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A Data Streaming Algorithm for Estimating Entropies of OD Flows

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Entropy: Definition

• Given $n$ flows of sizes $a_1, \ldots, a_n$. Let $s \equiv \sum_i a_i$. The empirical entropy is defined as

$$H \equiv - \sum_i \frac{a_i}{s} \log \left( \frac{a_i}{s} \right) = \log(s) - \frac{1}{s} \sum_i a_i \log(a_i).$$

• E.g. A stream of packets (anonymized): 1, 2, 1, 3, 2, 1, 3, 1

$$s = 8$$

$$\sum_i a_i \log(a_i) = 4 \log 4 + 2 \log 2 + 2 \log 2 = 12$$

$$H = \log(s) - \frac{1}{s} \sum_i a_i \log(a_i) = \log 8 - \frac{12}{8} = 1.5$$
OD Flow Entropy

• An OD flow is all the traffic between an ingress point and an egress point.

• Problem statement: measure the empirical entropies of all OD flows in an ISP network.

• We need to measure

  – the entropy norm $\sum_i a_i \log(a_i)$,
  – the volume of the OD flow (i.e., a traffic matrix element).
Motivation

- Entropy is a measurement of the diversity of the traffic
- Anomaly detection (profiling behavior; [Lakhina et al., 2005])
- DDoS attacks may not be detectable as simple volume changes, and may significantly increase traffic entropy across the whole network
- Entropy of OD flows may reveal more information than entropy of traffic at ingress routers
Solution Strategy

• Data streaming is concerned with processing a long stream of data items in one pass using a small working memory (called a sketch) in order to answer a class of queries regarding the stream.

• An \((\epsilon, \delta)\)-approximation algorithm for \(\theta\) is one that returns an estimate \(\hat{\theta}\) with relative error more than \(\epsilon\) with probability at most \(\delta\). That is \(\Pr[|\hat{\theta} - \theta| \geq \epsilon \theta] \leq \delta\).

• We want a solution with Intersection Measurable Property (IMP), which none of the existing entropy estimation algorithms has.

• [Indyk, 2006] stable distributions algorithm for \(L_p\) norm estimation has IMP. \(L_p\) norm of \(n\) flows of sizes \(a_1, \ldots, a_n\) is defined as \((\sum_i a_i^p)^{1/p}\).
Outline

• Indyk’s $L_p$ norm algorithm for single stream

• Using $L_p$ norm to approximate entropy norm

• Extending Indyk’s $L_p$ norm algorithm to OD flows (IMP)

• Enhancing accuracy of Indyk’s $L_p$ norm algorithm
Indyk’s $L_p$ Norm Algorithm: Big Picture

- Each packet causes increments of stable distribution values to an array of counters
- At end of epoch, $L_p$ norm can be inferred from the counter values
- More counters, more accurate the estimate.
Stable Distributions

• The p.d.f. of stable distribution $S(p), p \in (0, 2]$ is the Fourier transform of $e^{-|t|^p}$.

• $S(1)$ is the standard Cauchy distribution. $S(2)$ is the Gaussian distribution with mean 0 and standard deviation 2.

• $p$-stable property: for any constants $a_1, \ldots, a_n$ and random variables $X, X_1, \ldots, X_n$ with distribution $S(p)$

$$a_1X_1 + \ldots + a_nX_n \sim_d (|a_1|^p + \ldots + |a_n|^p)^{1/p} X$$
Indyk’s $L_p$ Norm Algorithm: Details

- For each potential flow $i$, draw a random number $X_i$ from the p-stable distribution.

- For each packet in flow $i$, increment a real-valued counter by $X_i$.

- At end of epoch, the value of the counter will be $\sum a_i X_i$, which is distributed as $(\sum a_i^p)^{1/p} X$, where $X$ is of distribution $S(p)$.

- To extract the quantity $(\sum a_i^p)^{1/p}$, do this independently for many counters, take the median of their absolute values, and divide by the median of $|X|$.
Using $L_p$ Norm to Approximate Entropy Norm

- We can approximate $x \ln x$ by linear combinations of $x^p$ for $x$ on a fixed interval $[0, N]$ within relative error $\epsilon$:

$$x \ln x \approx \frac{1}{2\alpha} (x^{1+\alpha} - x^{1-\alpha}), \text{ where } \alpha = \frac{\sqrt{6\epsilon}}{1+6\epsilon \ln N}$$

e.g. $N = 1000, \epsilon = 0.026, \alpha = 0.05$

- Proof: By Taylor expansion of $x^\alpha = e^{\alpha \ln x}$
Using $L_p$ norm to approximate Entropy Norm

- Therefore we can use $L_{1+\alpha}$ and $L_{1-\alpha}$ norms to estimate the entropy norm

\[
x \ln x \approx \frac{1}{2\alpha} \left( x^{1+\alpha} - x^{1-\alpha} \right)
\]

\[
\sum a_i \ln a_i \approx \frac{1}{2\alpha} \left( \sum a_i^{1+\alpha} - \sum a_i^{1-\alpha} \right)
\]

- In parallel, we have an elephant-detection module that handles (with high probability) all the flows of size greater than $N$. 

