On the Constant of Gravitation

A note by Zaman Akil, introduced by Jean-Claude Pecker (Collège de France, Paris)

Two documents are published hereafter: a paper by Zaman Akil which was earlier submitted for publication in the Proceedings of the French Academy of Sciences, and a note by myself, presented at the same time. The paper was rejected by an anonymous referee, and our efforts to secure a second referee failed. Since all this transpired in 1988, I feel the time is ripe to publish Akil's paper together with the note I prepared to introduce and, in a sense, legitimate the paper (of course this is a highly unusual procedure). I thank C. Roy Keys for agreeing to publish both documents in APEIRON.

The reader will permit me to trace some of the history of this paper. On January 2, 1985, Zaman Akil sent the Academy of Sciences a short summary of a longer work. At his request, the Perpetual Secretary of the Academy, Prof. Paul Germain, sent the letter to several members of the Academy, including myself. I was the only one who agreed to discuss it with the author. His strange result was dismissed *a priori* by my colleagues as being a purely spurious relation without justification, and which could not be understood, since Akil equated a dimensionless quantity to a physical quantity of dimensions $L^3M^{-1}T^{-2}$. A long correspondence then ensued between Mr. Akil and myself, notwithstanding the difficulties created by the fact that Mr. Akil divides his time between London and Kuwait. This correspondence resulted in the paper published below (which was submitted to the Academy in 1988-1989) together with my “note to the reader” in defence of Akil’s peculiar results.

After the paper was rejected, we prepared an envelope for permanent deposit (“pli cacheté) with the Academy of Sciences containing the documents published here, plus the correspondence exchanged over the past five years. This envelope was deposited on December 12, 1991. Of course, we do not exclude the possibility that some of the texts may be published in other journals.

Presentation

This paper is a study of the universal gravitational constant G. Its “theoretical value” is calculated from the ratio of the masses of the proton and muon to the mass of the electron. The equation

$$\left[\left(\frac{2p\, m_p}{m_e}\right)\cdot\left(\frac{2p\, m_m}{m_e}\right)\right]^{-1}$$

yields a value of $6.67187\,(\pm 0.00002)\times 10^{-8}$, whereas measurement gives us $G = 6.6714\,(\pm 0.0006)\times 10^{-8}$ cgs. The author provides profound reasons for this apparent contradiction in the “dimension” of the quantities in question. A model is presented which shows that material bodies attract one another according to laws that are comparable to other types of attractive forces, provided that care is taken to properly define the “constants” being applied depending upon whether “inertial” or “gravitational” masses are involved. This discussion suggests a plausible relationship between gravitation and the basic properties of fundamental particles.

Note to the reader

The paper by Zaman Akil published here offers a highly peculiar interpretation of Cavendish's constant "*G*". It is the sequel to an earlier paper that was sent to the French Academy of Sciences in 1985, which the Perpetual Secretary of the Academy asked me to examine. It incorporates a number of changes made further to an exchange of correspondence between Mr. Akil and myself.

The numerical result obtained by Mr. Akil associates the constant *G* with the ratios of the proton and muon masses to the electron mass. This finding caused the paper to be rejected after the first reading, since the author had equated a number *G*, with dimensions $L^3M^{-1}T^{-2}$ in the conventional system of units, to an essentially dimensionless number which arose from a ratio between quantities of the same dimension.

It then occurred to us that a ratio of identical quantities (e.g. charges) measured in the system of electromagnetic units and in the system of electrostatic units is a simple function of the numerical value of the speed of light, which has dimensions LT^{-1}. It was this same basic notion that earlier led Maxwell to identify light waves with electromagnetic waves.

Were we not dealing with an analogous finding to Maxwell's, of possibly great importance? In other words, is the ratio of inertial mass to gravitational mass connected to a quantity with a definite dimension, the *G* constant? This is the problem Zaman Akil tries to solve. And while one might be skeptical about the solution he arrives at (why are the proton and the muon so special, and not, say, the tau particle?), he at least asks an interesting question, which I believe could usher in further discoveries, if only by stimulating debate.

