Effect of Inductance on Interconnect Propagation Delay in VLSI Circuits

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Outline

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- RLC transmission line model
- Short, medium and long lines
- Interconnect propagation delay models
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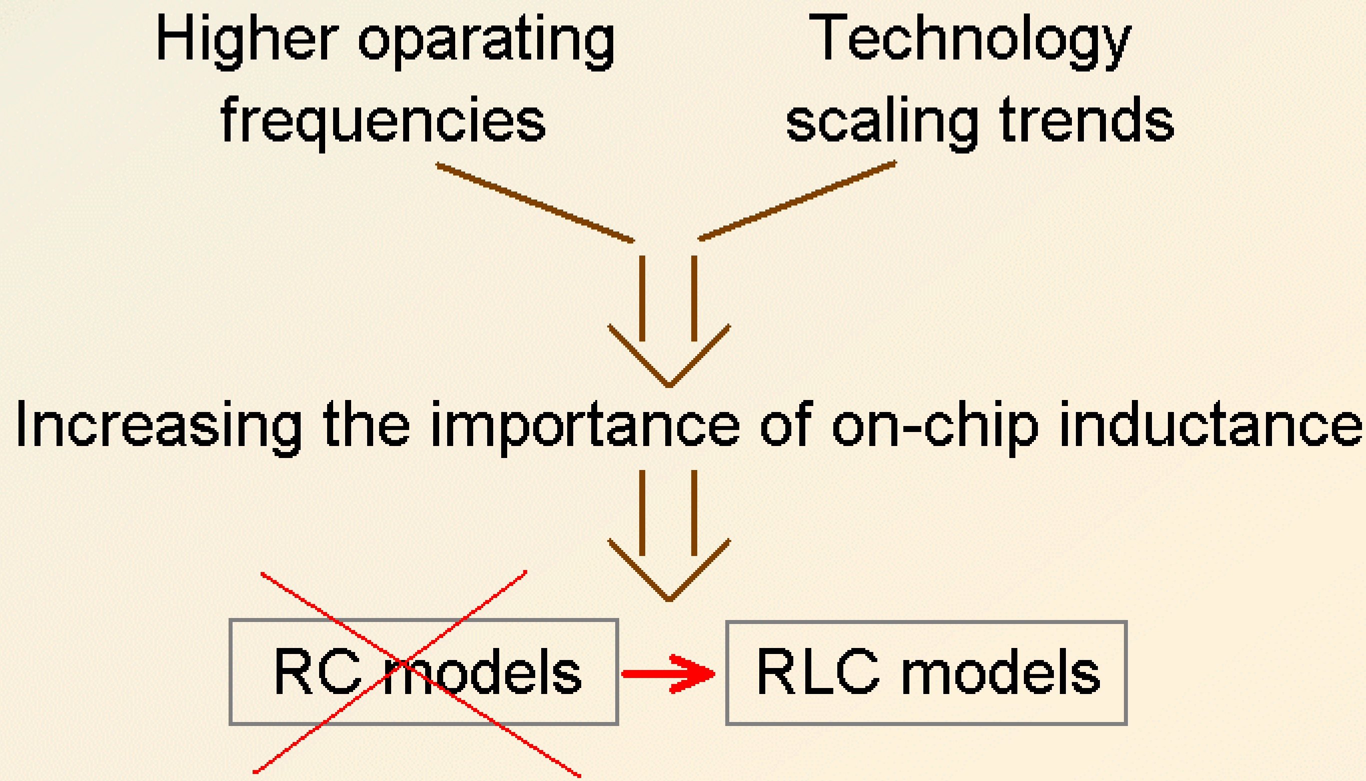
Introduction (1)



Introduction (2)

RC → RLC model conditions [2]

$$C_L << Cd \qquad \frac{Rd}{2Z_0} \leq 1 \qquad Z_{DRV} < nZ_0$$

$$n = 0.5 \div 1$$

Short, medium and long lines

- **Short lines**

$$R > 500 \frac{\Omega}{cm} \qquad l \ll \lambda \Leftrightarrow d\sqrt{LC} \ll T_r$$

