PI and PID controller tuning rules for time delay processes: a summary

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Abstract: The ability of proportional integral (PI) and proportional integral derivative (PID) controllers to compensate many practical industrial processes has led to their wide acceptance in industrial applications. The requirement to choose either two or three controller parameters is perhaps most easily done using tuning rules. A summary of tuning rules for the PI and PID control of single input, single output (SISO) processes with time delay are provided in this report. **Inevitably, this report is a work in progress and will be added to and extended regularly**.

Keywords: PI, PID, tuning rules, time delay.

1. Introduction

The ability of PI and PID controllers to compensate most practical industrial processes has led to their wide acceptance in industrial applications. Koivo and Tanttu [1], for example, suggest that there are perhaps 5-10% of control loops that cannot be controlled by SISO PI or PID controllers; in particular, these controllers perform well for processes with benign dynamics and modest performance requirements [2, 3]. It has been stated that 98% of control loops in the pulp and paper industries are controlled by SISO PI controllers [4] and that, in process control applications, more than 95% of the controllers are of PID type [3]. The PI or PID controller implementation has been recommended for the control of processes of low to medium order, with small time delays, when parameter setting must be done using tuning rules and when controller synthesis is performed either once or more often [5]. However, Ender [6] states that, in his testing of thousands of control loops in hundreds of plants, it has been found that more than 30% of installed controllers are operating in manual mode and 65% of loops operating in automatic mode produce less variance in manual than in automatic (i.e. the automatic controllers are poorly tuned); this is rather sobering, considering the wealth of information available in the literature for determining controller parameters automatically. It is true that this information is scattered throughout papers and books; the purpose of this paper is to bring together in summary form the tuning rules for PI and PID controllers that have been developed to compensate SISO processes with time delay. Tuning rules for the variations that have been proposed in the 'ideal' PI and PID controller structure are included. Considerable variations in the ideal PID controller structure, in particular, are encountered; these variations are explored in more detail in Section 2.

2. PID controller structures

The ideal continuous time domain PID controller for a SISO process is expressed in the Laplace domain as follows:

$$U(s) = G_c(s)E(s) \tag{1}$$

with
$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right) \tag{2}$$

and with K_c = proportional gain, T_i = integral time constant and T_d = derivative time constant. If $T_i = \infty$ and $T_d = 0$ (i.e. P control), then it is clear that the closed loop measured value, y, will always be less than the desired value, r (for processes without an integrator term, as a positive error is necessary to keep the measured value constant, and less than the desired value). The introduction of integral action facilitates the achievement of equality between the measured value and the desired value, as a constant error produces an increasing controller output. The introduction of derivative action means that changes in the desired value may be anticipated, and

thus an appropriate correction may be added prior to the actual change. Thus, in simplified terms, the PID controller allows contributions from present controller inputs, past controller inputs and future controller inputs.

Many tuning rules have been defined for the ideal PI and PID structures. Tuning rules have also been defined for other PI and PID structures, as detailed in Section 4.

3. Process modelling

Processes with time delay may be modelled in a variety of ways. The modelling strategy used will influence the value of the model parameters, which will in turn affect the controller values determined from the tuning rules. The modelling strategy used in association with each tuning rule, as described in the original papers, is indicated in the tables. Of course, it is possible to use the tuning rules proposed by the authors with a different modelling strategy than that proposed by the authors; applications where this occurs are not indicated (to date). The modelling strategies are referenced as indicated. The full details of these modelling strategies are provided in Appendix 2.

A. First order lag plus delay (FOLPD) model ($G_m(s) = \frac{K_m e^{-s\tau_m}}{1+sT_m}$):

Method 1: Parameters obtained using the tangent and point method (Ziegler and Nichols [8], Hazebroek and Van den Waerden [9]); Appendix 2.

Method 2: K_m, τ_m assumed known; T_m estimated from the open loop step response (Wolfe [12]); Appendix 2.

Method 3: Parameters obtained using an alternative tangent and point method (Murrill [13]); Appendix 2.

Method 4: Parameters obtained using the method of moments (Astrom and Hagglund [3]); Appendix 2.

Method 5: Parameters obtained from the closed loop transient response to a step input under proportional control (Sain and Ozgen [94]); Appendix 2.

Method 6: K_m, T_m, τ_m assumed known.

Method 7: Parameters obtained using a least squares method in the time domain (Cheng and Hung [95]); Appendix 2.

Method 8: Parameters obtained in the frequency domain from the ultimate gain, phase and frequency determined using a relay in series with the closed loop system in a master feedback loop. The model gain is obtained by the ratio of the integrals (over one period) of the process output to the controller output. The delay and time constant are obtained from the frequency domain data (Hwang [160]).

Method 9: Parameters obtained from the closed loop transient response to a step input under proportional control (Hwang [2]); Appendix 2.

Method 10: Parameters obtained from two points estimated on process frequency response using a relay and a relay in series with a delay (Tan *et al*. [39]); Appendix 2.

Method 11: T_m and τ_m are determined from the ultimate gain and period estimated using a relay in series with the process in closed loop; K_m assumed known (Hang and Cao [112]); Appendix 2.

Method 12: Parameters are estimated using a tangent and point method (Davydov *et al*. [31]); Appendix 2.

Method 13: Parameters estimated from the open loop step response and its first time derivative (Tsang and Rad [109]); Appendix 2.

Method 14: T_m and τ_m estimated from K_u, T_u determined using Ziegler-Nichols ultimate cycle method; K_m estimated from the process step response (Hang *et al*. [35]); Appendix 2.

Method 15: T_m and τ_m estimated from K_u, T_u determined using a relay autotuning method; K_m estimated from the process step response (Hang *et al*. [35]); Appendix 2.

Method 16: $|G_p(j\omega_{135^0})|$, ω_{135^0} and K_m are determined from an experiment using a relay in series with the process in closed loop; estimates for T_m and τ_m are subsequently calculated. (Voda and Landau [40]); Appendix 2.

Method 17: Parameter estimates back-calculated from discrete time identification method (Ferretti *et al*. [161]); Appendix 2.

* Method 18: Parameter estimates calculated from process reaction curve using numerical integration procedures (Nishikawa *et al*. [162]).

* Method 19: Parameter estimates determined graphically from a known higher order process (McMillan [58] *… also McMillan (1983), pp. 34-40*.

* Method 20: K_m estimated from the open loop step response. $T_{90\%}$ and τ_m estimated from the closed loop step response under proportional control (Astrom and Hagglund [93]?)

Method 21: Parameters estimated from linear regression equations in the time domain (Bi *et al*. [46]); Appendix 2.

Method 22: T_m and τ_m estimated from relay autotuning method (Lee and Sung [163]); K_m estimated from the closed loop process step response under proportional control (Chun *et al*. [57]); Appendix 2.

* Method 23: Parameters are estimated from a step response autotuning experiment – Honeywell UDC 6000 controller (Astrom *et al*. [30]).

Method 24: Parameters are estimated from the closed loop step response when process is in series with a PID controller (Morilla *et al*. [104a]); Appendix 2.

Method 25: τ_m and T_m obtained from an open loop step test as follows: $T_m = 1.4(t_{67\%} - t_{33\%})$, $\tau_m = t_{67\%} - 1.1T_m$. K_m assumed known (Chen and Yang [23a]).

Method 26: τ_m and T_m obtained from an open loop step test as follows: $T_m = 1.245(t_{70\%} - t_{33\%})$, $\tau_m = 1.498t_{33\%} - 0.498t_{70\%}$. K_m assumed known (Miluse *et al*. [27b]).

* Method 27: Data at the ultimate period is deduced from an open loop impulse response (Pi-Mira *et al*. [97a]).

B. Non-model specific

Method 1: Parameters K_u, K_m, ω_u are estimated from data obtained using a relay in series with the process in closed loop and from the process step response (Kristiansson and Lennartsson [157]).

need to check how the other methods define these parameters –

C. Integral plus time delay (IPD) model ($G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$)

Method 1: τ_m assumed known; K_m determined from the slope at start of the open loop step response (Ziegler and Nichols [8]); Appendix 2.

Method 2: K_m, τ_m assumed known.

Method 3: Parameters estimated from the ultimate gain and frequency values determined from an experiment using a relay in series with the process in closed loop (Tyreus and Luyben [75]); Appendix 2.

Method 4: Parameters are estimated from the servo or regulator closed loop transient response, under PI control (Rotach [77]); Appendix 2.

Method 5: Parameters are estimated from the servo closed loop transient response under proportional control (Srividya and Chidambaram [80]); Appendix 2.

Method 6: K_u and T_u are estimated from estimates of the ultimate and crossover frequencies. The ultimate frequency estimate is obtained by placing an amplitude dependent gain in series with the process in closed loop; the crossover frequency estimate is obtained by also using an amplitude dependent gain (Pecharroman and Pagola [165]); Appendix 2.

D. First order lag plus integral plus time delay (FOLIPD) model ($G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$)

* Method 1: Method of moments (Astrom and Hagglund [3]).

Method 2: K_m, T_m, τ_m assumed known.

Method 3: Parameters estimated from the open loop step response and its first and second time derivatives (Tsang and Rad [109]); Appendix 2.

Method 4: K_u and T_u are estimated from estimates of the ultimate and crossover frequencies (Pecharroman and Pagola [165]) – as in Method 6, IPD model.

E. Second order system plus time delay (SOSPD) model ($G_m(s) = \frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}, \frac{K_m e^{-s\tau_m}}{(1+T_{m1}s)(1+T_{m2}s)}$)

Method 1: $K_m, T_{m1}, T_{m2}, \tau_m$ or $K_m, T_{m1}, \xi_m, \tau_m$ assumed known.

Method 2: Parameters estimated using a two-stage identification procedure involving (a) placing a relay in series with the process in closed loop and (b) placing a proportional controller in series with the process in closed loop (Sung *et al*. [139]); Appendix 2.

* Method 3: Parameters obtained in the frequency domain from the ultimate gain, phase and frequency determined using a relay in series with the closed loop system in a master feedback loop. The model gain is obtained by the ratio of the integrals (over one period) of the process output to the controller output. The other parameters are obtained from the frequency domain data (Hwang [160]).

Method 4: T_m and τ_m estimated from K_u, T_u determined using a relay autotuning method; K_m estimated from the process step response (Hang *et al*. [35]); Appendix 2.

* Method 5: Parameter estimates back-calculated from discrete time identification method (Ferretti *et al*. [161]).

Method 6: Parameteres estimated from the underdamped or overdamped transient response in open loop to a step input (Jahanmiri and Fallahi [149]); Appendix 2.

* Method 7: Parameters estimated from a least squares time domain method (Lopez *et al*. [84]).

Method 8: Parameters estimated from data obtained when the process phase lag is -90^0 and -180^0, respectively (Wang *et al*. [143]); Appendix 2.

* Method 9: Parameter estimates back-calculated from discrete time identification method (Wang and Clements [147]).

Method 10: K_m, T_{m1} and τ_m are determined from the open loop time domain Ziegler-Nichols response (Shinskey [16], page 151); T_{m2} assumed known.

Method 11: Parameters estimated from two points determined on process frequency response using a relay and a relay in series with a delay (Tan *et al*. [39]); Appendix 2.

* Method 12: Parameter estimated back-calculated from discrete time identification method (Lopez *et al*. [84]).

