BLIND ITERATIVE DECISION FEEDBACK EQUALIZERS FOR BLOCK QAM TRANSMISSION SYSTEMS

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ABSTRACT

A novel blind initialization procedure for iterative decision feedback equalizers in block-based transmission systems is proposed and investigated. It relies on an initial stage using Regalia's block-based Constant Modulus iterative algorithm for the blind computation of a linear equalizer; then a switch to decision feedback mode is performed. It is shown how the building blocks of the decision feedback equalizer (feedforward and feedback filters, automatic gain control and phase rotation) can be blindly estimated. Due to the unknown lag introduced by the blind linear equalizer, delay synchronization of the feedforward and feedback filters is also required. These filters are then refined over successive decision feedback iterations. This approach can also be used as a blind channel identifier for other block receiver designs, such as soft ISI cancelers and decoder-aided (i.e. turbo) equalizers.

1. INTRODUCTION

As data rates increase in modern digital communications systems, intersymbol interference (ISI) becomes an inevitable consequence of the dispersive nature of bandlimited propagation channels. The receiver must include an equalizer to mitigate the effects of ISI. In applications in which the channel impulse response is unknown and no training sequence is available, the equalizer must be computed/updated blindly from the received signal and knowledge of the statistics of the data source alone. A common approach in continuous transmission systems is to blindly update a linear equalizer (LE) using the Constant Modulus Algorithm (CMA), and then switch to a decision directed (DD) mode once the symbol error rate (SER) is low enough [10]. Switching to a DD-based decision feedback equalizer (DFE) is also possible and desirable, since DFEs generally outperform LEs for the same complexity. A generic strategy for blind DFE initialization in such setting was proposed in [4]; see also [1].

We develope a similar strategy for blind iterative DFE computation in block transmission systems with quadrature amplitude modulation (QAM). An iterative CMA-based LE for packet-based systems was recently proposed by Regalia in [8]. Its computational simplicity makes it an obvious candidate for the first iterations, during which hard decisions would not be reliable. Once the eye has been opened the DFE mode is switched on, which should reduce the SER considerably. Several issues must be addressed if the switch is to be successful. First, gain and phase correction must be applied since the CM approach does not completely compensate for these impairments. Second, the equalization lag to which the CM-based LE has converged must be identified, in order to properly synchronize the DFE. Third, the feedforward and feedback filters of the DFE must be computed. We show how all these tasks can be carried out blindly.

The paper is organized as follows. Section 2 presents the channel model. Regalia's block-oriented CM scheme is briefly reviewed in section 3. Gain and phase correction are discussed in section 4, while blind computation of the DFE building blocks and their mutual synchronization in terms of time delay are considered in sections 5 and 6 respectively. Section 7 presents the operation of the iterative DFE. We close with some examples and conclusions.

2. PROBLEM STATEMENT

We consider a (complex-valued) baseband-equivalent single-input single-output channel model

$$u_n = \sum_{k=0}^{L} h_k a_{n-k} + \eta_n = H(z)a_n + \eta_n, \qquad (1)$$

where u_n is the received signal, $\{h_k\}_{k=0}^L$ is the channel impulse response, $\{a_n\}$ is the transmitted symbol sequence, and η_n is additive noise. $H(z) = \sum_{k=0}^L h_k z^{-k}$ is the z-transform of the channel response. It is assumed that $\{a_n\}$, $\{\eta_n\}$ are independent, zero-mean white sequences with variances σ_a^2 and σ_η^2 respectively. The symbols $\{a_n\}$ are drawn from a complex QAM constellation. We consider block-based transmissions, in which $N-\hat{L}$ symbols (with $\hat{L}$ an estimate of the channel order L) are sent through the channel so that N samples of the received signal $(u_1, u_2, \cdots, u_N)^T$ are available. During the transmission of a data block, it is assumed that the channel taps remain constant. The goal is to recover the transmitted symbols $\{a_n\}$ relying solely on knowledge of their probability distribution and the received samples.