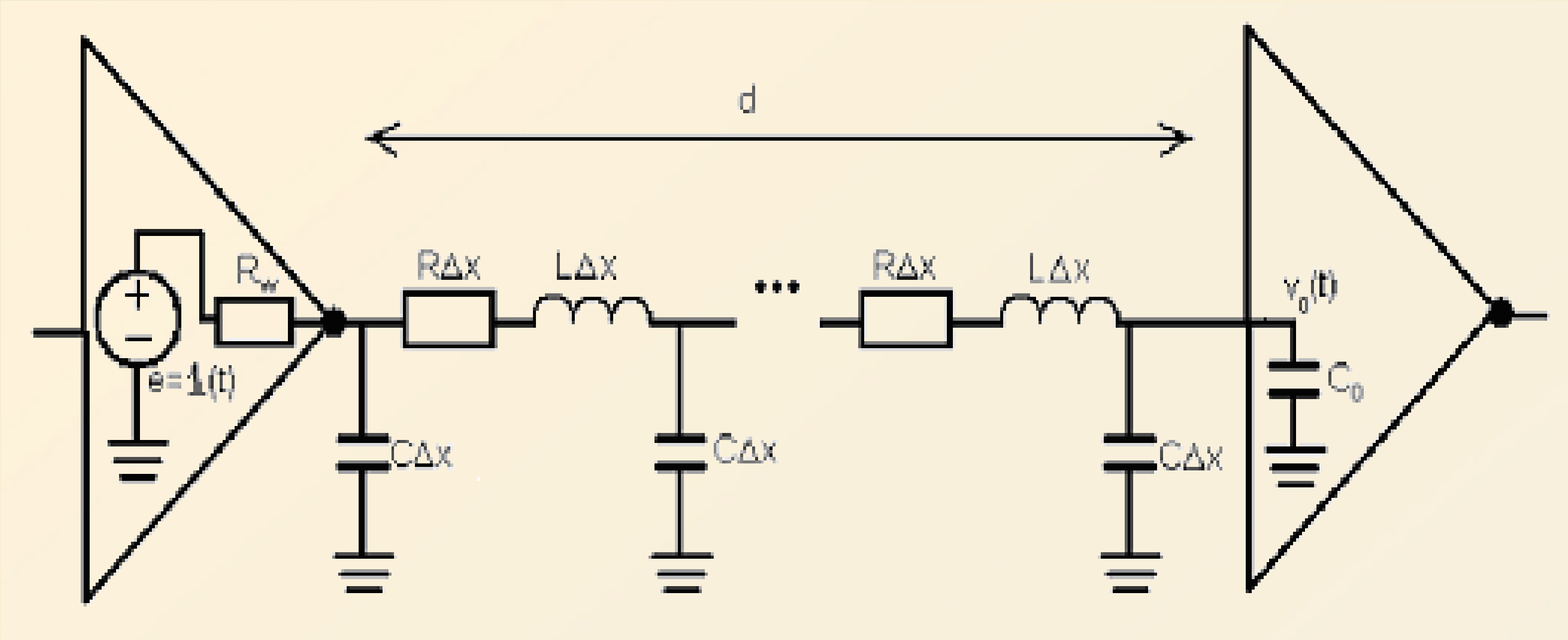
- **Medium lines**

$$R < 500 \frac{\Omega}{cm} \qquad l < \lambda \Leftrightarrow d\sqrt{LC} < T_r$$

- **Long lines**

$$R < 100 \frac{\Omega}{cm} \qquad l \approx \lambda \Leftrightarrow d\sqrt{LC} \approx T_r$$

RLC transmission line model



Interconnect propagation delay models

- **Ismail-Friedman**

$$T_{50\%} = \frac{e^{-2.9\zeta^{1.35}} + 1.48\zeta}{\omega_n}$$

$$\zeta = \zeta_{line} \frac{R_T + C_T + R_T C_T + 0.5}{\sqrt{1 + C_T}}$$

$$\zeta_{line} = \frac{R_t}{2}\sqrt{\frac{C_t}{L_t}}, \quad R_T = \frac{R_w}{R_t}, \quad C_T = \frac{C_0}{C_t}, \quad \omega_n = \frac{1}{\sqrt{L_t(C_t + C_0)}},$$

for $0 < C_T < 1, \quad 0 < R_T < 1$

R_t, L_t, C_t – total resistance, inductance and capacitance of transmission line (R_t = Rd where d-line length, etc.).