* Method 13: Parameters estimated from a step response autotuning experiment – Honeywell UDC 6000 controller (Astrom *et al*. [30]).

Method 14: $T_{m1} = T_{m2}$. τ_m and T_{m1} obtained from an open loop step test as follows: $T_{m1} = 0.794(t_{70\%} - t_{33\%})$, $\tau_m = 1.937\, t_{33\%} - 0.937\, t_{70\%}$. K_m assumed known (Miluse *et al*. [27b]).

Method 15: K_u and T_u are estimated from estimates of the ultimate and crossover frequencies (Pecharroman and Pagola [165]) – as in Method 6, IPD model.

F. <u>Integral squared plus time delay (I^2PD) model</u> ($G_m(s) = \frac{K_m e^{-s\tau_m}}{s^2}$)

Method 1: K_m, T_m, τ_m assumed known.

<u>G. Second order system (repeated pole) plus integral plus time delay (SOSIPD) model</u> ($G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)^2}$)

Method 1: K_u and T_u are estimated from estimates of the ultimate and crossover frequencies (Pecharroman and Pagola [165]) – as in Method 6, IPD model.

Method 2: K_m, T_m, τ_m assumed known.

H. <u>Third order system plus time delay (TOLPD) model</u> ($G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})(1+sT_{m3})}$).

Method 1: $K_m, T_{m1}, T_{m2}, T_{m3}, \tau_m$ known.

I. <u>Unstable first order lag plus time delay model</u> ($G_m(s) = \frac{K_m e^{-s\tau_m}}{1 - sT_m}$)

Method 1: K_m, T_m, τ_m known.

Method 2: The model parameters are obtained by least squares fitting from the open loop frequency response of the unstable process; this is done by determining the closed loop magnitude and phase values of the (stable) closed loop system and using the Nichols chart to determine the open loop response (Huang and Lin [154], Deshpande [164]).

J. Unstable second order system plus time delay model ($G_m(s) = \dfrac{K_m e^{-s\tau_m}}{(1 - sT_{m1})(1 + sT_{m2})}$)

Method 1: $K_m, T_{m1}, T_{m2}, \tau_m$ known.

Method 2: The model parameters are obtained by least squares fitting from the open loop frequency response of the unstable process; this is done by determining the closed loop magnitude and phase values of the (stable) closed loop system and using the Nichols chart to determine the open loop response (Huang and Lin [154], Deshpande [164]).

K. Second order system plus time delay model with a positive zero ($G_m(s) = \dfrac{K_m(1 - sT_{m3})e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})}$)

Method 1: $K_m, T_{m1}, T_{m2}, T_{m3}, \tau_m$ known.

L. Second order system plus time delay model with a negative zero ($G_m(s) = \dfrac{K_m(1 + sT_{m3})e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})}$)

Method 1: $K_m, T_{m1}, T_{m2}, T_{m3}, \tau_m$ known.

M. Fifth order system plus delay model ($G_m(s) = \dfrac{K_m(1 + b_1 s + b_2 s^2 + b_3 s^3 + b_4 s^4 + b_5 s^5)e^{-s\tau_m}}{(1 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5)}$)

Method 1: $K_m, b_1, b_2, b_3, b_4, b_5, a_1, a_2, a_3, a_4, a_5, \tau_m$ known.

N. General model with a repeated pole ($G_m(s) = \dfrac{K_m e^{-s\tau_m}}{(1 + sT_m)^n}$)

* Method 1: Strejc's method

O. General stable non-oscillating model with a time delay

P. Delay model ($G_m(s) = e^{-s\tau_m}$)

Note: * means that the procedure has not been fully described to date.

4. Organisation of the report

The tuning rules are organised in tabular form, as is indicated in the list of tables below. Within each table, the tuning rules are classified further; the main subdivisions made are as follows:

(i) Tuning rules based on a measured step response (also called process reaction curve methods).

(ii) Tuning rules based on minimising an appropriate performance criterion, either for optimum regulator or optimum servo action.

(iii) Tuning rules that gives a specified closed loop response (direct synthesis tuning rules). Such rules may be defined by specifying the desired poles of the closed loop response, for instance, though more generally, the desired closed loop transfer function may be specified. The definition may be expanded to cover techniques that allow the achievement of a specified gain margin and/or phase margin.

(iv) Robust tuning rules, with an explicit robust stability and robust performance criterion built in to the design process.

(v) Tuning rules based on recording appropriate parameters at the ultimate frequency (also called ultimate cycling methods).

(vi) Other tuning rules, such as tuning rules that depend on the proportional gain required to achieve a quarter decay ratio or magnitude and frequency information at a particular phase lag.

Some tuning rules could be considered to belong to more than one subdivision, so the subdivisions cannot be considered to be mutually exclusive; nevertheless, they provide a convenient way to classify the rules. Tuning rules for the variations that have been proposed in the ‘ideal’ PI and PID controller structure are included in the appropriate table. In all cases, one column in the tables summarise the conditions under which the tuning rules are designed to operate, if appropriate ($Y(s)$ = closed loop system output, $R(s)$ = closed loop system input).

Tables 1-3: PI tuning rules – FOLPD model - $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{1+sT_m}$

Table 1: Ideal controller $G_c(s) = K_c\left(1+\dfrac{1}{T_i s}\right)$. Eighty-five such tuning rules are defined; the references are

(a) Process reaction methods: Ziegler and Nichols [8], Hazebroek and Van der Waerden [9], Astrom and Hagglund [3], Chien *et al*. [10], Cohen and Coon [11], Wolfe [12], Murrill [13] – *page 356*, McMillan [14] – *page 25*, St. Clair [15] – *page 22* and Shinskey [15a]. Twelve tuning rules are defined.

(b) Performance index minimisation (regulator tuning): Minimum IAE - Murrill [13] – *pages 358-363*, Shinskey [16] – *page 123*, ** Shinskey [17], Huang *et al*. [18], Yu [19]. Minimum ISE - Hazebroek and Van der Waerden [9], Murrill [13]– *pages 358-363*, Zhuang and Atherton [20], Yu [19]. Minimum ITAE - Murrill [13] – *pages 358-363*, Yu [19]. Minimum ISTSE - Zhuang and Atherton [20]. Minimum ISTES – Zhuang and Atherton [20]. Thirteen tuning rules are defined.

(c) Performance index minimisation (servo tuning): Minimum IAE – Rovira *et al*. [21], Huang *et al*. [18]. Minimum ISE - Zhuang and Atherton [20], han and Lehman [22]. Minimum ITAE - Rovira *et al*. [21]. Minimum ISTSE - Zhuang and Atherton [20]. Minimum ISTES – Zhuang and Atherton [20]. Seven tuning rules are defined.

(d) Direct synthesis: Haalman [23], Chen and Yang [23a], Pemberton [24], Smith and Corripio [25], Smith *et al*. [26], Hang *et al*. [27], Miluse *et al*. [27a], Gorecki *et al*. [28], Chiu *et al*. [29], Astrom *et al*. [30], Davydov *et al*. [31], Schneider [32], McAnany [33], Leva *et al*. [34], Khan and Lehman [22], Hang *et al*. [35, 36], Ho *et al*. [37], Ho *et al* [104], Tan *et al*. [39], Voda and Landau [40], Friman and Waller [41], Smith [42], Cox *et al*. [43], Cluett and Wang [44], Abbas [45], Bi *et al*. [46], Wang and Shao [47]. Thirty-one tuning rules are defined.

(e) Robust: Brambilla *et al*. [48], Rivera *et al*. [49], Chien [50], Thomasson [51], Fruehauf *et al*. [52], Chen *et al*. [53], Ogawa [54], Lee *et al*. [55], Isaksson and Graebe [56], Chun *et al*. [57]. Ten tuning rules are defined.

(f) Ultimate cycle: McMillan [58], Shinskey [59] – *page 167*, **Shinskey [17], Shinskey [16] – *page 148*, Hwang [60], Hang *et al*. [65], Zhuang and Atherton [20], Hwang and Fang [61]. Twelve tuning rules are defined.

Table 2: Controller $G_c(s) = K_c\left(b+\dfrac{1}{T_i s}\right)$ Two direct synthesis tuning rules are defined by Astrom and Hagglund [3] - *page 205-208*.

Table 3: Two degree of freedom controller: $U(s) = K_c\left(1+\dfrac{1}{T_i s}\right)E(s) - \alpha K_c R(s)$. One performance index minimisation tuning rule is defined by Taguchi and Araki [61a].

Tables 4-7: PI tuning rules - non-model specific

Table 4: Controller $G_c(s) = K_c\left(1+\dfrac{1}{T_i s}\right)$. Nineteen such tuning rules are defined; the references are:

(a) Ultimate cycle: Ziegler and Nichols [8], Hwang and Chang [62], ** Hang *et al*. [36], McMillan [14] – *page 90*, Pessen [63], Astrom and Hagglund [3] – *page 142*, Parr [64] – *page 191*, Yu [122] – *page 11*. Seven tuning rules are defined.

(b) Other tuning rules: Parr [64] – *page 191*, McMillan [14] – *pages 42-43*, Parr [64] – *page 192*, Hagglund and Astrom [66], Leva [67], Astrom [68], Calcev and Gorez [69], Cox *et al*. [70]. Eight tuning rules are defined.

(c) Direct synthesis: Vrancic *et al*. [71], Vrancic [72], Friman and Waller [41], Kristiansson and Lennartson [158a]. Four tuning rules are defined.

Table 5: Controller $G_c(s) = K_c\left(b + \frac{1}{T_i s}\right)$. One direct synthesis tuning rule is defined by Astrom and Hagglund [3] – *page 215*.

Table 6: Controller $U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) + K_c(b-1)R(s)$. One direct synthesis tuning rule is defined by Vrancic [72].

Table 7: Controller $U(s) = K_c Y(s) - \frac{K_c}{T_i s}E(s)$. One direct synthesis tuning rule is defined by Chien *et al*. [74].

Tables 8-11: PI tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Table 8: Controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)$. Twenty such tuning rules are defined; the references are:

(a) Process reaction methods: Ziegler and Nichols [8], Wolfe [12], Tyreus and Luyben [75], Astrom and Hagglund [3] – *page 138*. Four tuning rules are defined.
(b) Regulator tuning – performance index minimisation: Minimum ISE – Hazebroek and Van der Waerden [9]. Minimum IAE - Shinskey [59] – *page 74*. Minimum ITAE - Poulin and Pomerleau [82]. Four tuning rules are defined.
(c) Ultimate cycle: Tyreus and Luyben [75], ** Shinskey [17]. Two tuning rules are defined.
(d) Robust: Fruehauf *et al*. [52], Chien [50], Ogawa [54]. Three tuning rules are defined.
(e) Direct synthesis: Wang and Cluett [76], Cluett and Wang [44], Rotach [77], Poulin and Pomerleau [78], Kookos *et al*. [38]. Five tuning rules are defined.
(f) Other methods: Penner [79], Srividya and Chidambaram [80]. Two tuning rules are defined.

Table 9: Controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)\frac{1}{1 + T_f s}$. One robust tuning rule is defined by Tan *et al*. [81].

Table 10: Controller $U(s) = K_c Y(s) - \frac{K_c}{T_i s}E(s)$. One direct synthesis tuning rule is defined by Chien *et al*. [74].