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On the Constant of Gravitation

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An analysis is made of the meaning of the gravitational constant, *G*. Its "theoretical" value is derived very precisely employing a straightforward and original expression involving only the absolute mass ratio of two fundamental particles: the proton and the muon:

$$G_{cgs} = \left(4p^{2} m_{p} m_{m} \Big/ m_{e}^{2} \right)^{-1}$$

A model is presented which shows that material bodies gravitationally attract one another in a manner that duplicates other known microsystems. Thus, it has been possible to suggest a plausible relationship between gravitation and the basic properties of fundamental particles. The dimensional problem posed by the expression above is discussed; it is explained why it results from incomplete expressions.

Introduction

We shall examine the original basic expression of Newton's law of universal gravitation and try to establish possible links to other areas of physics with the intention of inter-relating the gravitational theory to other known natural forces and the properties of fundamental particles. We shall also investigate whether, in the final analysis, the constant G contains, implicitly as a factor, the constant that determines the proportionality of equivalence between gravitational and inertial mass.

Defining equations and dimensions

Our units in the field of electromagnetics are defined in terms of a mechanical force. But in Newton's expression for the gravitational force:

$$F = \frac{Gm_a m_b}{r^2}$$

Here both masses are directly related to force, and since there already exists a standard unit for mass, we do not need a defining equation for any particular "gravitational" mass unit. However, the inertial mass, M, as defined by Newton's second law $F = kMg$ is a kinematic quantity governed by motion while in the gravitational expression, m_a and m_b, are akin to "charges", being proportional to the quantity of mass in an object. Consequently, the gravitational constant, G, ends up having the arbitrary dimensions $L^3M^{-1}T^{-2}$ without, in the meantime, improving our understanding of the mechanism involved in gravitation, or the important principle of mass equivalence. On the other hand, to the practical physicist and engineer, the introduction of a new system of units to measure such "gravitational charges", would be superfluous. Nevertheless, here we propose—and for a good heuristic reason—to do just this, and find out whether a new meaning for the G constant can be identified by linking it to other physical relations. As we shall see, the "gravitational unit" so defined, even though it remains dormant in the background, may help us read more into the original expression and clarify some aspects of the phenomenon of gravitation that some consider ambiguous, and others even see as a great mystery.

An analogous implicit unit for gravitational charge

Keeping with the cgs system of units, we next *define* the magnitude of this proposed gravitational unit of "gravitational charge". Thus if Ψ represents the cgs unit of the "charge" placed at a unit distance R (1 cm) from another similar "charge", and K_m, a constant reflecting the nature of the intervening shielding medium, then by definition:

$$F = \frac{K_m \Psi\Psi}{R^2} (= 1\,\text{dyne}) \tag{1}$$

Our next step is to find out the *corresponding magnitude of mass, proportional to one defined unit of this "gravitational charge"*. We replace m_a and m_b, in eqn (1) with m_p and m_m, such that if they were placed one unit distance apart ($R = 1$ cm), then the gravitational force attracting one mass to the other would be exactly equal to a unit force (*i.e.* one dyne). Thus:

$$F = \frac{K_m \Psi^2}{R^2} = \frac{G m_p m_\text{m}}{R^2}$$

or

$$G = \frac{K_m \Psi^2}{\left(K_p K_\text{m} m_g^2\right)} \tag{2}$$

by letting $m_p = K_p m_g$ and $m_\text{m} = K_\text{m} m_g$, where m_g is the usual unit of inertial mass (gram) and where K_p and K_m are two dimensionless numerical constants. But K_m should normally be put equal to unity, if we want to *define* a "gravitational mass" (which we called "gravitational charge", to avoid any confusion). Finally, re-arranging and inserting the "new" interpretation of the G constant in the original equation (1) we arrive at:

$$F = \frac{\Psi^2 \left(m_e / K_p m_g \right) \left(m_b / K_m m_g \right)}{R^2} \tag{3}$$