Proposed solution (1)

Exact formula for voltage across Co (1)

$$U_2(s) = H(s)\frac{1}{s}$$

$$H(s) = \frac{Z_c(s)}{Z_c(s)+R_w} \quad \frac{\left(1+\rho_o(s)\right)\exp(-\gamma(s)d)}{1-\rho_o(s)\rho_w(s)\exp(-2\gamma(s)d)}$$

$\rho_w(s)$, $\rho_o(s)$ – input and output reflection coefficients respectively,
$Z_c(s)$ – characteristic impedance of the line,
$\gamma(s)$ - propagation coefficient,
d – length of the line

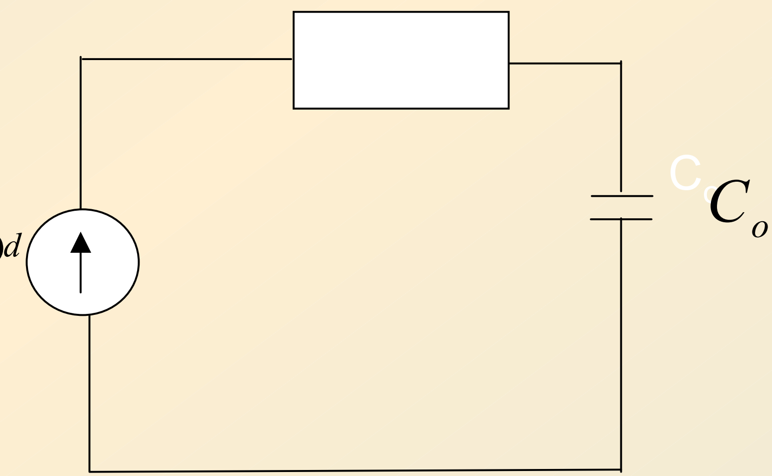
$$H_0(s) = \frac{Z_c(s)}{Z_c(s)+R_w}\left(1+\rho_o(s)\right)\exp(-\gamma(s)d) \qquad 0 \le t < 3T$$

Proposed solution (1b)

C

$$Z_c(s) = Z_o\sqrt{1+\frac{R}{sL}}$$

$$E_w(s) = \frac{2E_o}{s}\,\frac{Z_c(s)}{Z_c(s)+R_w}\,e^{-\gamma(s)d}$$



Proposed solution (2)

Exact formula for voltage across Co (2)

$$U_{20}(p)=\frac{2e^{\frac{-p}{\sqrt{1+C_T}}}\exp\left(-\frac{p}{\sqrt{1+C_T}}\left(F(p)-1\right)\right)}{p\left[1+p\frac{2R_T C_T \zeta_{line}}{\sqrt{1+C_T}}+pF(p)\frac{C_T}{\sqrt{1+C_T}}+\frac{2R_T\zeta_{line}}{F(p)}\right]}$$

where

$$F(p)=\sqrt{1+\frac{2\zeta_{line}\sqrt{1+C_T}}{p}}, \qquad p=\frac{s}{\omega_n}$$

Proposed solution (3)

Approximation of the voltage across Co

$$U_{2a}(p) = e^{\frac{-p}{\sqrt{1+C_T}}}\left[a_0 + a_1\frac{1}{p} + a_2\frac{1}{p^2} + a_3\frac{1}{p^3}\ldots\right]$$

where

$$a_n = \frac{1}{n!}\frac{d^n U_{2a}(z)}{dz^n}\bigg|_{z=0} \qquad z = \frac{1}{p}.$$

Proposed solution (4)

Approximation coefficients

$$a_0 = a_1 = 0$$

$$a_3 = a_2\left[\frac{1}{2}\zeta_{line}^2 - \frac{\left((2R_T + C_T)\zeta_{line} + 1\right)}{(2R_T\zeta_{line} + 1)C_T}\right] \qquad a_2 = \frac{\exp(-\zeta_{line})\sqrt{1+C_T}}{2R_T C_T \zeta_{line} + C_T}$$

$$a_4 = a_2\left[\begin{array}{l}\frac{1}{2}\zeta_{line}^3\left(\left(\frac{1}{4}\zeta_{line} - 1\right)(1+C_T)\right) - \frac{\zeta_{line}^2(1+C_T)}{(2R_T\zeta_{line}+1)C_T}\left(\frac{C_T}{2} + 2R_T\right) + \\ (1+C_T)\left(\begin{array}{l}2R_T C_T \zeta_{line}^{\ 2} + C_T\zeta_{line} \\ -2 - 2C_T\zeta_{line} - 4R_T\end{array}\right) \cdot \frac{\zeta_{line}\left(1+\zeta_{line}(C_T + R_T)\right)}{2C_T^{\ 2}(2R_T\zeta_{line}+1)^2}\end{array}\right]$$