Table 11: Two degree of freedom controller: $U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - \alpha K_c R(s)$. Two performance index minimisation - servo/regulator tuning rules are defined by Taguchi and Araki [61a] and Pecharroman and Pagola [134b].

Tables 12-14: PI tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1 + sT_m)}$

Table 12: Ideal controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)$. Six such tuning rules are defined; the references are:

(a) Ultimate cycle: McMillan [58]. One tuning rule is defined.
(b) Regulator tuning – minimum performance index: Minimum IAE – Shinskey [59] – *page 75*. Shinskey [59] – *page 158*. Minimum ITAE - Poulin and Pomerleau [82]. Four tuning rules are defined.
(c) Direct synthesis - Poulin and Pomerleau [78]. One tuning rule is defined.

Table 13: Controller $G_c(s) = K_c\left(b + \frac{1}{T_i s}\right)$. One direct synthesis tuning rule is defined by Astrom and Hagglund [3] – *pages 210-212*.

Table 14: Two degree of freedom controller: $U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - \alpha K_c R(s)$. Two performance index minimisation tuning rules are defined by Taguchi and Araki [61a] and Pecharroman and Pagola [134b].

Tables 15-16: PI tuning rules – SOSPD model $\frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$ or $\frac{K_m e^{-s\tau_m}}{(1 + T_{m1}s)(1 + T_{m2}s)}$

Table 15: Ideal controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)$. Ten tuning rules are defined; the references are:

(a) Robust: Brambilla *et al*. [48]. One tuning rule is defined.
(b) Direct synthesis: Tan *et al*. [39]. One tuning rule is defined.
(c) Regulator tuning – minimum performance index: Minimum IAE - Shinskey [59] – *page 158*, ** Shinskey [17], Huang *et al*. [18], Minimum ISE – McAvoy and Johnson [83], Minimum ITAE – Lopez *et al*. [84]. Five tuning rules are defined.
(d) Servo tuning – minimum performance index: Minimum IAE - Huang *et al*. [18]. One tuning rule is defined.
(e) Ultimate cycle: Hwang [60], ** Shinskey [17]. Two tuning rules are defined.

Table 16: Two degree of freedom controller: $U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - \alpha K_c R(s)$. Three performance index minimisation tuning rules are defined by Taguchi and Araki [61a] and Pecharroman and Pagola [134a], [134b].

Table 17: <u>PI tuning rules – SOSIPD model (repeated pole)</u> $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1 + sT_m)^2}$

Table 17: Two degree of freedom controller: $U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - \alpha K_c R(s)$. Two performance index minimisation tuning rules are defined by Taguchi and Araki [61a] and Pecharroman and Pagola [134b].

Tables 18-19: <u>PI tuning rules – third order lag plus delay (TOLPD) model</u> $\frac{K_m e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})(1 + sT_{m3})}$

Table 18: Ideal controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)$. One *** tuning rule is defined. The reference is Hougen [85].

Table 19: Two degree of freedom controller: $U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - \alpha K_c R(s)$. One performance index minimisation tuning rule is defined by Taguchi and Araki [61a].

Table 20: <u>PI tuning rules - unstable FOLPD model</u> $\frac{K_m e^{-s\tau_m}}{1 - sT_m}$

Table 20: Ideal controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)$. Six tuning rules are defined; the references are:

(a) Direct synthesis: De Paor and O'Malley [86], Venkatashankar and Chidambaram [87], Chidambaram [88], Ho and Xu [90]. Four tuning rules are defined.
(b) Robust: Rotstein and Lewin [89]. One tuning rule is defined.
(c) Ultimate cycle: Luyben [91]. One tuning rule is defined.

Table 21: <u>PI tuning rules - unstable SOSPD model</u> $\frac{K_m e^{-s\tau_m}}{(1 - sT_{m1})(1 + sT_{m2})}$

Table 21: Ideal controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)$. Three tuning rules are defined; the references are:

(a) Ultimate cycle: McMillan [58]. One tuning rule is defined.
(b) Minimum performance index – regulator tuning: Minimum ITAE – Poulin and Pomerleau [82]. Two tuning rules are defined.

Table 22: <u>PI tuning rules – delay model</u> $e^{-s\tau_m}$

Table 22: Ideal controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)$. Two tuning rules are defined; the references are:

(a) Direct synthesis: Hansen [91a].
(b) Minimum performance index – regulator tuning: Shinskey [57].

Tables 23-40: PID tuning rules - FOLPD model $\dfrac{K_m e^{-s\tau_m}}{1+sT_m}$

Table 23: Ideal controller $G_c(s) = K_c\left(1+\dfrac{1}{T_i s}+T_d s\right)$. Fifty-seven tuning rules are defined; the references are:

(a) Process reaction: Ziegler and Nichols [8], Astrom and Hagglund [3] – *page 139*, Parr [64] – *page 194*, Chien *et al.* [10], Murrill [13]- *page 356*, Cohen and Coon [11], Astrom and Hagglund [93]- *pages 120-126*, Sain and Ozgen [94]. Eight tuning rules are defined.
(b) Minimum performance index – regulator tuning: Minimum IAE – Murrill [13] – *pages 358-363*, Cheng and Hung [95]. Minimum ISE - Murrill [13] – *pages 358-363*, Zhuang and Atherton [20]. Minimum ITAE - Murrill [13] – *pages 358-363*. Minimum ISTSE - Zhuang and Atherton [20]. Minimum ISTES - Zhuang and Atherton [20]. Minimum error - step load change - Gerry [96]. Eight tuning rules are defined.
(c) Minimum performance index – servo tuning: Minimum IAE - Rovira *et al*. [21], Wang *et al*. [97]. Minimum ISE - Wang *et al*. [97], Zhuang and Atherton [20]. Minimum ITAE - Rovira *et al*. [21], Cheng and Hung [95], Wang *et al*. [97]. Minimum ISTSE – Zhuang and Atherton [20]. Minimum ISTES – Zhuang and Atherton [20]. Nine tuning rules are defined.
(d) Ultimate cycle: Pessen [63], Zhuang and Atherton [20], Pi-Mira *et al*. [97a], Hwang [60], Hwang and Fang [61], McMillan [58], Astrom and Hagglund [98], Li *et al*. [99], Tan *et al*. [39], Friman and Waller [41]. Fourteen tuning rules are defined.
(e) Direct synthesis: Gorecki *et al*. [28], Smith and Corripio [25], Suyama [100], Juang and Wang [101], Cluett and Wang [44], Zhuang and Atherton [20], Abbas [45], Camacho *et al*.[102, Ho *et al*. [103], Ho *et al* [104], Morilla *et al*. [104a]. Fourteen tuning rules are defined.
(f) Robust: Brambilla *et al*. [48], Rivera *et al*. [49], Fruehauf *et al*. [52], Lee *et al*. [55]. Four tuning rules.

Table 24: Ideal controller with first order filter $G_c(s) = K_c\left(1+\dfrac{1}{T_i s}+T_d s\right)\dfrac{1}{T_f s+1}$. Three robust tuning rules are defined by ** Morari and Zafiriou [105], Horn *et al*. [106] and Tan *et al*. [81].

Table 25: Ideal controller with second order filter $G_c(s) = K_c\left(1+\dfrac{1}{T_i s}+T_d s\right)\dfrac{1+b_1 s}{1+a_1 s+a_2 s^2}$. One robust tuning rule is defined by Horn *et al*. [106].

Table 26: Ideal controller with set-point weighting $G_c(s) = K_c\left(b+\dfrac{1}{T_i s}+T_d s\right)$. One direct synthesis tuning rule is defined by Astrom and Hagglund [3] – *pages 208-210*.

Table 27. Ideal controller with first order filter and set-point weighting:

$U(s) = K_c\left(1+\dfrac{1}{T_i s}+T_d s\right)\dfrac{1}{T_f s+1}\left[R(s)\dfrac{1+0.4T_r s}{1+sT_r}-Y(s)\right]$. One direct synthesis tuning rule is defined by Normey-Rico *et al*. [106a].

Table 28: Classical controller $G_c(s) = K_c\left(1+\dfrac{1}{T_i s}\right)\dfrac{1+sT_d}{1+s\dfrac{T_d}{N}}$. Twenty tuning rules are defined; the references are:

(a) Process reaction: Hang *et al*. [36] – *page 76*, Witt and Waggoner [107], St. Clair [15] – *page 21*, Shinskey [15a]. Five tuning rules are defined.
(b) Minimum performance index – regulator tuning: Minimum IAE - Kaya and Scheib [108], Witt and Waggoner [107]. Minimum ISE - Kaya and Scheib [108]. Minimum ITAE - Kaya and Scheib [108], Witt and Waggoner [107]. Five tuning rules are defined.
(c) Minimum performance index – servo tuning: Minimum IAE - Kaya and Scheib [108], Witt and Waggoner [107]. Minimum ISE - Kaya and Scheib [108]. Minimum ITAE - Kaya and Scheib [108], Witt and Waggoner [107]. Five tuning rules are defined.
(d) Direct synthesis: Tsang and Rad [109], Tsang *et al*. [111]. Two tuning rules are defined.
(e) Robust: Chien [50]. One tuning rule is defined.

(f) Ultimate cycle: Shinskey [59] – *page 167*, Shinskey [16] – *page 143*. Two tuning rules are defined.

Table 29: Non-interacting controller $U(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(E(s) - \frac{T_d s}{1+\frac{T_d s}{N}}Y(s)\right)$. Two tuning rules are defined; the references are:

(a) Minimum performance index – regulator tuning: Minimum IAE - Huang *et al*. [18].
(b) Minimum performance index – servo tuning: Minimum IAE - Huang *et al*. [18].

Table 30: Non-interacting controller $U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - \frac{T_d s}{1+\frac{sT_d}{N}}Y(s)$. Five tuning rules are defined; the references are:

(a) Minimum performance index – servo tuning: Minimum ISE - Zhuang and Atherton [20], Minimum ISTSE - Zhuang and Atherton [20]. Minimum ISTES - Zhuang and Atherton [20]. Three tuning rules are defined.
(b) Ultimate cycle: Zhuang and Atherton [20], Shinskey [16] – *page 148*. Two tuning rules are defined.

Table 31: Non-interacting controller $U(s) = \left(K_c + \frac{1}{T_i s}\right)E(s) - \frac{T_d s}{1+\frac{sT_d}{N}}Y(s)$. Six tuning rules are defined; the references are:

(a) Minimum performance index – regulator tuning: Minimum IAE – Kaya and Scheib [108]. Minimum ISE – Kaya and Scheib [108]. Minimum ITAE – Kaya and Scheib [108].
(b) Minimum performance index – servo tuning: Minimum IAE – Kaya and Scheib [108]. Minimum ISE – Kaya and Scheib [108]. Minimum ITAE – Kaya and Scheib [108].

Table 32: Non-interacting controller with setpoint weighting:

$U(s) = K_c\left(b + \frac{1}{T_i s}\right)E(s) - \frac{K_c T_d s}{1+T_d s/N}Y(s) + K_c(b-1)Y(s)$. Three ultimate cycle tuning rules are defined by Hang and Astrom [111], Hang *et al*. [65] and Hang and Cao [112].

