F}$.

6. EXAMPLES

To illustrate these points, we present some examples. In all of these $\{d_n\}$ is assumed real, zero-mean, white and symmetrically distributed around the origin (thus **A4** holds), with $\mu_k = E[d_n^k]$. First, consider a channel with quadratic and cubic nonlinearities

$$x_n = \sum_{i=0}^{R_1} g_i^{(1)} d_{n-i} + \sum_{i=0}^{R_2}\sum_{j=i}^{R_2} g_{ij}^{(2)} d_{n-i}d_{n-j} + \sum_{i=0}^{R_3}\sum_{j=i}^{R_3}\sum_{k=j}^{R_3} g_{ijk}^{(3)} d_{n-i}d_{n-j}d_{n-k} + \eta_n, \quad (27)$$

which matches the model in (1). Assumption **A3** holds provided that $\mu_4 \neq \mu_2^2$ and $\mu_2\mu_6 \neq \mu_4^2$. Assume that $R_1 > R_2$, $R_1 > R_3$; then **A5** holds. Define

$$\begin{aligned} k_i &= N + R_2 - i, \qquad 1 \le i \le R_2, \\ k_{ij} &= N + R_3 - j, \qquad 0 \le i \le j \le R_3. \end{aligned}$$

One can show that the matrix $A_0^{-1}B_0$ is block diagonal, having as diagonal blocks

$$J_{k_i}, \qquad 1 \le i \le R_2, \qquad J_{k_{ij}}, \qquad 0 \le i \le j \le R_3.$$

Therefore $A_0^{-1}B_0$ is already in Jordan form. We see that the conditions of lemma 2 hold, since $k_a > k_i$ and $k_a > k_{ij}$ because of the memory requirements $R_1 > R_2$, $R_1 > R_3$. Therefore the ambiguity matrix for this structure must be as in (6).

As a second example, consider for any $M_0 \geq 0$.

$$x_n = \sum_{j=0}^{M_0} h_{0j} d_{n-j} + h_1 d_n^2 d_{n-1}^2 + h_2 d_n^2 d_{n-2}^2. \quad (28)$$

Observe, by suitably choosing M_0, *the linear kernel can be forced to have the same length as another kernel.* In this case, regardless of M_0, B_0 is nonsingular. Thus, $A_0^{-1}B_0$ does not have any Jordan block associated with the zero eigenvalue. Thus, blind equalizability obtains from second order statistics alone.

As a final example, assume the channel is

$$x_n = \sum_{j=0}^{M_0} h_{0j} d_{n-j} + \sum_{j=0}^{M_1} h_{1j} d_{n-j}^2, \quad (29)$$

with $M_0 < M_1$, i.e. the linear kernel no longer has the largest size. The vectors $S_n^{(i)}$ are for this case

$$\begin{aligned} S_n^{(1)} &= [\; d_n \;\; \cdots \;\; d_{n-N-M_0+1} \;]^T, \\ S_n^{(2)} &= [\; d_n^2 \;\; \cdots \;\; d_{n-N-M_1+1}^2 \;]^T. \end{aligned}$$

Assumption **A3** holds provided that $\mu_4 \neq \mu_2^2$; then one has $Q = \Lambda \otimes I_{N+M}$ with $\Lambda = \mathrm{diag}(\sqrt{\mu_2}, \sqrt{\mu_4 - \mu_2^2})$. Observe that **A5** is satisfied. The normalized covariance matrix $\bar{C}_s(1)$ is block diagonal, with diagonal blocks, J_{k_a} and J_{k_1}, $k_1 > k_a$. Thus, as $\bar{C}_s(1)$ is similar to $C_s^{-1}(0)C_s(1)$, the conditions of lemma 2 are met. Thus blind equalizability holds despite the fact that the linear kernel is not the longest. Strictly speaking the algorithm in the previous Section does not work for this case. However, a minor variation of this algorithm does yield the desired equalizer.

7. CONCLUSIONS

The problem of blindly identifying/equalizing a SIMO nonlinear FIR channel using second-order statistics of the observed signal has been considered. We have presented sufficient conditions on the statistics of the symbol sequence allowing the design of linear FIR zero-forcing equalizers. An algorithm was given to obtain an estimate of the channel matrix, from which the equalizers can be extracted.

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