Proposed solution (5)

Aproximate formula for voltage across Co (3)

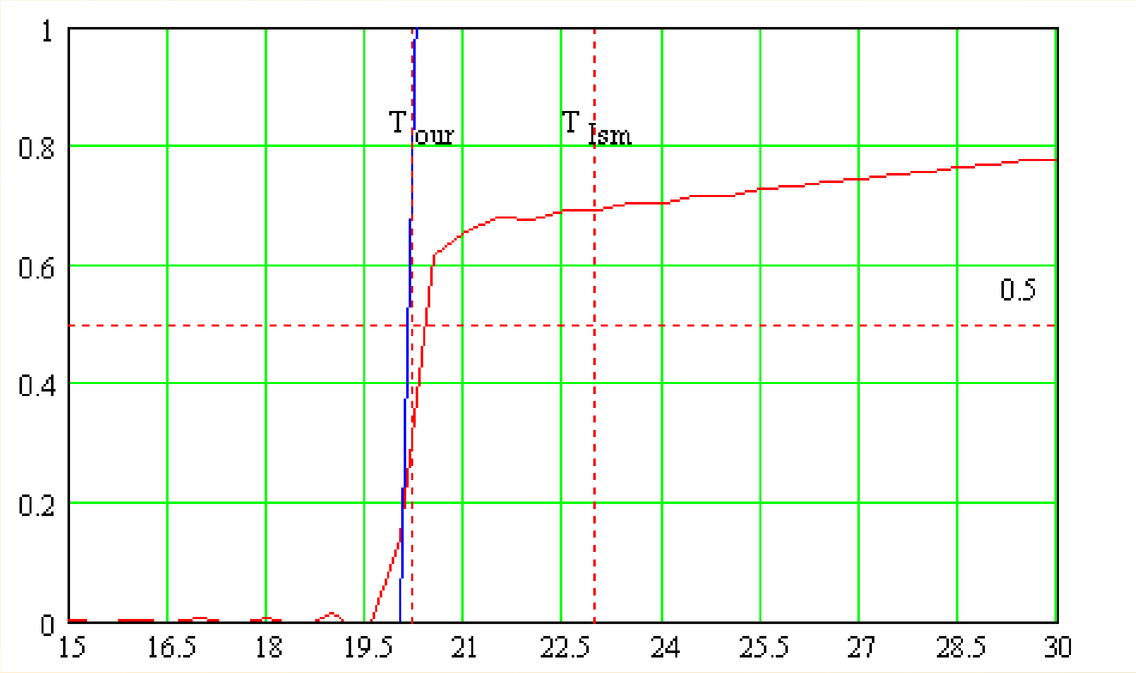
$$u_{2a}(\tau)=2\left[a_2\left(\tau-\frac{1}{\sqrt{1+C_T}}\right)+\frac{a_3}{2}\left(\tau-\frac{1}{\sqrt{1+C_T}}\right)^2+\frac{a_3}{6}\left(\tau-\frac{1}{\sqrt{1+C_T}}\right)^3+\ldots\right]$$

$$u_{2a}(T_{our})=\frac{1}{2}$$

$$\tau=t\omega_n$$

Simulation results (1)