Table 33: Industrial controller $U(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(R(s) - \frac{1+T_d s}{1+\frac{T_d s}{N}}Y(s)\right)$. Six tuning rules are defined: the reference are:

(a) Minimum performance index – regulator tuning: Minimum IAE - Kaya and Scheib [108]. Minimum ISE - Kaya and Scheib [108]. Minimum ITAE - Kaya and Scheib [108]. Three tuning rules are defined.
(b) Minimum performance index – servo tuning: Minimum IAE - Kaya and Scheib [108]. Minimum ISE - Kaya and Scheib [108]. Minimum ITAE - Kaya and Scheib [108]. Three tuning rules are defined.

Table 34: Series controller $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)(1+sT_d)$. Three tuning rules are defined; the references are:

(a) ******: Astrom and Hagglund [3] – *page 246*.
(b) Ultimate cycle: Pessen [63].
(c) Direct synthesis: Tsang *et al*. [110].

Table 35: Series controller with filtered derivative $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(1+\frac{sT_d}{1+\frac{sT_d}{N}}\right)$. One robust tuning rule is defined by Chien [50].

Table 36: Controller with filtered derivative $G_c(s) = K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+s\frac{T_d}{N}}\right)$. Three tuning rules are defined; the references are:

(a) Robust: Chien [50], Gong *et al*. [113]. Two tuning rules are defined.
(b) Direct synthesis: Davydov *et al*. [31]. One tuning rule is defined.

Table 37: Alternative non-interacting controller 1 - $U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - K_c T_d s Y(s)$. Six ultimate cycle tuning rules are defined; the references are: Shinskey [59] – *page 167*, ** Shinskey [17], Shinskey [16] – *page 143*, VanDoren [114].

Table 38: Alternative filtered derivative controller - $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+0.5\tau_m s + 0.0833\tau_m{}^2 s^2}{[1+0.1\tau_m s]^2}\right)$. One direct synthesis tuning rule is defined by Tsang *et al*. [110].

Table 39: I-PD controller $U(s) = \frac{K_c}{T_i s}E(s) - K_c(1+T_d s)Y(s)$. Two direct synthesis tuning rules are defined by Chien *et al*. [74] and Argelaguet *et al*. [114a].

Table 40: Two degree of freedom controller: $U(s) = K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+\frac{T_d}{N}s}\right)E(s) - K_c\left(\alpha+\frac{\beta T_d s}{1+\frac{T_d}{N}s}\right)R(s)$. One performance index minimisation tuning rule is defined by Taguchi and Araki [61a].

Tables 41-48: <u>PID tuning rules - non-model specific</u>

Table 41: Ideal controller $G_c(s) = K_c\left(1+\frac{1}{T_i s}+T_d s\right)$. Twenty five tuning rules are defined; the references are

(a) Ultimate cycle: Ziegler and Nichols [8], Blickley [115], Parr [64] – *pages 190-191*, De Paor [116], Corripio [117] – *page 27*, Mantz andTacconi [118], Astrom and Hagglund [3] – *page 142*, Astrom and Hagglund [93], Atkinson and Davey [119], ** Perry and Chilton [120], Yu [122] – *page 11*, Luo *et al*. [121], McMillan [14] – *page 90*, McAvoy and Johnson [83], Karaboga and Kalinli [123], Hang and Astrom [124], Astrom *et al*. [30], St. Clair [15] - *page 17*, Shin *et al*. [125]. Nineteen tuning rules are defined.
(b) Other tuning: Harriott [126], Parr [64] – *pages 191, 193*, McMillan [14] - *page 43*, Calcev and Gorez [69], Zhang *et al*. [127], Garcia and Castelo [127a]. Six tuning rules are defined.

Table 42: Controller with filtered derivative $G_c(s) = K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+\frac{T_d s}{N}}\right)$. Eight tuning rules are defined; the references are:

(a) Direct synthesis: Vrancic [72], Vrancic [73], Lennartson and Kristiansson [157], Kristiansson and Lennartson [158], Kristiansson and Lennartson [158a]. Six tuning rules are defined.
(b) Other tuning: Leva [67], Astrom [68]. Two tuning rules are defined.

Table 43: Ideal controller with set-point weighting:

$U(s) = K_c\left(F_p R(s) - Y(s)\right) + \frac{1}{T_i s}\left(F_i R(s) - Y(s)\right) + T_d s\left(F_d R(s) - Y(s)\right)$. One ultimate cycle tuning rule is defined by Mantz and Tacconi [118].

Table 42: Ideal controller with proportional weighting $G_c(s) = K_c\left(b+\frac{1}{T_i s}+T_d s\right)$. One direct synthesis tuning rule is defined by Astrom and Hagglund [3] – *page 217*.

Table 44: Non-interacting controller $U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - \frac{K_c T_d s}{1+\frac{s T_d}{N}}Y(s)$. One ultimate cycle tuning rule is defined by Fu *et al*. [128].

Table 45: Series controller $U(s) = K_c\left(1+\frac{1}{T_i s}\right)(1+sT_d)$. Three ultimate cycle tuning rules are defined by Pessen [131], Pessen [129] and Grabbe *et al*. [130].

Table 46: Series controller with filtered derivative $U(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(1+\frac{sT_d}{1+\frac{sT_d}{N}}\right)$. One ultimate cycle tuning rule is defined by Hang *et al*. [36] - *page 58*.

Table 47: Classical controller $U(s) = K_c\left(1+\frac{1}{T_i s}\right)\frac{1+sT_d}{1+s\frac{T_d}{N}}$. One ultimate cycle tuning rule is defined by Corripio [117].

Table 48: Non-interacting controller $U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - K_c T_d sY(s)$. One ultimate cycle tuning rule is defined by VanDoren [114].

Tables 49-58: <u>PID tuning rules - IPD model</u> $\frac{K_m e^{-s\tau_m}}{s}$

Table 49: Ideal controller $G_c(s) = K_c\left(1+\frac{1}{T_i s}+T_d s\right)$. Five tuning rules are defined; the references are:

(a) Process reaction: Ford [132], Astrom and Hagglund [3] – *page 139*. Two tuning rules are defined.
(b) Direct synthesis: Wang and Cluett [76], Cluett and Wang [44], Rotach [77]. Three tuning rules are defined.

Table 50: Ideal controller with first order filter, set-point weighting and output feedback:
$U(s) = K_c\left(1+\frac{1}{T_i s}+T_d s\right)\frac{1}{T_f s+1}\left[R(s)\frac{1+0.4T_r s}{1+sT_r} - Y(s)\right] - K_0 Y(s)$. One direct synthesis tuning rule has been defined by Normey-Rico *et al*. [106a].

Table 51: Ideal controller with filtered derivative $G_c(s) = K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+\frac{sT_d}{N}}\right)$. One robust tuning rule has been defined by Chien [50].

Table 52: Series controller with filtered derivative $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(1+\frac{T_d s}{1+\frac{T_d s}{N}}\right)$. One robust tuning rule has been defined by Chien [50].

Table 53: Classical controller $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+T_d s}{1+\frac{T_d s}{N}}\right)$. Five tuning rules have been defined; the references are:

(a) Ultimate cycle: Luyben [133], Belanger and Luyben [134]. Two tuning rules have been defined.
(b) Robust: Chien [50]. One tuning rule has been defined.
(c) Performance index minimisation – regulator tuning: ** Minimum IAE - Shinskey [17], Shinskey [59] – *page 74*. Two tuning rules have been defined.

Table 54: Alternative non-interacting controller 1: $U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - K_c T_d sY(s)$. Two performance index minimisation rules – minimum IAE regulator tuning have been defined by Shinskey [59] – *page 74* and ** Shinskey [17].

Table 55: I-PD controller $U(s) = \frac{K_c}{T_i s} E(s) - K_c(1 + T_d s)Y(s)$. One direct synthesis tuning rule has been defined by Chien *et al*. [74].

Table 56: Controller $U(s) = K_c(1 + \frac{1}{T_i s})E(s) + K_c(b - 1)R(s) - K_c T_d s Y(s)$. One direct synthesis tuning rule has been defined by Hansen [91a].

Table 57: Two degree of freedom controller: $U(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N}s}\right)E(s) - K_c\left(\alpha + \frac{\beta T_d s}{1 + \frac{T_d}{N}s}\right)R(s)$. Two minimum performance index – servo/regulator tuning have been defined by Taguchi and Araki [61a] and Pecharroman and Pagola [134a].

Tables 58-67: PID tuning rules - FOLIPD model $\frac{K_m e^{-s\tau_m}}{s(1 + sT_m)}$

Table 58: Ideal controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$. One ultimate cycle tuning rule has been defined by Millan [58].

Table 59: Ideal controller with filter $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{1 + T_f s}$. Three robust tuning rules have been defined by Tan *et al*. [81], Zhang *et al*. [135] and Tan *et al*. [136].

Table 60: Ideal controller with set-point weighting $G_c(s) = K_c\left(b + \frac{1}{T_i s} + T_d s\right)$. One direct synthesis tuning rule has been defined by Astrom and Hagglund [3] - *pages 212-213*.

Table 61: Classical controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)\left(\frac{1 + T_d s}{1 + \frac{T_d s}{N}}\right)$. Five tuning rules have been defined; the references are as follows:

(a) Robust: Chien [50]. One tuning rule is defined.
(b) Minimum performance index – regulator tuning: Minimum IAE – Shinskey [59] – *page 75*, Shinskey [59] – *pages 158-159*, Minimum ITAE - Poulin and Pomerleau [82], [92]. Four tuning rules are defined.

Table 62: Series controller with derivative filtering $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)\left(1 + \frac{T_d s}{1 + \frac{T_d s}{N}}\right)$. One robust tuning rule has been defined by Chien [50].

Table 63: Alternative non-interacting controller 1: $U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - K_c T_d s Y(s)$. Two minimum performance index (minimum IAE) – regulator tuning rules have been defined by Shinskey [59] – *page 75*, *page 159*.

Table 64: Ideal controller with filtered derivative: $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + s\frac{T_d}{N}}\right)$. One robust tuning rule has been defined by Chien [50].

Table 65: Ideal controller with set-point weighting:

$U(s) = K_c\left(F_p R(s) - Y(s)\right) + \frac{K_c}{T_i s}\left[F_i R(s) - Y(s)\right] + K_c T_d s\left[F_d R(s) - Y(s)\right]$. One ultimate cycle tuning rule has been defined by Oubrahim and Leonard [138].

Table 66: Alternative classical controller: $G_c(s) = K_c\left(\frac{1+T_i s}{1+\frac{T_d s}{N}}\right)\left(\frac{1+T_d s}{1+\frac{T_d s}{N}}\right)$. One direct synthesis tuning rule has been defined by Tsang and Rad [109].

Table 67: Two degree of freedom controller:

$U(s) = K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+\frac{T_d}{N}s}\right)E(s) - K_c\left(\alpha+\frac{\beta T_d s}{1+\frac{T_d}{N}s}\right)R(s)$. Two minimum performance index – servo/regulator tuning have been defined by Taguchi and Araki [61a] and Pecharroman and Pagola [134a].