3. LINEAR EQUALIZATION STAGE

Error propagation would preclude blind adaptation of a DFE from a cold start (unless the channel is sufficiently mild), and therefore we initially resort to an LE. In particular, P iterations of Regalia's CM block-oriented algorithm [8] are run initially. We briefly describe the process here.

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Let $\mathbf{g}$ be the $M \times 1$ vector comprising the LE taps, so that the equalizer output is

$$y_n = [\ u_n \quad u_{n-1} \quad \cdots \quad u_{n-M+1}\ ]\mathbf{g} = \mathbf{u}_n^T \mathbf{g}.$$

Let $\mathbf{U} = [\ \mathbf{u}_1 \quad \mathbf{u}_2 \quad \cdots \quad \mathbf{u}_N\ ]^T$, and let $\mathbf{U} = \mathbf{QR}$ be its QR decomposition (i.e. $\mathbf{Q}$ is unitary and $\mathbf{R}$ is upper triangular). By stacking all y_n in a vector $\mathbf{y}$, one can write

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \mathbf{Ug} = \mathbf{QRg} = \mathbf{Qw},$$

where $\mathbf{w} = \mathbf{Rg}$. The k-th iteration of the algorithm can be summarized as follows:

1. Let $\mathbf{y}_k = \mathbf{Q}\mathbf{w}_k$.
2. Let $\mathbf{z}_k = \mathbf{y}_k \odot \mathbf{y}_k^*$, where $\odot$ denotes componentwise (Hadamard) product.
3. Let $F_k = ||\mathbf{z}_k||^2$, and $\mu_k = \beta/F_k$ with $\frac{1}{2} \leq \beta \leq \frac{2}{3}$.
4. $\mathbf{v}_{k+1} = \mathbf{w}_k - \mu_k[\mathbf{Q}^H(\mathbf{z}_k \odot \mathbf{y}_k) - F_k\mathbf{w}_k]$.
5. $\mathbf{w}_{k+1} = \mathbf{v}_{k+1}/||\mathbf{v}_{k+1}||$.

It is common to use a 'center-spike' initialization for the equalizer, i.e. $\mathbf{w}$ is set initially to the middle column of $\mathbf{R}$. Basically, this iterative process seeks a minimum of the cost function $F(\mathbf{w}) = (\sum_{n=1}^N |y_n|^4)/(\sum_{n=1}^N |y_n|^2)^2$ subject to $||\mathbf{w}|| = 1$. If successful, after a few iterations the LE has 'opened the eye' so that hard decisions on the LE output are reliable enough for DFE adaptation purposes.

Instead of running the LE stage for a prespecified number P of iterations, the value of F_k could also be used as an indicator of eye opening: we can approximate $F \approx \kappa_y/N$, where $\kappa_y = E[|y_n|^4]/E^2[|y_n|^2]$ is the normalized kurtosis of $\{y_n\}$. Therefore, the switch could be made whenever $F_k < \zeta\kappa_a/N$ is detected, where κ_a is the kurtosis of the source and ζ is a number slightly greater than one. Alternatively, the switch criterion $|F_k - F_{k-1}| < \epsilon$, with ϵ a small positive number, could also be used.

4. GAIN AND PHASE CORRECTION

Note that the CM scheme outlined in section 3 does not provide phase correction (since the cost F is insensitive to complex phase rotations) or gain control (since the norm of $\mathbf{w}$ is constrained to 1). Let $\mathbf{w}$, $\mathbf{y}$ be the equalizer tap and output vectors, respectively, after the P prespecified CM iterations with the LE. If the eye has been opened, the distribution of the elements of $\mathbf{y}$ will resemble the constellation of the original symbols, possibly rotated and scaled. To compensate for these effects, both $\mathbf{w}$, $\mathbf{y}$ are multiplied by a complex scalar $re^{-j\phi}$. The scaling r is easily computed in order to match the variances of the equalizer output and the original constellation:

$$r = \sqrt{\frac{\sigma_a^2}{||\mathbf{y}||^2/N}} = \sqrt{N\sigma_a^2}, \tag{2}$$

since the vector $\mathbf{y}$ has unit norm.