Clearly, dividing m_a by m_p and m_b by m_m results in two dimensionless numerical magnitudes, say N_a and N_b such that we end up, in effect, with $F = Z_a Z_b / R^2$, where $Z_a = N_a \Psi$ and $Z_b = N_b \Psi$ correspond to the magnitudes of the implicit "gravitational charges". Hence, mass is now dimensionally "neutralized" in (3) and its presence is only necessary to numerically process the magnitude of this gravitational "charge". This exercise, though obviously tautological, nevertheless helps us treat the subject in a more systematic manner. From the dimensional point of view, the above expression is correct and is now identical to expressions employed in electromagnetism. The question now arises: has the product of the two ratios K_p and K_m, any physical significance? Or, is the constant G, as often treated in the literature, just an arbitrary numerical ratio dependent mainly upon our choice of a particular system of units to carry out measurements? Indeed, it is remarkable that after the product $K_p K_m$ is divided by the factor $4p^2$ (to balance static and centripetal forces), it turns out to be, to a very high degree of precision, equal to the product of the absolute mass of the proton multiplied by that of the muon. Therefore if $K'_p = K_p / 2p$ and $K'_m = K_m / 2p$ are respectively the ratios in these basic units of the proton to the electron mass [= 1836. 15152 (70)] (RMP 1984) and the muon mass to the electron mass [= 206.76833(50)] (RMP 1984) then from (2) and bearing in mind that both Ψ and $m_g = 1$ by definition, we arrive at

$$G = \frac{\Psi^2}{\left(4p^2 m_g^2 K'_p K'_m\right)} = 6.67187(\pm 0.00002)\times 10^{-8}\ \mathrm{cm^3 g^{-1} s^2} \tag{4}$$

This "theoretical" value of G compares remarkably well with a recent experimental precise determination of its value by method of resonance which gives: 6.6714 ($\pm$0.0006)$\times 10^{-8}$. (Pontikis 1972) But this happy ending is not without its difficulties: Some may promptly object that if we change to another system of units (*e.g.* British system), we arrive at a different numerical value for G which is not compatible with these combination of ratios of particular masses. In the following section we shall try to show why these difficulties are unfounded.

Natural microscopic units and dimensionless constants

Let D be the ratio of the strength of the electrostatic to the gravitational forces between two electrons. This ratio is independent of distance and is assumed to be a fundamental dimensionless natural constant not subject to changes of units we may arbitrarily make (Dirac 1976). Thus, if F_e and F_g stand, respectively, for the strengths of the electric and gravitational forces, then:

$$D = \frac{F_e}{F_g} = \frac{q_a^2}{G_a m_a^2} \tag{5}$$

where q_a and m_a represent the charge and mass of electron and G_a is the gravitational constant, all expressed in the same system of units. The constant D is presumably a purely numerical ratio which is determined by some naturally fixed properties of the electron. It follows that G_a, the gravitational constant, must also have the dimensions of , *i.e.* $L^3M^{-1}T^{-2}$. Clearly, for another system of units (*e.g* the British system), although neither the numerical value of G_a nor the ratio of charge to mass in that system is the same, we must still keep:

$$\frac{q_a^2}{G_a m_a^2} = \frac{q_z^2}{G_z m_z^2} = D \tag{6}$$

where q_z, m_z and G_z are parameters as measured in different system of units. Also, since some other fundamental particles carry the same constant quantity of charge, q_a, but have different masses, we can also consider the case in the same system of units (say in cgs) but now for other combinations of particles, such as a proton-electron or proton-muon pairs, as this would also lead to a similar dimensionless natural constant. The argument is equally valid for any practical system of units (*e.g.* the cgs), if we are able to provide stable and permanently fixed charges that can be taken as a semi-”standard” natural unit. Needless to say, presently, this is rather practically difficult to realize and quite unnecessary. Nonetheless, the relative strength F_e/F_g in the cgs units (for a charge of one esu, e_s, and a one gram mass, m_g) would simply be D_m. and must have the numerical value of $1/G_a$, where G_a is the value of the gravitational constant as measured in the cgs system of units . Thus from the above we arrive at:

$$G_a = \frac{e_s^2}{m_g^2 D_m}$$

Comparing the form of G_a (or G) as given by equation (2) with this result leads to $D_m = K_p K_m$ if we assimilate e_s^2 to K_m^{-2}. Whereas, if we start with a “natural microscopic system” of units, based on the charge and mass of the electron, we arrive at:

$$\frac{e''^2}{G''m''^2} = \frac{q_a^2}{G_a m_a^2} = D$$

Since in the "natural system" we take $e'' = m'' = 1$, G'' is then the gravitational constant in these natural microscopic units. Finally, substituting for G_a and rearranging:

$$G'' = \left[\frac{e''^2}{m''^2}\right]\left[\left(\frac{m_a^2}{m_g^2}\right)\left(\frac{e_s^2}{q_a^2 4p^2 K_p' K_m'}\right)\right] \tag{7}$$