Tables 68-79: PID tuning rules - SOSPD model $\frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ or $\frac{K_m e^{-s\tau_m}}{T_{m1}{}^2 s^2 + 2\xi_m T_{m1}s+1}$

Table 68: Ideal controller $G_c(s) = K_c\left(1+\frac{1}{T_i s}+T_d s\right)$. Twenty seven tuning rules have been defined; the references are:

(a) Minimum performance index – servo tuning: Minimum ITAE – Sung *et al*. [139]. One tuning rule is defined.
(b) Minimum performance index – regulator tuning: Minimum ITAE – Sung *et al*. [139], Lopez *et al*. [84]. One tuning rule is defined.
(c) Ultimate cycle: Hwang [60], Shinskey [16] – *page 151*. Three tuning rules are defined.
(d) Direct synthesis: Hang *et al*. [35], Ho *et al*. [140], Ho *et al*. [141], Ho *et al*. [142], Wang *et al*. [143], Leva *et al*. [34], Wang and Shao [144], Pemberton [145], Pemberton [24], Suyama [100], Smith *et al*. [146], Chiu *et al*. [29], Wang and Clemens [147], Gorez and Klan [147a], Miluse *et al*. [27a], Miluse *et al*. [27b], Seki *et al*. [147b], Landau and Voda [148]. Nineteen tuning rules are defined.
(e) Robust: Brambilla *et al*. [48], Chen *et al*. [53], Lee *et al*. [55]. Three tuning rules are defined.

Table 69: Filtered controller $G_c(s) = K_c\left(1+\frac{1}{T_i s}+T_d s\right)\frac{1}{T_f s+1}$. One robust tuning rule has been defined by Hang *et al*. [35].

Table 70: Filtered controller $G_c(s) = K_c\left(1+\frac{1}{T_i s}+T_d s\right)\frac{b_1 s+1}{a_1 s+1}$. One robust tuning rule has been defined by Jahanmiri and Fallahi [149].

Table 71: Classical controller $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+T_d s}{1+\frac{T_d s}{N}}\right)$. Seven tuning rules have been defined; the references are:

(a) Minimum performance index – regulator tuning: Minimum IAE - Shinskey [59] – *page 159*, ** Shinskey [59], ** Shinskey [17], ** Shinskey [17]. Minimum ISE – McAvoy and Johnson [83]. Five tuning rules are defined.
(b) Direct synthesis: Astrom *et al*. [30], Smith *et al*. [26]. Two tuning rules are defined.

Table 72: Alternative classical controller $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+NT_d s}{1+T_d s}\right)$. One ***** tuning rule has been defined by Hougen [85].

Table 73: Alternative non-interacting controller 1: $U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - K_c T_d s Y(s)$. Three minimum performance index (minimum IAE) – regulator tuning rules have been defined by Shinskey [59] – *page 158*, ** Shinskey [17], ** Shinskey [17].

Table 74: Series controller $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)(1+T_d s)$. One minimum performance index - regulator tuning rule has been defined by Haalman [23].

Table 75: Non-interacting controller $U(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(E(s) - \frac{T_d s}{1+\frac{T_d s}{N}}Y(s)\right)$. Two tuning rules have been defined. The references are:

(a) Minimum performance index – regulator tuning: Minimum IAE - Huang *et al*. [18].
(b) Minimum performance index – servo tuning: Minimum IAE - Huang *et al*. [18].

Table 76: Ideal controller with set-point weighting:

$U(s) = K_c\left(F_p R(s) - Y(s)\right) + \frac{K_c}{T_i s}\left[F_i R(s) - Y(s)\right] + K_c T_d s\left[F_d R(s) - Y(s)\right]$. One ultimate cycle tuning rule has been defined by Oubrahim and Leonard [138].

Table 77: Non-interacting controller $U(s) = K_c\left(b+\frac{1}{T_i s}\right)\left[R(s) - Y(s)\right] - \left(c + T_d s\right)Y(s)$. One direct synthesis tuning rule has been defined by Hansen [150].

Table 78: Ideal controller with filtered derivative $G_c(s) = K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+\frac{s T_d}{N}}\right)$. Two tuning rules are defined; the references are:

(a) Direct synthesis: Hang *et al*. [151].
(b) Robust: Hang *et al*. [151].

Table 79: Two degree of freedom controller: $U(s) = K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+\frac{T_d}{N}s}\right)E(s) - K_c\left(\alpha+\frac{\beta T_d s}{1+\frac{T_d}{N}s}\right)R(s)$. Three minimum performance index – servo/regulator tuning rules have been defined by Taguchi and Araki [61a] and Pecharroman and Pagola [134a], [134b].

Table 80: <u>PID tuning rules - I^2PD model</u> $G_m(s) = \frac{K_m e^{-s\tau_m}}{s^2}$

Table 80: Controller $U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) + K_c(b-1)R(s) - K_c T_d s Y(s)$. One direct synthesis tuning rule has been defined by Hansen [91a].

Table 81: <u>PID tuning rules – SOSIPD model (repeated pole)</u> $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)^2}$

Table 81: Two degree of freedom controller: $U(s) = K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+\frac{T_d}{N}s}\right)E(s) - K_c\left(\alpha+\frac{\beta T_d s}{1+\frac{T_d}{N}s}\right)R(s)$. Two minimum performance index – servo/regulator tuning rules have been defined by Taguchi and Araki [61a] and Pecharroman and Pagola [134a].

Tables 82-84: PID tuning rules - SOSPD model with a positive zero $\frac{K_m(1-sT_{m3})e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$

Table 82: Controller with filtered derivative $G_c(s)=K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+\frac{T_d s}{N}}\right)$. One robust tuning rule has been defined by Chien [50].

Table 83: Classical controller $G_c(s)=K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+T_d s}{1+\frac{T_d s}{N}}\right)$. One robust tuning rule has been defined by Chien [50].

Table 84: Series controller with filtered derivative $G_c(s)=K_c\left(1+\frac{1}{T_i s}\right)\left(1+\frac{T_d s}{1+\frac{T_d s}{N}}\right)$. One robust tuning rule has been defined by Chien [50].

Tables 85-88: PID tuning rules - SOSPD model with a negative zero $\frac{K_m(1+sT_{m3})e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$

Table 85: Ideal controller $G_c(s)=K_c\left(1+\frac{1}{T_i s}+T_d s\right)$. One minimum performance index tuning rule has been defined by Wang *et al*. [97].

Table 86: Ideal controller with filtered derivative $G_c(s)=K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+\frac{T_d s}{N}}\right)$. One robust tuning rule has been defined by Chien [50].

Table 87: Classical controller $G_c(s)=K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+T_d s}{1+\frac{T_d s}{N}}\right)$. One robust tuning rule has been defined by Chien [50].

Table 88: Series controller with filtered derivative $G_c(s)=K_c\left(1+\frac{1}{T_i s}\right)\left(1+\frac{T_d s}{1+\frac{T_d s}{N}}\right)$. One robust tuning rule has been defined by Chien [50].

Table 89-90: PID tuning rules - TOLPD model $\frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})(1+sT_{m3})}$

Table 89: Ideal controller $G_c(s)=K_c\left(1+\frac{1}{T_i s}+T_d s\right)$. Two minimum performance index tuning rules have been defined by Polonyi [153].

Table 90: Two degree of freedom controller: $U(s)=K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+\frac{T_d}{N}s}\right)E(s)-K_c\left(\alpha+\frac{\beta T_d s}{1+\frac{T_d}{N}s}\right)R(s)$ .One minimum performance index tuning rule has been defined by Taguchi and Araki [61a].

Tables 91-93: PID tuning rules - unstable FOLPD model $\frac{K_m e^{-s\tau_m}}{1 - sT_m}$

Table 91: Ideal controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$. Three direct synthesis tuning rules are defined by De Paor and O'Malley [86], Chidambaram [88] and Valentine and Chidambaram [154].

Table 92: Non-interacting controller $U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - \frac{K_c T_d s}{1 + \frac{sT_d}{N}}Y(s)$. Two tuning rules have been defined; the references are:

(a) Minimum performance index – servo tuning: Minimum IAE - Huang and Lin [155]
(b) Minimum performance index – servo tuning: Minimum IAE - Huang and Lin [155]

Table 93: Classical controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)\left(\frac{1 + T_d s}{1 + \frac{T_d s}{N}}\right)$. One performance index minimisation – regulator tuning rule has been defined by Shinskey [16]– *page 381*.

Tables 94-97: PID tuning rules - unstable SOSPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{(1 - sT_{m1})(1 + sT_{m2})}$

Table 94: Ideal controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$. Two tuning rules have been defined; the references are

(a) Ultimate cycle: McMillan [58]
(b) Robust: Rotstein and Lewin [89].

Table 95: Classical controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)\left(\frac{1 + T_d s}{1 + \frac{T_d}{N}s}\right)$. Two minimum performance index tuning rules (regulator - minimum ITAE) have been defined by Poulin and Pomerleau [82], [92].

Table 96: Series controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)(1 + T_d s)$. One direct synthesis tuning rule has been defined by Ho and Xu [90].

Table 97: Non-interacting controller $U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - \frac{K_c T_d s}{1 + \frac{sT_d}{N}}Y(s)$. Two tuning rules have been defined; the references are

(a) Minimum performance index – servo tuning: Minimum IAE - Huang and Lin [155]
(b) Minimum performance index – regulator tuning: Minimum IAE - Huang and Lin [155]

Table 98: PID tuning rules – general model with a repeated pole $G_m(s) = \frac{K_m e^{-s\tau_m}}{(1 + sT_m)^n}$

Table 98: Ideal controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$. One direct synthesis tuning rule has been defined by Skoczowski and Tarasiejski [156]

Table 99: PID tuning rules – general stable non-oscillating model with a time delay

Table 99: Ideal controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$. One direct synthesis tuning rule has been defined by Gorez and Klan [147a].

Tables 100-101: PID tuning rules – fifth order model with delay

$$G_m(s) = \frac{K_m(1 + b_1 s + b_2 s^2 + b_3 s^3 + b_4 s^4 + b_s s^5)e^{-s\tau_m}}{\left(1 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5\right)}$$

Table 100: Ideal controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$. One direct synthesis tuning rule is defined by Vrancic *et al*. [159].

Table 101: Controller with filtered derivative $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d s}{N}}\right)$. One direct synthesis tuning rule is defined by Vrancic *et al*. [159].

** some more information needed.

The number of tuning rules in each table is included in the data. Servo and regulator tuning rules are counted separately; otherwise, rules in which different tuning parameters are provided for a number of variations in process parameters or desired response parameters (such as desired gain margin, phase margin or closed loop response time constant) are counted as one tuning rule. Tabular summaries are provided below.