Several block-based methods are available for blind phase estimation. For example, the fourth power estimator is well suited for QAM constellations, and it is known to yield almost maximum likelihood performance in the limit of small SNR (see [11] for a performance analysis of several blind phase estimation techniques). It is given by

$$\phi = \frac{1}{4}\,\text{angle}\left[E[a_n^{*4}]\frac{1}{N}\sum_{n=1}^N y_n^4\right]. \tag{3}$$

As with any blind procedure, a residual phase ambiguity of $k\pi/2$ remains as a consequence of the quadrant symmetry of the QAM constellation. This is usually sidestepped by using differential encoding of the data.

We should note that for M-PSK constellations with $M > 4$, one has $E[a_n^{*4}] = 0$ and therefore the fourth power estimator is not adequate. PSK-specific phase compensation techniques could be used in such cases [2].

5. COMPUTATION OF THE MMSE DFE

Once amplitude and phase offsets have been corrected in the received signal, the minimum mean square error (MMSE) DFE is to be computed. Since the approach is block-based and iterative, the feedforward and feedback filters (FFF and FBF) of the DFE (which we denote by $P(z)$ and $Q(z)$ respectively) need not be causal, i.e. 'future' decisions can be fed back since they are available from the previous iteration. Thus we let

$$P(z) = \sum_{k=-\infty}^{\infty} p_k z^{-k}, \qquad Q(z) = \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} q_k z^{-k}. \tag{4}$$

The DFE output at iteration k is therefore

$$y_n^{(k)} = P(z)u_n - Q(z)\hat{a}_{n-d}^{(k-1)},$$

where $\hat{a}_n^{(k-1)}$ are the hard decisions from the previous iteration, and d is some equalization lag. In this way, $y_n^{(k)}$ is an estimate of the symbol a_{n-d}. Now $P(z)$, $Q(z)$ are chosen to minimize the MSE $E[|a_{n-d} - y_n^{(k)}|^2]$, assuming that 'future' as well as 'past' decisions from the previous iteration are correct, i.e. $\hat{a}_m^{(k-1)} = a_m$ for $m \neq n - d$. The constraint $q_0 = 0$ on the feedback filter seen in (4) is imposed in order to avoid the trivial solution $P(z) = 0$, $Q(z) = -1$. The solution to this minimization problem can be given in terms of the channel $H(z)$ [5]:

$$P(z) = \frac{1}{\lambda + \alpha}\tilde{H}(z)z^{-d}, \tag{5}$$

$$Q(z) = \frac{1}{\lambda + \alpha}\left[\tilde{H}(z)H(z) - \alpha\right], \tag{6}$$

where $\tilde{H}(z)$ denotes the 'paraconjugate' of $H(z)$, i.e. $\tilde{H}(z) = H^*(1/z^*)$, and λ, α denote respectively the noise to transmitted power ratio and the channel gain:

$$\lambda = \frac{\sigma_\eta^2}{\sigma_a^2}, \qquad \alpha = \sum_{k=0}^{L} |h_k|^2 = ||H(z)||^2. \tag{7}$$

It is seen from (5)-(6) that the FFF and FBF of the MMSE DFE do have a finite number of coefficients (since $H(z)$ is FIR), and that the optimum FFF is a filter matched to the channel. In order to blindly obtain these filters, it will be useful to introduce the power spectral density (psd) $S_u(z)$ of the received signal:

$$S_u(z) = \sum_{k=-\infty}^{\infty} r_k z^{-k}, \qquad \text{with} \qquad r_k = E[u_n u_{n-k}^*]. \quad (8)$$