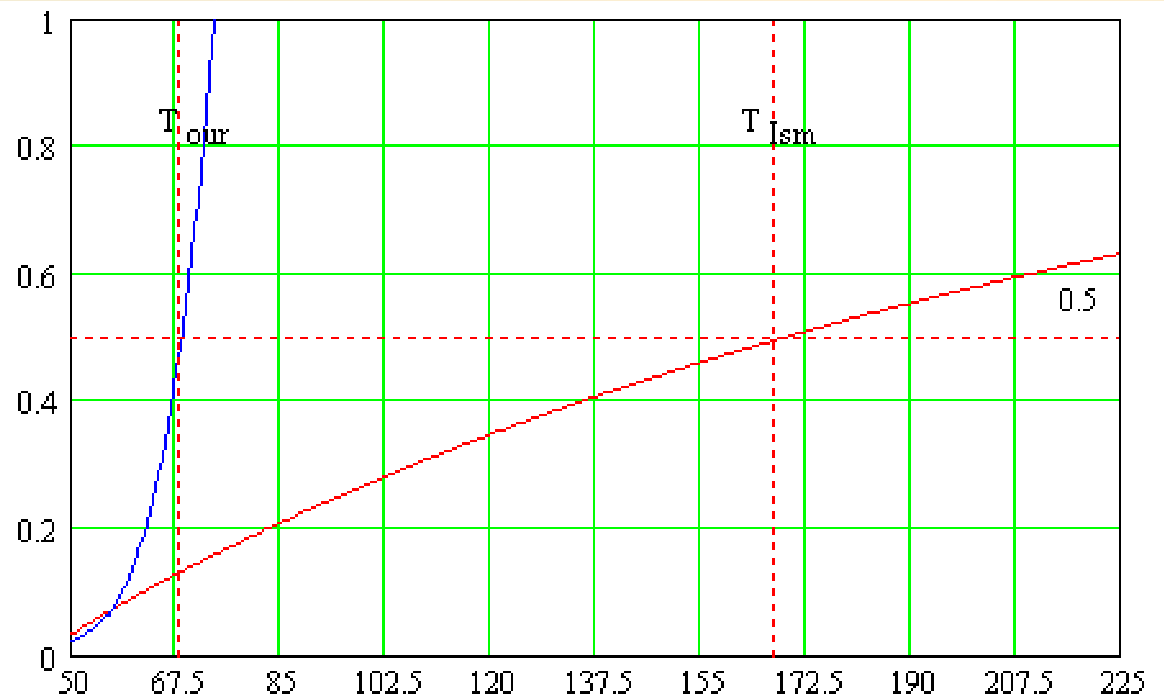
Ismail-Friedman method examination



Parameters: R=100[Ohm], L=1.6[nH/cm], C=1[pF/cm], Zdrv=25[Ohm] d=0.5[cm], (C_T=0.01, R_T=0.5$\Rightarrow$$C_o$=0.01pF); data taken from [2]

Simulation results (1)