Table A: Model structure and tuning rules – a summary for PI controllers

Model	Process reaction	Minimise Performance index	Direct Synthesis	Ultimate cycle	Robust tuning	Other rules	Total
Stable FOLPD	12	21	33	10	12	0	88 (53%)
Non-model specific	0	0	7	7	0	8	23 (14%)
IPD	4	6	6	2	4	2	24 (14%)
FOLIPD	0	6	2	1	0	0	9 (5%)
SOSPD	0	9	1	2	1	0	13 (7%)
SOSIPD	0	2	0	0	0	0	2 (1%)
TOLPD	0	1	0	0	0	1	2 (1%)
Unstable FOLPD	0	0	4	1	1	0	6 (4%)
Unstable SOSPD	0	2	0	1	0	0	3 (2%)
Delay model	0	1	1	0	0	0	2 (1%)
TOTAL	**16**	**48**	**54**	**24**	**18**	**11**	**171**

Table B: Model structure and tuning rules – a summary for PID controllers

Model	Process reaction	Minimise Performance index	Direct Synthesis	Ultimate cycle	Robust tuning	Other rules	Total
Stable FOLPD	13	45	24	28	12	1	123 (44%)
Non-model specific	0	0	7	27	0	8	42 (15%)
IPD	2	6	6	2	3	0	19 (7%)
FOLIPD	0	8	2	2	6	0	18 (6%)
SOSPD	0	16	23	4	6	1	50 (17%)
I^2PD	0	0	1	0	0	0	1 (0%)
SOSIPD	0	2	0	0	0	0	2 (1%)
SOSPD – pos. zero	0	0	0	0	3	0	3 (1%)
SOSPD – neg. zero	0	1	0	0	3	0	4 (1%)
TOLPD	0	3	0	0	0	0	3 (1%)
Unstable FOLPD	0	3	3	0	0	0	6 (2%)
Unstable SOSPD	0	4	1	1	1	0	7 (2%)
Higher order	0	0	4	0	0	0	4 (1%)
TOTAL	**15**	**88**	**71**	**64**	**34**	**10**	**282**

Table C: Model structure and tuning rules – a summary for PI/PID controllers

Model	Process reaction	Minimise Performance index	Direct Synthesis	Ultimate cycle	Robust tuning	Other rules	Total
Stable FOLPD	25	66	57	38	24	1	211 (47%)
Non-model specific	0	0	14	34	0	16	65 (15%)
IPD	6	12	12	4	7	2	43 (9%)
FOLIPD	0	14	4	3	6	0	27 (6%)
SOSPD	0	25	24	6	7	1	63 (14%)
I^2PD	0	0	1	0	0	0	1 (0%)
SOSIPD	0	4	0	0	0	0	4 (1%)
SOSPD – pos. zero	0	0	0	0	3	0	3 (1%)
SOSPD – neg. zero	0	1	0	0	3	0	4 (1%)
TOLPD	0	4	0	0	0	1	5 (1%)
Unstable FOLPD	0	3	7	0	1	0	12 (3%)
Unstable SOSPD	0	6	1	2	1	0	10 (2%)
Delay model	0	1	1	0	0	0	2 (0%)
Higher order	0	0	4	0	0	0	4 (1%)
TOTAL	**31**	**136**	**125**	**88**	**52**	**21**	**453**

Table D: PI controller structure and tuning rules – a summary

Controller structure	Stable FOLPD	Non-model specific	IPD	FOLIPD	SOSPD	Other	Total
$G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)$	85	19	20	6	10	12	**152 (92%)**
$G_c(s) = K_c\left(b + \frac{1}{T_i s}\right)$	2	1	0	1	2	0	**6 (4%)**
$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - \alpha K_c R(s)$	1	1	2	2	1	3	**10 (4%)**
$U(s) = K_c Y(s) - \frac{K_c}{T_i s} E(s)$	0	1	1	0	0	0	**2 (1%)**
$G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)\frac{1}{1 + T_f s}$	0	0	1	0	0	0	**1 (0%)**
Total	**88**	**21**	**23**	**8**	**12**	**15**	**167**

Table E: PID controller structure and tuning rules – a summary

Controller structure	Stable FOLPD	Non-model specific	IPD	FOLIPD	SOSPD	Other	Total
$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$	57	25	5	1	27	11	**126 (45%)**
$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + s\frac{T_d}{N}}\right)$	3	8	1	1	2	3	**18 (6%)**
$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{T_f s + 1}$	3	0	0	3	1	0	**7 (3%)**
$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{b_1 s + 1}{a_1 s + 1}$	0	0	0	0	1	0	**1 (0%)**
$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1 + b_1 s}{1 + a_1 s + a_2 s^2}$	3	0	0	0	0	0	**3 (1%)**
$G_c(s) = K_c\left(b + \frac{1}{T_i s} + T_d s\right)$	1	1	0	1	0	0	**3 (1%)**
Subtotal	***67***	***34***	***6***	***6***	***31***	***14***	***158 (56%)***
$G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)\frac{1 + sT_d}{1 + s\frac{T_d}{N}}$	20	1	5	5	7	5	**43 (15%)**
$G_c(s) = K_c\left(\frac{1 + T_i s}{1 + \frac{T_d s}{N}}\right)\left(\frac{1 + T_d s}{1 + \frac{T_d s}{N}}\right)$	0	0	0	1	0	0	**1 (0%)**
$G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)\left(\frac{1 + NT_d s}{1 + T_d s}\right)$	0	0	0	0	1	0	**1 (0%)**
$G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)(1 + sT_d)$	3	3	0	0	1	1	**8 (3%)**
$G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)\left(1 + \frac{sT_d}{1 + \frac{sT_d}{N}}\right)$	1	1	1	1	0	2	**6 (2%)**
$G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)\left(\frac{1 + 0.5\tau_m s + 0.0833\tau_m^2 s^2}{[1 + 0.1\tau_m s]^2}\right)$	1	0	0	0	0	0	**1 (0%)**
Subtotal	***25***	***5***	***6***	***7***	***9***	***8***	***60 (22%)***
$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - \frac{T_d s}{1 + \frac{sT_d}{N}}Y(s)$	5	0	0	0	0	0	**5 (2%)**
$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)\left(E(s) - \frac{T_d s}{1 + \frac{T_d s}{N}}Y(s)\right)$	2	0	0	0	4	0	**6 (2%)**

Controller structure	Stable FOLPD	Non-model specific	IPD	FOLIPD	SOSPD	Other	Total
$U(s)=\left(K_c+\frac{1}{T_i s}\right)E(s)-\frac{T_d s}{1+\frac{sT_d}{N}}Y(s)$	6	0	0	0	0	0	**6 (2%)**
$U(s)=K_c\left(1+\frac{1}{T_i s}\right)E(s)-\frac{K_c T_d s}{1+\frac{sT_d}{N}}Y(s)$	0	1	0	0	1	0	**6 (2%)**
$U(s)=K_c\left(b+\frac{1}{T_i s}\right)E(s)-\frac{K_c T_d s}{1+T_d s/N}Y(s)+K_c(b-1)Y(s)$	3	0	0	0	0	0	**3 (1%)**
$U(s)=K_c\left(1+\frac{1}{T_i s}\right)\left(R(s)-\frac{1+T_d s}{1+\frac{T_d s}{N}}Y(s)\right)$	6	0	0	0	0	0	**6 (2%)**
$U(s)=K_c\left(1+\frac{1}{T_i s}\right)E(s)-K_c T_d sY(s)$	6	1	2	2	3	0	**14 (5%)**
$U(s)=K_c\left(b+\frac{1}{T_i s}\right)[R(s)-Y(s)]-(c+T_d s)Y(s)$	0	0	0	0	1	0	**1 (0%)**
$U(s)=\frac{K_c}{T_i s}E(s)-K_c(1+T_d s)Y(s)$	2	0	1	0	0	0	**3 (1%)**
$U(s)=K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+\frac{T_d}{N}s}\right)E(s)-K_c\left(\alpha+\frac{\beta T_d s}{1+\frac{T_d}{N}s}\right)R(s)$	1	0	1	1	3	3	**9 (3%)**
$U(s)=K_c\left(F_p R(s)-Y(s)\right)+\frac{1}{T_i s}\left(F_i R(s)-Y(s)\right)+T_d s\left(F_d R(s)-Y(s)\right)$	0	1	0	1	1	0	**3 (1%)**
$U(s)=K_c(1+\frac{1}{T_i s})E(s)+K_c(b-1)R(s)-K_c T_d sY(s)$	0	0	1	0	0	1	**2 (1%)**
$U(s)=K_c\left(1+\frac{1}{T_i s}+T_d s\right)\frac{1}{T_f s+1}\left[R(s)\frac{1+0.4T_r s}{1+sT_r}-Y(s)\right]$	1	0	0	0	0	0	**1 (0%)**
$U(s)=K_c\left(1+\frac{1}{T_i s}+T_d s\right)\frac{1}{T_f s+1}\left[R(s)\frac{1+0.4T_r s}{1+sT_r}-Y(s)\right]-K_0 Y(s)$	0	0	1	0	0	0	**1 (0%)**
Subtotal	***32***	***3***	***6***	***4***	***8***	***8***	***61 (22%)***
Total	**124**	**42**	**18**	**17**	**49**	**30**	**280**

3. Tuning rules for PI and PID controllers

Table 1: PI tuning rules - FOLPD model - $G_m(s) = \frac{K_m e^{-s\tau_m}}{1+sT_m}$ – ideal controller $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)$. *84 tuning rules*