We observe that this psd is given by

$$S_u(z) = \sigma_a^2 H(z)\tilde{H}(z) + \sigma_\eta^2. \quad (9)$$

From (6) and (9), and noting that $\sigma_a^2(\lambda+\alpha) = r_0$ (the variance of the received signal), it follows that

$$Q(z) = \frac{1}{r_0} S_u(z) - 1. \quad (10)$$

Therefore, the FBF taps can be directly (and blindly) computed from the autocorrelation sequence of the received signal. This autocorrelation sequence can be very efficiently estimated due to the fact that the QR decomposition of the data matrix $\mathbf{U}$ is already available from the LE stage. In particular, if we denote $\mathbf{R} = [R_{ij}]_{1\le i,j\le M}$, then the estimate of the lag-k coefficient becomes

$$\begin{aligned} \hat{r}_k &= \frac{1}{N}\sum_{n=1}^{N} u_n u_{n-k}^* \\ &= \frac{1}{N} R_{11} R_{1,k+1}^*, \qquad 0 \le k \le \hat{L}. \quad (11) \end{aligned}$$

With these, the estimated FBF is given by

$$\hat{Q}(z) = \frac{1}{r_0}(\hat{r}_{\hat{L}}^* z^{\hat{L}} + \cdots + \hat{r}_1^* z + \hat{r}_1 z^{-1} + \cdots + \hat{r}_{\hat{L}} z^{-\hat{L}}). \quad (12)$$

In order to estimate the FFF taps, let $G(z)$ be the z-transform of the linear equalizer impulse response $\mathbf{g}$ at the end of the LE stage (recall that $\mathbf{Rg} = \mathbf{w}$), and after gain and phase correction have been applied as discussed in section 4. Then the cascade of channel and linear equalizer approximates a pure delay:

$$G(z)H(z) \approx z^{-d},$$

and therefore

$$\begin{aligned} \frac{\sigma_a^2}{r_0} G(z)H(z)\tilde{H}(z) &\approx \frac{\sigma_a^2}{r_0}\tilde{H}(z)z^{-d} \\ &= P(z), \quad (13) \end{aligned}$$

which is the desired matched filter. Now, by making the approximation $\sigma_a^2 H(z)\tilde{H}(z) \approx S_u(z)$ (which ignores the contribution of the additive noise to the received signal psd) in (13), we finally obtain our FFF estimate as

$$\hat{P}(z) = \frac{1}{r_0} G(z) S_u(z). \quad (14)$$

6. DELAY SYNCHRONIZATION

As shown in section 5, the estimate FFF (14) satisfies

$$\hat{P}(z) \approx \frac{\sigma_a^2}{r_0}\tilde{H}(z)z^{-d}.$$

It is important to note that the delay d is unknown (it corresponds to the delay associated to the CM-based linear equalizer). In a serial (i.e. noniterative) implementation of the DFE this would not be a problem. However, for the iterative approach to work, the value of d must be determined in order to synchronize the operations of the FFF and FBF (since the latter operates on decisions from the previous iteration). To do so, we note that since the LE $G(z)$ had M taps, the convolution $G(z)S_u(z)$ has a total of $2L+M$ coefficients. (Usually, in order to achieve acceptable performance, it is necessary to take M considerably larger than L.) Thus only $L+1$ of the $2L+M$ coefficients of $G(z)S_u(z)$ will be significant, and their position in this convolution will be given by the unknown delay. Therefore d can be estimated by computing the energies of the coefficients of $G(z)S_u(z)$ over all windows of size $\hat{L}+1$. The taps of the feedforward filter are determined by picking the $\hat{L}+1$ coefficients of the window with maximum energy. In this way, if the resulting FFF $\hat{P}(z)$ is seen as a degree-$\hat{L}$, causal FIR filter, we would have

$$\hat{P}(z) \approx \frac{\sigma_a^2}{r_0}\tilde{H}(z)z^{-\hat{L}}. \quad (15)$$

A similar energy maximization approach was suggested in [6] in order to resolve delay ambiguities in a blind channel identification setting.