Here, e'', e_s, m'' and m_g all = 1 by definition. Therefore, G'', the gravitational constant in the natural "microscopic system of units" is numerically = $1/D = (2.39978\times 10^{-43})$. Thus, the ratios K_p and K_m are *implicit no matter how we change our units.*

A physical model for an improved understanding of the gravitational mechanism

The above analysis discloses strong self-evident inter-relations and, consequently, tempts us to consider the following tentative conclusions:

In his derivation of the gravitational expression, Newton may have used Kepler's third law constant, $K_s = R^3/T^2$, to substitute for T in his assumption that we deal with a centripetal force which attracts, say, a planet to the Sun. Thus for the force of attraction between Sun and Earth

$$F_s = \frac{M_e V^2}{R} = \frac{M_e 4p^2 R}{T}$$

he arrived at:

$$F_s = \frac{4p^2 K_s M_e}{R^2}$$

(Obviously M_e and V stand for the mass and orbital velocity of Earth; R and T are respectively radius and period of the orbital gravitational motion.) Hence, $4p^2K_s$ is a constant that depends on the gravitational properties of the Sun. Finally, to introduce the inertial mass of the Sun into the equation, he assumed $4p^2K_s = GM_s$ where M_s is the inertial mass of the Sun and G is a universal constant. In my view, the current practice of identifying m_a and m_b as they appear in Newton's expression (1) as gravitational masses, is misleading, and not rigorous. They are indeed both equal to the inertial masses by definition, and we must avoid unnecessary confusion! Obviously, it is this constant, $4p^2K_s$, which we can consider as representative of the "gravitational" property of the mass of the Sun; and is, consequently, found to be proportional to its inertial mass. Likewise, all material bodies in the universe must have a similar equivalent constant. But, if we let $G'M'_s = 4p^2K_s$, we arrive at $G'M'_s = GM_s$ where M'_s denotes the "gravitational mass" and G' is dimensionally, though not numerically, the same as G, if *gravitational* mass and *inertial* mass are assumed to be dimensionally identical. Hence, we assign to G' the numerical value of *unity*, thereby uniquely *defining gravitational mass*. Therefore, the ratio of the inertial to gravitational mass for any body, M_x, is given by

$$\frac{M_s}{M_s'} = \frac{G'}{g}$$

It is important that we relate both the gravitational and the inertial mass of the body, to the same constant *i.e* $4p^2K_p$. It follows that writing $M'_x = GM_x$, as is often done, is dimensionally wrong and may lead to false assumptions. We also expect the so-called principle of equivalence to be valid, as long as G/G' or M_x/M'_x is a universal dimensionless constant. However, after re-writing (1) and rearranging by substituting the full form of G, as outlined above, and remembering that $G'/G = 4p^2 K'_p K'_m$, we get:

$$F = \frac{4p^2 MR}{T^2} = \frac{G\left(m'_a \frac{G'}{G}\right)\left(m'_b \frac{G'}{G}\right)}{R^2}$$
$$= \frac{\left(\frac{\Psi^2}{m_g^2}\right) 4p^2 \left(m'_a K'_p\right)\left(m'_b K'_m\right)}{R^2} \quad (8)$$

We see now the reason why the $4p^2$ factor appears on the right hand side of the expression. It is, perhaps, the property of space-time-matter structure that the gravitational force acts in this way and that this numerical ratio, *M/M'* is a truly fundamental quantity.

It is interesting, and puzzling to note that the mass ratios of two fundamental particles—the proton and muon to that of the electron—come to play such a strange role. A number of investigators had already expected the proton to play such a part: but why the muon and not, say, a tauon? This intriguing question certainly merits further investigation.

References

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