<table>
<tr><th>Rule</th><th colspan="3">K_c</th><th colspan="3">T_i</th><th colspan="3">Comment</th></tr>
<tr><td>**Process reaction**</td><td colspan="9"></td></tr>
<tr><td>Ziegler and Nichols [8]
Model: Method 1.</td><td colspan="3">$\frac{0.9T_m}{K_m\tau_m}$</td><td colspan="3">$3.33\tau_m$</td><td colspan="3">Quarter decay ratio. $\frac{\tau_m}{T_m} \le 1$</td></tr>
<tr><td rowspan="12">Hazebroek and Van der Waerden [9]
Model: Method 1</td><td colspan="3">$\frac{\alpha T_m}{K_m\tau_m}$</td><td colspan="3">$\beta\tau_m$</td><td colspan="3"></td></tr>
<tr><td>τ_m/T_m</td><td>α</td><td>β</td><td>τ_m/T_m</td><td>α</td><td>β</td><td>τ_m/T_m</td><td>α</td><td>β</td></tr>
<tr><td>0.2</td><td>0.68</td><td>7.14</td><td>1.1</td><td>0.90</td><td>1.49</td><td>2.0</td><td>1.20</td><td>1.00</td></tr>
<tr><td>0.3</td><td>0.70</td><td>4.76</td><td>1.2</td><td>0.93</td><td>1.41</td><td>2.2</td><td>1.28</td><td>0.95</td></tr>
<tr><td>0.4</td><td>0.72</td><td>3.70</td><td>1.3</td><td>0.96</td><td>1.32</td><td>2.4</td><td>1.36</td><td>0.91</td></tr>
<tr><td>0.5</td><td>0.74</td><td>3.03</td><td>1.4</td><td>0.99</td><td>1.25</td><td>2.6</td><td>1.45</td><td>0.88</td></tr>
<tr><td>0.6</td><td>0.76</td><td>2.50</td><td>1.5</td><td>1.02</td><td>1.19</td><td>2.8</td><td>1.53</td><td>0.85</td></tr>
<tr><td>0.7</td><td>0.79</td><td>2.17</td><td>1.6</td><td>1.06</td><td>1.14</td><td>3.0</td><td>1.62</td><td>0.83</td></tr>
<tr><td>0.8</td><td>0.81</td><td>1.92</td><td>1.7</td><td>1.09</td><td>1.10</td><td>3.2</td><td>1.71</td><td>0.81</td></tr>
<tr><td>0.9</td><td>0.84</td><td>1.75</td><td>1.8</td><td>1.13</td><td>1.06</td><td>3.4</td><td>1.81</td><td>0.80</td></tr>
<tr><td>1.0</td><td>0.87</td><td>1.61</td><td>1.9</td><td>1.17</td><td>1.03</td><td></td><td></td><td></td></tr>
<tr><td colspan="3">$\alpha = 0.5\frac{\tau_m}{T_m} + 0.1$</td><td colspan="3">$\beta = \frac{\tau_m}{1.6\tau_m - 1.2T_m}$</td><td colspan="3">$\frac{\tau_m}{T_m} > 3.5$</td></tr>
<tr><td>Astrom and Hagglund [3] – *page 138*
Model: Not relevant</td><td colspan="3">$\frac{0.63T_m}{K_m\tau_m}$</td><td colspan="3">$3.2\tau_m$</td><td colspan="3">Ultimate cycle Ziegler-Nichols equivalent</td></tr>
<tr><td rowspan="2">Chien *et al*. [10] - regulator
Model: Method 1</td><td colspan="3">$\frac{0.6T_m}{K_m\tau_m}$</td><td colspan="3">$4\tau_m$</td><td colspan="3">0% overshoot - $0.11 < \frac{\tau_m}{T_m} < 1.0$</td></tr>
<tr><td colspan="3">$\frac{0.7T_m}{K_m\tau_m}$</td><td colspan="3">$2.33\frac{\tau_m}{K_m}$</td><td colspan="3">20% overshoot - $0.11 < \frac{\tau_m}{T_m} < 1.0$</td></tr>
<tr><td>Astrom and Hagglund [3] – regulator – *page 150*
Model: Method 1</td><td colspan="3">$\frac{0.7T_m}{K_m\tau_m}$</td><td colspan="3">$2.3\tau_m$</td><td colspan="3">20% overshoot</td></tr>
<tr><td rowspan="2">Chien *et al*. [10] - servo
Model: Method 1</td><td colspan="3">$\frac{0.35T_m}{K_m\tau_m}$</td><td colspan="3">$1.17T_m$</td><td colspan="3">0% overshoot - $0.11 < \frac{\tau_m}{T_m} < 1.0$</td></tr>
<tr><td colspan="3">$\frac{0.6T_m}{K_m\tau_m}$</td><td colspan="3">T_m</td><td colspan="3">20% overshoot- $0.11 < \frac{\tau_m}{T_m} < 1.0$</td></tr>
<tr><td>Cohen and Coon [11] process reaction
Model: Method 1.</td><td colspan="3">$\frac{1}{K_m}\left(0.9\frac{T_m}{\tau_m} + 0.083\right)$</td><td colspan="3">$T_m\left(\frac{3.33\frac{\tau_m}{T_m} + 0.31\left(\frac{\tau_m}{T_m}\right)^2}{1 + 2.22\frac{\tau_m}{T_m}}\right)$</td><td colspan="3">Quarter decay ratio</td></tr>
</table>

<table>
<tr><th>Rule</th><th colspan="3">K_c</th><th colspan="3">T_i</th><th>Comment</th></tr>
<tr><td rowspan="4">Two constraints criterion - Wolfe [12]

Model: Method 2.</td><td colspan="3">$\frac{\alpha T_m}{K_m \tau_m}$</td><td colspan="3">$\beta \tau_m$</td><td rowspan="4">Decay ratio = 0.4; minimum error integral (regulator mode).</td></tr>
<tr><td>τ_m/T_m</td><td>α</td><td>β</td><td>τ_m/T_m</td><td>α</td><td>β</td></tr>
<tr><td>0.2</td><td>4.4</td><td>3.23</td><td>1.0</td><td>0.78</td><td>1.28</td></tr>
<tr><td>0.5</td><td>1.8</td><td>2.27</td><td>5.0</td><td>0.30</td><td>0.53</td></tr>
<tr><td>Two constraints criterion - Murrill [13] – *page 356*

Model: Method 3</td><td colspan="3">$\frac{0.928}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.946}$</td><td colspan="3">$\frac{T_m}{1.078}\left(\frac{\tau_m}{T_m}\right)^{0.583}$</td><td>Quarter decay ratio; minimum error integral (servo mode).
$0.1 \le \frac{\tau_m}{T_m} \le 1.0$</td></tr>
<tr><td>McMillan [14] – *page 25*
Model: Method 3</td><td colspan="3">$\frac{K_m}{3}$</td><td colspan="3">τ_m</td><td>Time delay dominant processes</td></tr>
<tr><td>St. Clair [15] – *page 22*
Model: Method 3</td><td colspan="3">$\frac{0.333 T_m}{K_m \tau_m}$</td><td colspan="3">$T_m$</td><td>$\frac{T_m}{\tau_m} \le 3.0$</td></tr>
<tr><td>Shinskey [15a]
Model: Method 1</td><td colspan="3">$\frac{0.667 T_m}{K_m \tau_m}$</td><td colspan="3">$3.78\tau_m$</td><td>$\frac{T_m}{\tau_m} = 0.167$</td></tr>
<tr><td>**Regulator tuning**</td><td colspan="7">**Performance index minimisation**</td></tr>
<tr><td>Minimum IAE - Murrill [13] – *pages 358-363*
Model: Method 3</td><td colspan="3">$\frac{0.984}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.986}$</td><td colspan="3">$\frac{T_m}{0.608}\left(\frac{\tau_m}{T_m}\right)^{0.707}$</td><td>$0.1 \le \frac{\tau_m}{T_m} \le 1.0$</td></tr>
<tr><td rowspan="4">Minimum IAE - Shinskey [16] – *page 123*
Model: Method 6</td><td colspan="3">$1.00 T_m/K_m\tau_m$</td><td colspan="3">$3.0\tau_m$</td><td>$\tau_m/T_m = 0.2$</td></tr>
<tr><td colspan="3">$1.04 T_m/K_m\tau_m$</td><td colspan="3">$2.25\tau_m$</td><td>$\tau_m/T_m = 0.5$</td></tr>
<tr><td colspan="3">$1.11 T_m/K_m\tau_m$</td><td colspan="3">$1.45\tau_m$</td><td>$\tau_m/T_m = 1$</td></tr>
<tr><td colspan="3">$1.39 T_m/K_m\tau_m$</td><td colspan="3">τ_m</td><td>$\tau_m/T_m = 2$</td></tr>
<tr><td rowspan="2">Minimum IAE - Shinskey [17] – page 38
Model: Method 6</td><td colspan="3">$0.95 T_m/K_m\tau_m$</td><td colspan="3">$3.4\tau_m$</td><td>$\tau_m/T_m = 0.1$</td></tr>
<tr><td colspan="3">$0.95 T_m/K_m\tau_m$</td><td colspan="3">$2.9\tau_m$</td><td>$\tau_m/T_m = 0.2$</td></tr>
<tr><td>Minimum IAE – Huang *et al*. [18]
Model: Method 6</td><td colspan="3">$K_c^{(1)}$ [1]</td><td colspan="3">$T_i^{(1)}$</td><td>$0.1 \le \frac{\tau_m}{T_m} \le 1$</td></tr>
<tr><td rowspan="2">Minimum IAE – Yu [19]
(Load model = $\frac{K_L e^{-s\tau_L}}{1+sT_L}$)

Model: Method 6</td><td colspan="3">$\frac{0.685}{K_m}\left(\frac{T_L}{T_m}\right)^{0.214-0.346\frac{\tau_m}{T_m}}\left(\frac{\tau_m}{T_m}\right)^{-1.256}$</td><td colspan="3">$\frac{T_m}{0.214}\left(\frac{T_L}{T_m}\right)^{1.977\frac{\tau_m}{T_m}-0.55}\left(\frac{\tau_m}{T_m}\right)^{1.123}$</td><td>$\frac{T_L}{T_m} \le 2.641\frac{\tau_m}{T_m} + 0.16$;
$\frac{\tau_m}{T_m} \le 0.35$</td></tr>
<tr><td colspan="3">$\frac{0.874}{K_m}\left(\frac{T_L}{T_m}\right)^{-0.099+0.159\frac{\tau_m}{T_m}}\left(\frac{\tau_m}{T_m}\right)^{-1.041}$</td><td colspan="3">$\frac{T_m}{0.415}\left(\frac{T_L}{T_m}\right)^{-4.515\frac{\tau_m}{T_m}+0.067}\left(\frac{\tau_m}{T_m}\right)^{0.876}$</td><td>$2.641\frac{\tau_m}{T_m} + 0.16 \le \frac{T_L}{T_m} \le 1$;
$\frac{\tau_m}{T_m} \le 0.35$</td></tr>
</table>

[1] $$K_c^{(1)} = \frac{1}{K_m}\left[6.4884 + 4.6198\frac{\tau_m}{T_m} + 0.8196\left(\frac{\tau_m}{T_m}\right)^{-0.9077} - 5.2132\left(\frac{\tau_m}{T_m}\right)^{-0.063} - 7.2712\left(\frac{\tau_m}{T_m}\right)^{0.5961} - 0.7241 e^{\frac{\tau_m}{T_m}}\right]$$

$$T_i^{(1)} = T_m\left[0.0064 + 3.9574\frac{\tau_m}{T_m} - 6.4789\left(\frac{\tau_m}{T_m}\right)^2 + 9.4348\left(\frac{\tau_m}{T_m}\right)^3 - 10.7619\left(\frac{\tau_m}{T_m}\right)^4 + 7.5146\left(\frac{\tau_m}{T_m}\right)^5 - 2.2236\left(\frac{\tau_m}{T_m}\right)^6\right]$$

<table>
<tr><th>Rule</th><th colspan="3">K_c</th><th colspan="3">T_i</th><th colspan="3">Comment</th></tr>
<tr><td rowspan="2">Minimum IAE – Yu [19]
(Load model = $\frac{K_L e^{-s\tau_L}}{1+sT_L}$)
(continued)
Model: Method 6</td><td colspan="3">$\frac{0.871}{K_m}\left(\frac{T_L}{T_m}\right)^{-0.015+0.384\frac{\tau_m}{T_m}}\left(\frac{\tau_m}{T_m}\right)^{-1.055}$</td><td colspan="3">$\frac{T_m}{0.444}\left(\frac{T_L}{T_m}\right)^{-0.217\frac{\tau_m}{T_m}-0.213}\left(\frac{\tau_m}{T_m}\right)^{0.867}$</td><td colspan="3">$1<\frac{T_L}{T_m}\le 3$; $\frac{\tau_m}{T_m}\le 0.35$</td></tr>
<tr><td colspan="3">$\frac{0.513}{K_m}\left(\frac{T_L}{T_m}\right)^{0.218\frac{\tau_m}{T_m}}\left(\frac{\tau_m}{T_m}\right)^{-1.451}$</td><td colspan="3">$\frac{T_m}{0.670}\left(\frac{T_L}{T_m}\right)^{-0.003\frac{\tau_m}{T_m}-0.084}\left(\frac{\tau_m}{T_m}\right)^{0.56}$</td><td colspan="3">$\frac{\tau_m}{T_m}>0.35$</td></tr>
<tr><td rowspan="6">Minimum ISE - Hazebroek and Van der Waerden [9]