7. DECISION FEEDBACK EQUALIZATION STAGE

With the initialization procedure described in sections 5 and 6, the DFE operates iteratively over the data block. A decision directed approach could be used in order to refine the FFF and FBF estimates at every iteration. However, in order to reduce the computational complexity by taking advantage of the QR decomposition available from the LE stage, and to reduce the effect of error propagation, we propose to keep the FBF fixed at its initial value (12) while updating the FFF using the same CM approach, although suitably modified as explained next.

Let $\bar{\mathbf{R}}$ be the $(\hat{L}+1)\times(\hat{L}+1)$ principal submatrix of $\mathbf{R}$, and $\bar{\mathbf{Q}}$ the matrix formed by the first $\hat{L}+1$ columns of $\mathbf{Q}$. Also let the vector $\mathbf{p}$ comprise the $\hat{L}+1$ coefficients of the initial FFF $\hat{P}(z)$ (15). Then we initialize $\bar{\mathbf{w}} = \bar{\mathbf{R}}\mathbf{p}$. Similarly, let $\mathbf{q}$ be the FBF vector

$$\mathbf{q} = [\ \hat{r}_{\hat{L}}^* \quad \cdots \quad \hat{r}_1^* \quad 0 \quad \hat{r}_1 \quad \cdots \quad \hat{r}_{\hat{L}}\ ]^T,$$

and $\mathcal{A}_k$ the $N\times(2\hat{L}+1)$ Toeplitz matrix formed by hard decisions from iteration k:

$$\mathcal{A}_k = \begin{bmatrix} \hat{a}_1^{(k)} & 0 & \cdots & 0 \\ \hat{a}_2^{(k)} & \hat{a}_1^{(k)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \hat{a}_N^{(k)} & \hat{a}_{N-1}^{(k)} & \cdots & \hat{a}_{N-2\hat{L}}^{(k)} \end{bmatrix}.$$

The following procedure illustrates the computation of the DFE output at iteration k, $\mathbf{y}_k = [\ y_1^{(k)} \quad y_2^{(k)} \quad \cdots \quad y_N^{(k)}\ ]^T$, and the update of the FFF:

1. Let $\mathbf{y}_k = \bar{\mathbf{Q}}\bar{\mathbf{w}}_k - \mathcal{A}_{k-1}\mathbf{q}$.
2. Let $\mathbf{z}_k = \mathbf{y}_k \odot \mathbf{y}_k^*$.
3. Let $F_k = ||\mathbf{z}_k||^2/||\mathbf{y}_k||^4$, and $\mu_k = \beta/||\mathbf{z}_k||^2$ with β a constant.
4. $\bar{\mathbf{w}}_{k+1} = \bar{\mathbf{w}}_k - \mu_k \bar{\mathbf{Q}}^H[(\mathbf{z}_k \odot \mathbf{y}_k) - F_k||\mathbf{y}_k||^2\mathbf{y}_k]$.

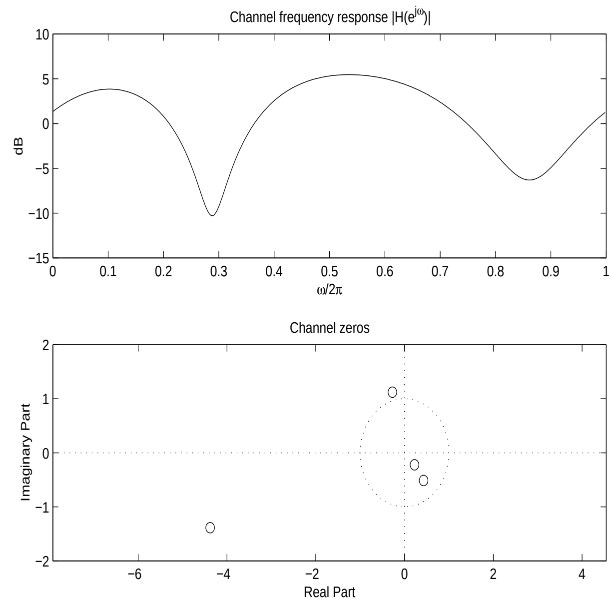

Figure 1: Channel characteristics.