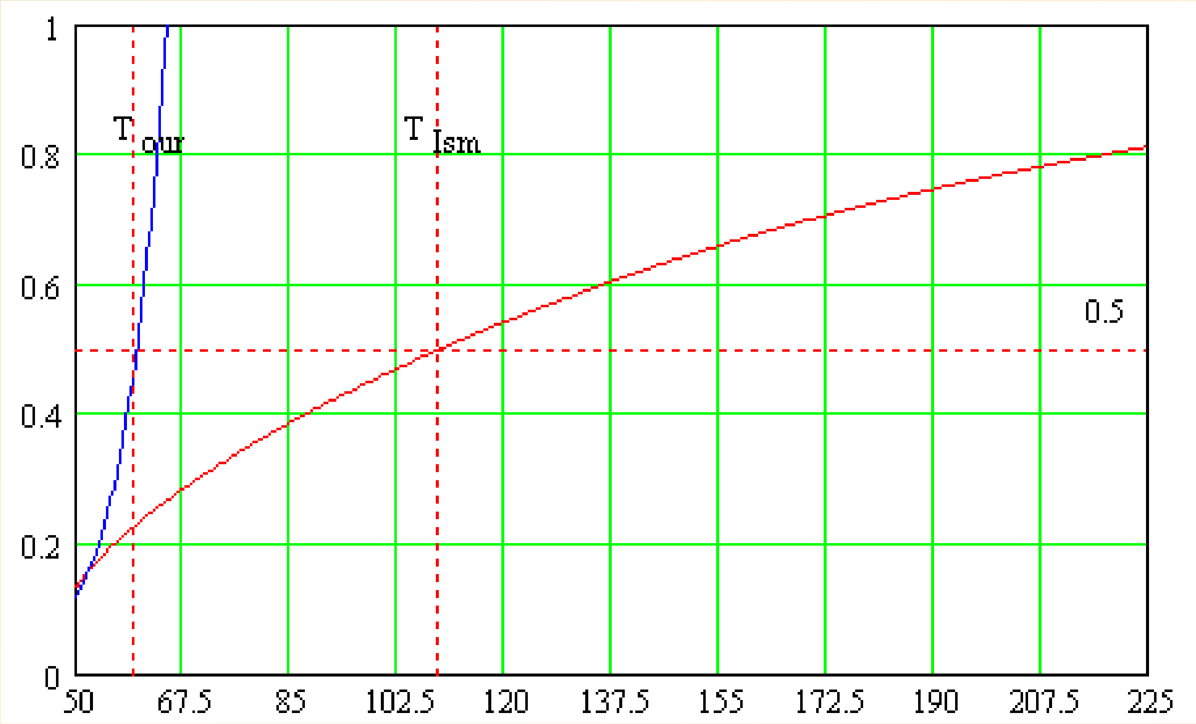
Short lines



Parameters: R=494[Ohm], L=4.75[nH/cm], C=1.73[pF/cm], Zdrv=50[Ohm] d=0.5[cm], data taken from [1]

Simulations results (2)

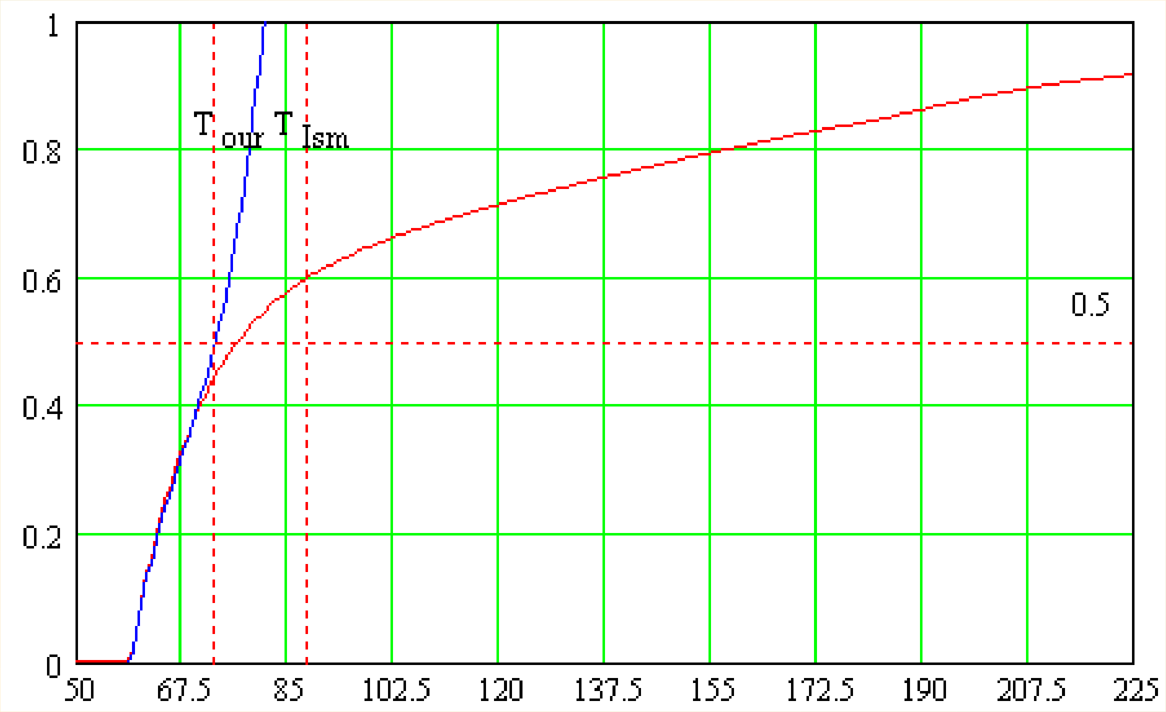
Medium lines



Parameters: R=248[Ohm], L=3.7[nH/cm], C=1.85[pF/cm], Zdrv=50[Ohm], d=0.5[cm], date taken from [1]

Simulations results (3)