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Estimating $L_p$ Norm of OD Flows

- Indyk’s algorithm has Intersection Measurable Property (IMP).
  \[
  \vec{O} = \text{origin } L_p \text{ sketch}
  \]
  \[
  \vec{D} = \text{destination } L_p \text{ sketch}
  \]
  \[
  \vec{O} - \vec{D} = \text{component-wise subtraction of the sketches}
  \]

- The OD flow $L_p$ norm estimator is
  \[
  \left( \frac{\Lambda(\vec{O})^p + \Lambda(\vec{D})^p - \Lambda(\vec{O} - \vec{D})^p}{2} \right)^{1/p},
  \]
  where \( \Lambda \) is the operator to extract $L_p$ norm from sketch.

- $L_{1+\alpha}$ and $L_{1-\alpha}$ norm estimations of OD flows give us entropy estimation of OD flows.
Estimating $L_p$ Norm of OD Flows

$$\Lambda(\tilde{O})^p \approx |a_1|^p + \ldots + |a_k|^p + |b_1|^p + \ldots + |b_l|^p$$
$$\Lambda(\tilde{D})^p \approx |a_1|^p + \ldots + |a_k|^p + |c_1|^p + \ldots + |c_m|^p$$
$$\Lambda(\tilde{O} - \tilde{D})^p \approx |b_1|^p + \ldots + |b_l|^p + |-c_1|^p + \ldots + |-c_m|^p$$
$$= |b_1|^p + \ldots + |b_l|^p + |c_1|^p + \ldots + |c_m|^p$$

Hence,

$$\frac{\Lambda(\tilde{O})^p + \Lambda(\tilde{D})^p - \Lambda(\tilde{O} - \tilde{D})^p}{2} \approx |a_1|^p + \ldots + |a_k|^p.$$
Estimating $L_1$ Norm of OD Flows (Traffic Matrix)

- We can apply IMP to $L_1$ sketches

- Or we can utilize $L_{1+\alpha}$ and $L_{1-\alpha}$ norm estimations to avoid the overhead of $L_1$ sketch.

\[
x \approx \frac{1}{2} (x^{1+\alpha} + x^{1-\alpha})
\]

\[
L_1(\vec{a}) = \sum a_i \approx \frac{1}{2} \left( \sum a_i^{1+\alpha} + \sum a_i^{1-\alpha} \right)
\]
Modifications to Indyk’s Sketch

• Note that for every packet we have to perform hundreds or thousands of updates per packet (infeasible at line speeds).

• Solution:
  – Hash packets into many (thousands of) buckets.
  – Apply Indyk’s algorithm to each bucket with a small number (tens) of counters.
  – Combine the $L_p$ norm of all buckets.

• The overall relative error is much smaller than the relative error of each bucket. (Think of Central Limit Theorem.)

• We also use large lookup tables for the stable distribution RV’s.
Experiment Setup

- We collect full packet trace on a 1Gbit/s ingress link of a large Tier-1 ISP.

- We use routing table dump to figure out the flows going to different egress link.

- At egress link we add dummy flows to simulate the full egress trace.

- We run our algorithm on the ingress trace and egress trace many times, and compare with the actual OD flow entropy.

- Typical parameters: 50,000 buckets, 20 counters, $\alpha = 0.05$, elephant detection sampling rate 0.001.
Experiment Results

Varying numbers of buckets

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Experiment Results

Varying fraction of traffic from ingress

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Experiment Results

Varying fraction of traffic from ingress
Experiment Results

Varying fraction of traffic from egress traffic

Relative error vs Ratio of OD traffic to egress traffic (%)
Experiment Results

Error distribution for actual entropy
Summary

• Approximation of the entropy norm using the $L_p$ norms. (A new algorithm to estimate entropy of a single stream.)

• $L_p$ norms for OD flows. (A new algorithm for traffic matrix since it is the $L_1$ norm of OD flows.)

• The first algorithm to estimate entropy of OD flows by combining the above two.

• Improvement to Indyk’s sketch structure.
Thank you!

Questions?
References