5. With $\mathcal{D}(\cdot)$ denoting the hard decision device, let

$$\hat{a}_n^{(k)} = \begin{cases} \mathcal{D}(y_{n+\hat{L}}^{(k)}), & n = 1, \ldots, N - \hat{L}, \\ 0, & n = N - \hat{L} + 1, \ldots, N. \end{cases}$$

Again, this iteration represents a gradient descent on the cost function $F(\bar{\mathbf{w}}) = (\sum_{n=1}^{N} |y_n|^4)/(\sum_{n=1}^{N} |y_n|^2)^2$, the main difference with respect to the LE stage being that $\bar{\mathbf{w}}$ cannot be constrained to have unit norm due to the effect of the FBF in the output vector $\mathbf{y}$.

It should be noted that the FBF vector $\mathbf{q}$ could also be updated, by noting that the MMSE FBF (6) can be written in terms of the MMSE FFF (5) as

$$\begin{aligned} Q(z) &= (\lambda + \alpha) P(z) \tilde{P}(z) - \frac{\alpha}{\lambda + \alpha} \\ &= \frac{\sigma_u^2}{\sigma_a^2} \left[P(z)\tilde{P}(z) - \|P(z)\|^2 \right]. \end{aligned} \tag{16}$$

Thus, given $\bar{\mathbf{w}}_k$, one could solve for $\mathbf{p}_k$ in $\bar{\mathbf{R}}\mathbf{p}_k = \bar{\mathbf{w}}_k$ via backsubstitution, and then obtain $\mathbf{q}_k$ by convolving $\mathbf{p}_k$ with its reversed-conjugate version, multiplying the result by $\hat{r}_0/\sigma_a^2$ and setting the central tap to zero.

8. SIMULATION EXAMPLES

Here we consider a rather severe complex channel of order $L = 4$ given by

$$\begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} = \begin{bmatrix} 0.0 + j0.2 \\ -0.2 + j0.8 \\ 0.3 - j0.1 \\ -0.8 + j0.3 \\ 0.1 - j0.2 \end{bmatrix} \tag{17}$$

whose frequency response and zero pattern are shown in Fig. 1.

In the first simulation the symbols were drawn from a QPSK constellation. The SNR at the channel output was 18 dB, and a linear equalizer with $M = 9$ taps was used. The packet size was $N = 500$ symbols. The evolution of the cost F_k in one run is shown in Fig. 2 using $\beta = 2/3$ in the stepsize of both stages. The switch to DFE mode is done after $P = 20$ iterations of the LE stage; the assumed channel order was $\hat{L} = L = 4$.

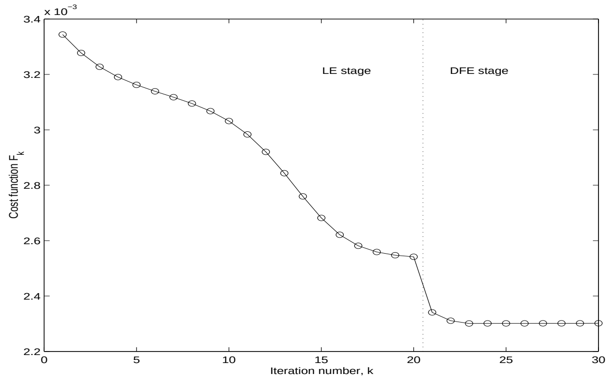

Figure 2: Evolution of the CM cost F_k. QPSK symbols, packet size $N = 500$, SNR = 18 dB.