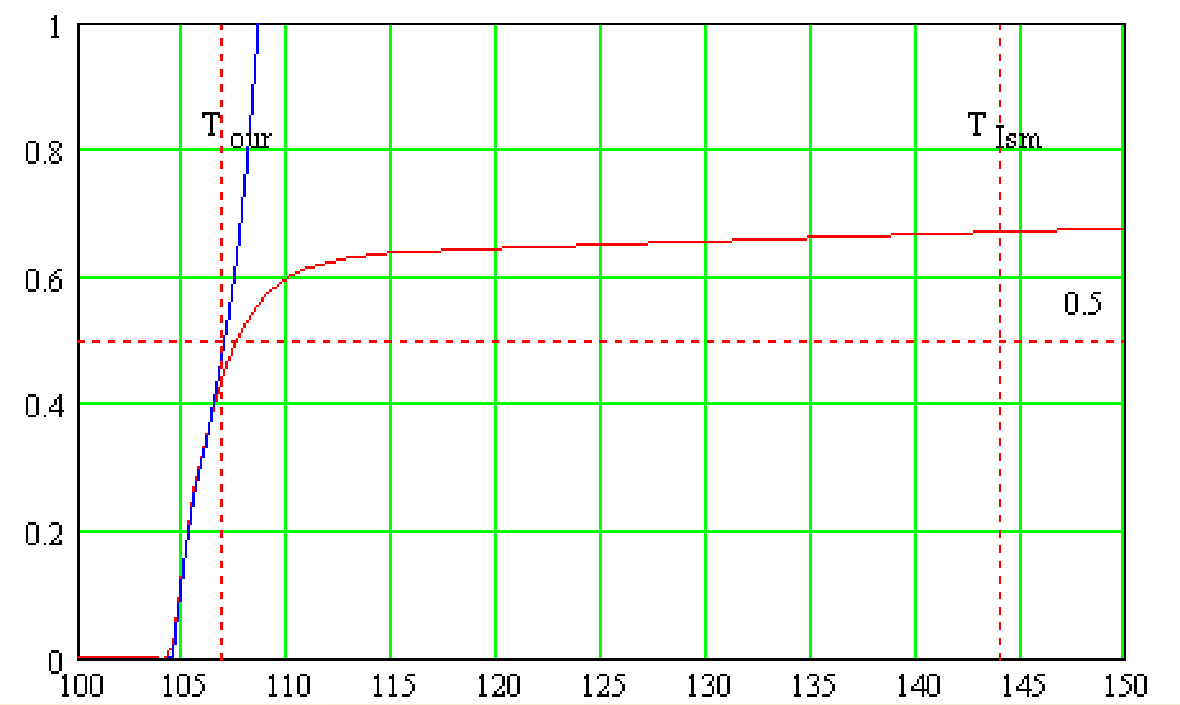
Long lines (1)



Parameters: R=76[Ohm], L=5.3[nH/cm], C=2.6[pF/cm], Zdrv=50[Ohm], d=0.5[cm], data taken from [1]

Simulations results (1)

Long lines (2)



Parameters: R=8[Ohm], L=1.6[nH/cm], C=27.3[pF/cm], Zdrv=11[Ohm], d=0.5[cm], data taken from [1]

Conclusions

- Moments of the voltage step response can be calculated in analytical way relatively easy
- The closed-form Ismail formula for propagation delay gives inaccurate results in some cases (long lines)
- Our approach gives better results for long lines.
- The accuracy of our approach can be increased by taking more terms in series

References

1. Y.I.Ismail, E.G.Friedman, Effects of inductance on the propagation delay and repeater insertion in VLSI circuits, IEEE Tran. VLSI Sys., vol. 8, No. 2, April 2000, pp. 195-206.
2. A. Deutsch *et al*., When are transmission-line effects important for on-chip interconnections, IEEE Tran.on MTT., vol. 45, No. 10, Octob.1997, pp. 1836-1846.
3. K.Banerjee, A.Mehrotra, Analysis on-chip inductance effects for distributed RLC interconnects, IEEE Tran. On CAD of Integr. Circuits and Syst., vol.16, No. 8, august 2002, pp. 904-915.
4. B. Chen, H.Young, R.Luo, H.Wang, A novel method for worst-case interconnect estimation, IEEE CAS-I, vol. 50, No.6, June 2003, pp. 778-781.
5. A.K.Palita, W. Anheier, J.Schloeffel, Estimation of signal integrity loss through reduced order interconnect model, Proc. of 7th Workshop on Signal Propagation on Interconnects (SPI), Siena, Italy, 11-14 May 2003, pp.163-166.