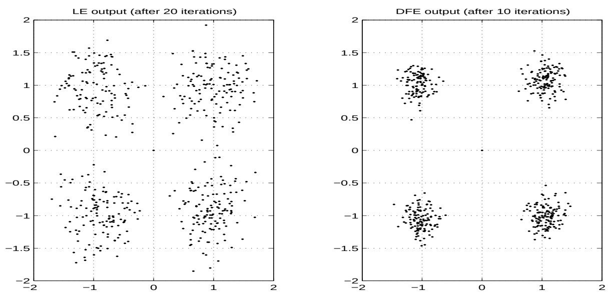

Figure 3: Scatter plots after the LE and DFE stages for QPSK. Packet size $N = 500$, SNR = 18 dB.

Observe the drop in the cost F_k once the DFE is switched on, which implies a significant increase in the eye opening as can be seen in Fig. 3. In the DFE stage, the FBF was kept fixed at its initial value, although a similar behavior was observed when it was updated as outlined in section 7. We should note that if $\hat{L} > L$, then the FFF will have some extra leading or trailing taps of small magnitude which introduce some additional noise enhancement; however, the DFE initialization procedure is robust to this effect, and the synchronization scheme of section 6 automatically compensates for the additional delay introduced by small FFF leading taps. Thus knowledge of the channel order is not critical.

We repeated the experiment using a 16QAM constellation for the source. In that case, the LE order and packet size had to be increased to $M = 16$ and $N = 1000$ symbols respectively. Similarly, convergence of the LE stage was significantly slowed down as can be observed in Fig. 4, so that it was run for $P = 40$ iterations before switching to DFE mode. The resulting eye diagrams are shown in Fig. 5.

Since the feedforward filter of the MMSE DFE reduces to a filter matched to the channel, is it clear that the relation (13) can be alternatively seen as a blind channel estimator. It can be viewed as a block-based counterpart of the Gooch-Harp method of on-line blind channel identification [3, 9], which estimates the channel im-

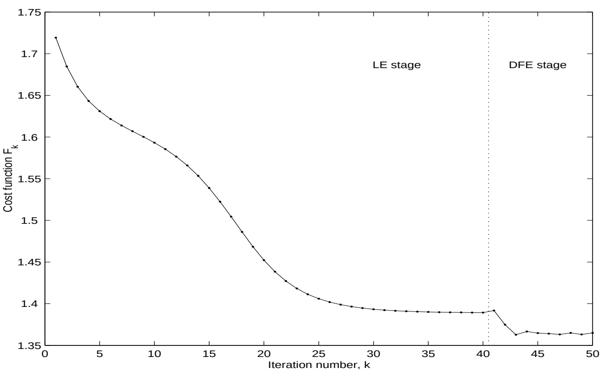

Figure 4: Evolution of the CM cost F_k. 16QAM symbols, packet size $N = 1000$, SNR = 18 dB.

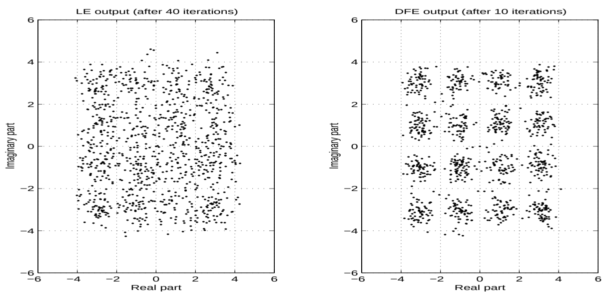

Figure 5: Scatter plots after the LE and DFE stages for 16QAM. Packet size $N = 1000$, SNR = 18 dB.

pulse response by correlating the received signal with the symbol estimates obtained by a constant-modulus based blind equalizer.

In order to test the quality of this estimator, we run some simulations to compute the normalized root-mean-square error (NRMSE), defined as

$$\mathrm{NRMSE} = \frac{1}{||\mathbf{h}||}\sqrt{\frac{1}{R}\sum_{i=1}^{R}||\mathbf{h}-\hat{\mathbf{h}}(i)||^2},$$

where $\mathbf{h}$ is the vector of channel coefficients, $\hat{\mathbf{h}}(i)$ is the channel estimate in the ith run, and R is the number of runs. We considered the same channel (17), using an equalizer of order $M = 16$. Fig. 6 shows the NRMSE as a function of the SNR α/λ at the channel output, for several values of the packet size N and QPSK transmitted symbols (Averaging was made over 500 independent runs). Fig. 7 is similar but considering a 16QAM constellations. As it could be expected, the latter scheme requires considerably more symbols to achieve similar performance.

9. CONCLUSIONS

We have presented a procedure for blind initialization of iterative DFEs in block based transmission systems. The key idea is to open the eye with an iterative, blind linear equalizer, and then switch to decision feedback mode. Several issues must be taken care of in this switch, such as correcting the gain and phase of the linear

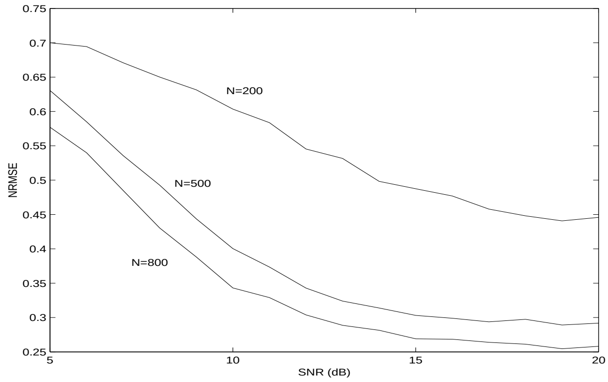

Figure 6: Channel NRMSE as a function of SNR, for different packet sizes (OQPSK modulation).

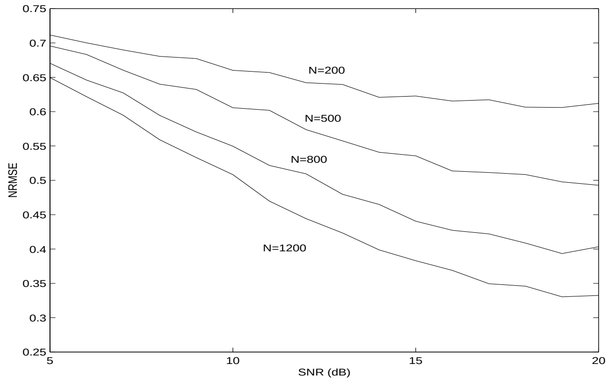

Figure 7: Channel NRMSE as a function of SNR, for different packet sizes (16QAM modulation).

equalizer, computing the building blocks of DFE, and compensating for the unknown delay introduced by the linear equalizer. It has been shown how to perform all these tasks blindly and with a low computational cost.

The iterative DFE based on hard decisions is sensitive to error propagation from one iteration to the next, due to wrong decisions being made. Given the iterative nature of the scheme, it appears to be well suited for more sophisticated approaches. For example, Pollock and Kennedy [7] have proposed a block-oriented bidirectional DFE, which uses a data-dependent soft nonlinearity instead of a hard decision device. In systems incorporating channel coding, turbo equalizers [5, 12]), which exploit information from the error correction decoder, can also be used. Both approaches are characterized by a symbol-by-symbol recomputation of the FFF of the equalizer, which amounts to having a time-varying DFE, and both require knowledge of the channel impulse response. Thus the method presented here constitutes a good candidate for an initial blind channel estimation stage in these schemes. This first blind channel estimate can be further refined along successive iterations of the turbo equalizer [13].

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