Model: Method 1</td><td colspan="3">$\frac{T_m}{K_m\tau_m}\left(0.74+0.3\frac{\tau_m}{T_m}\right)$</td><td colspan="3">$1.43T_m$</td><td colspan="3">$\tau_m/T_m<0.2$</td></tr>
<tr><td colspan="3">$\frac{\alpha T_m}{K_m\tau_m}$</td><td colspan="3">$\beta\tau_m$</td><td colspan="3"></td></tr>
<tr><td>τ_m/T_m</td><td>α</td><td>β</td><td>τ_m/T_m</td><td>α</td><td>β</td><td>τ_m/T_m</td><td>α</td><td>β</td></tr>
<tr><td>0.2</td><td>0.80</td><td>7.14</td><td>0.7</td><td>0.96</td><td>2.44</td><td>2.0</td><td>1.46</td><td>1.18</td></tr>
<tr><td>0.3</td><td>0.83</td><td>5.00</td><td>1.0</td><td>1.07</td><td>1.85</td><td>3.0</td><td>1.89</td><td>0.95</td></tr>
<tr><td>0.5</td><td>0.89</td><td>3.23</td><td>1.5</td><td>1.26</td><td>1.41</td><td>5.0</td><td>2.75</td><td>0.81</td></tr>
<tr><td>Minimum ISE - Murrill [13] – pages 358-363
Model: Method 3</td><td colspan="3">$\frac{1.305}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.959}$</td><td colspan="3">$\frac{T_m}{0.492}\left(\frac{\tau_m}{T_m}\right)^{0.739}$</td><td colspan="3">$0.1\le\frac{\tau_m}{T_m}\le 1.0$</td></tr>
<tr><td rowspan="2">Minimum ISE – Zhuang and Atherton [20]

Model: Method 6</td><td colspan="3">$\frac{1.279}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.945}$</td><td colspan="3">$\frac{T_m}{0.535}\left(\frac{\tau_m}{T_m}\right)^{0.586}$</td><td colspan="3">$0.1\le\frac{\tau_m}{T_m}\le 1.0$</td></tr>
<tr><td colspan="3">$\frac{1.346}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.675}$</td><td colspan="3">$\frac{T_m}{0.552}\left(\frac{\tau_m}{T_m}\right)^{0.438}$</td><td colspan="3">$1.1\le\frac{\tau_m}{T_m}\le 2.0$</td></tr>
<tr><td rowspan="4">Minimum ISE – Yu [19]
(Load model = $\frac{K_L e^{-s\tau_L}}{1+sT_L}$)

Model: Method 6</td><td colspan="3">$\frac{0.921}{K_m}\left(\frac{T_L}{T_m}\right)^{0.181-0.205\frac{\tau_m}{T_m}}\left(\frac{\tau_m}{T_m}\right)^{-1.214}$</td><td colspan="3">$\frac{T_m}{0.430}\left(\frac{T_L}{T_m}\right)^{0.954\frac{\tau_m}{T_m}-0.49}\left(\frac{\tau_m}{T_m}\right)^{0.639}$</td><td colspan="3">$\frac{T_L}{T_m}\le 2.310\frac{\tau_m}{T_m}+0.077$;
$\frac{\tau_m}{T_m}\le 0.35$</td></tr>
<tr><td colspan="3">$\frac{1.157}{K_m}\left(\frac{T_L}{T_m}\right)^{-0.045+0.344\frac{\tau_m}{T_m}}\left(\frac{\tau_m}{T_m}\right)^{-1.014}$</td><td colspan="3">$\frac{T_m}{0.359}\left(\frac{T_L}{T_m}\right)^{-2.532\frac{\tau_m}{T_m}-0.292}\left(\frac{\tau_m}{T_m}\right)^{0.899}$</td><td colspan="3">$2.310\frac{\tau_m}{T_m}+0.077\le\frac{T_L}{T_m}\le 1$;
$\frac{\tau_m}{T_m}\le 0.35$</td></tr>
<tr><td colspan="3">$\frac{1.07}{K_m}\left(\frac{T_L}{T_m}\right)^{-0.065+0.234\frac{\tau_m}{T_m}}\left(\frac{\tau_m}{T_m}\right)^{-1.047}$</td><td colspan="3">$\frac{T_m}{0.347}\left(\frac{T_L}{T_m}\right)^{-1.112\frac{\tau_m}{T_m}-0.094}\left(\frac{\tau_m}{T_m}\right)^{0.898}$</td><td colspan="3">$1<\frac{T_L}{T_m}\le 3$; $\frac{\tau_m}{T_m}\le 0.35$</td></tr>
<tr><td colspan="3">$\frac{1.289}{K_m}\left(\frac{T_L}{T_m}\right)^{0.04+0.067\frac{\tau_m}{T_m}}\left(\frac{\tau_m}{T_m}\right)^{-0.889}$</td><td colspan="3">$\frac{T_m}{0.596}\left(\frac{T_L}{T_m}\right)^{0.372\frac{\tau_m}{T_m}-0.44}\left(\frac{\tau_m}{T_m}\right)^{0.46}$</td><td colspan="3">$\frac{\tau_m}{T_m}>0.35$</td></tr>
<tr><td>Minimum ITAE - Murrill [13] – pages 358-363
Model: Method 3</td><td colspan="3">$\frac{0.859}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.977}$</td><td colspan="3">$\frac{T_m}{0.674}\left(\frac{\tau_m}{T_m}\right)^{0.680}$</td><td colspan="3">$0.1\le\frac{\tau_m}{T_m}\le 1.0$</td></tr>
<tr><td rowspan="3">Minimum ITAE – Yu [19]
(Load model = $\frac{K_L e^{-s\tau_L}}{1+sT_L}$)

Model: Method 6</td><td colspan="3">$\frac{0.598}{K_m}\left(\frac{T_L}{T_m}\right)^{0.272-0.254\frac{\tau_m}{T_m}}\left(\frac{\tau_m}{T_m}\right)^{-1.341}$</td><td colspan="3">$\frac{T_m}{0.805}\left(\frac{T_L}{T_m}\right)^{0.304\frac{\tau_m}{T_m}-0.112}\left(\frac{\tau_m}{T_m}\right)^{0.196}$</td><td colspan="3">$\frac{T_L}{T_m}\le 2.385\frac{\tau_m}{T_m}+0.112$;
$\frac{\tau_m}{T_m}\le 0.35$</td></tr>
<tr><td colspan="3">$\frac{0.735}{K_m}\left(\frac{T_L}{T_m}\right)^{-0.011-1.945\frac{\tau_m}{T_m}}\left(\frac{\tau_m}{T_m}\right)^{-1.055}$</td><td colspan="3">$\frac{T_m}{0.425}\left(\frac{T_L}{T_m}\right)^{-5.809\frac{\tau_m}{T_m}+0.241}\left(\frac{\tau_m}{T_m}\right)^{0.901}$</td><td colspan="3">$2.385\frac{\tau_m}{T_m}+0.112\le\frac{T_L}{T_m}\le 1$;
$\frac{\tau_m}{T_m}\le 0.35$</td></tr>
<tr><td colspan="3">$\frac{0.787}{K_m}\left(\frac{T_L}{T_m}\right)^{0.084+0.154\frac{\tau_m}{T_m}}\left(\frac{\tau_m}{T_m}\right)^{-1.042}$</td><td colspan="3">$\frac{T_m}{0.431}\left(\frac{T_L}{T_m}\right)^{-0.148\frac{\tau_m}{T_m}-0.365}\left(\frac{\tau_m}{T_m}\right)^{0.901}$</td><td colspan="3">$1<\frac{T_L}{T_m}\le 3$; $\frac{\tau_m}{T_m}\le 0.35$</td></tr>
</table>

Rule	K_c	T_i	Comment
Minimum ITAE – Yu [19] (Load model = $\frac{K_L e^{-s\tau_L}}{1+sT_L}$) (continued) Model: Method 6	$\frac{0.878}{K_m}\left(\frac{T_L}{T_m}\right)^{0.172-0.057\frac{\tau_m}{T_m}}\left(\frac{\tau_m}{T_m}\right)^{-0.909}$	$\frac{T_m}{0.794}\left(\frac{T_L}{T_m}\right)^{0.228\frac{\tau_m}{T_m}-0.257}\left(\frac{\tau_m}{T_m}\right)^{0.489}$	$\frac{\tau_m}{T_m} > 0.35$
Minimum ISTSE - Zhuang and Atherton [20] *Model: Method 6*	$\frac{1.015}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.957}$	$\frac{T_m}{0.667}\left(\frac{\tau_m}{T_m}\right)^{0.552}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
	$\frac{1.065}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.673}$	$\frac{T_m}{0.687}\left(\frac{\tau_m}{T_m}\right)^{0.427}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$
Minimum ISTES – Zhuang and Atherton [20] *Model: Method 6*	$\frac{1.021}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.953}$	$\frac{T_m}{0.629}\left(\frac{\tau_m}{T_m}\right)^{0.546}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
	$\frac{1.076}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.648}$	$\frac{T_m}{0.650}\left(\frac{\tau_m}{T_m}\right)^{0.442}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$
Servo tuning	**Performance index minimisation**		
Minimum IAE – Rovira *et al*. [21] *Model: Method 3*	$\frac{0.758}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.861}$	$\frac{T_m}{1.020-0.323\frac{\tau_m}{T_m}}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
Minimum IAE - Huang *et al*. [18] *Model: Method 6*	$K_c^{(2)}$ [2]	$T_i^{(2)}$	$0.1 \le \frac{\tau_m}{T_m} \le 1$
Minimum ISE - Zhuang and Atherton [20] *Model: Method 6*	$\frac{0.980}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.892}$	$\frac{T_m}{0.690-0.155\frac{\tau_m}{T_m}}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
	$\frac{1.072}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.560}$	$\frac{T_m}{0.648-0.114\frac{\tau_m}{T_m}}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$
Minimum ISE – Khan and Lehman [22] *Model: Method 6*	$\left(\frac{0.7388}{\tau_m}+\frac{0.3185}{T_m}\right)\frac{T_m}{K_m}$	$\frac{K_c K_m}{\left(\frac{0.5291}{T_m\tau_m}-\frac{0.0003082}{\tau_m^{\ 2}}\right)}$	$0.01 \le \frac{\tau_m}{T_m} \le 0.2$
	$\left(\frac{0.808}{\tau_m}+\frac{0.511}{T_m}-\frac{0.255}{\sqrt{T_m\tau_m}}\right)\frac{T_m}{K_m}$	$\frac{K_c K_m}{\left(\frac{0.095}{\tau_m^{\ 2}}+\frac{0.846}{\tau_m T_m}-\frac{0.381}{\tau_m\sqrt{T_m\tau_m}}\right)}$	$0.2 \le \frac{\tau_m}{T_m} \le 20$
Minimum ITAE – Rovira *et al*. [21] *Model: Method 3*	$\frac{0.586}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.916}$	$\frac{T_m}{1.030-0.165\frac{\tau_m}{T_m}}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$

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[2] $$K_c^{(2)} = \frac{1}{K_m}\left[-13.0454-9.0916\frac{\tau_m}{T_m}+0.3053\left(\frac{\tau_m}{T_m}\right)^{-1.0169}+1.1075\left(\frac{\tau_m}{T_m}\right)^{3.5959}-2.2927\left(\frac{\tau_m}{T_m}\right)^{3.6843}+4.8259e^{\frac{\tau_m}{T_m}}\right]$$

$$T_i^{(2)} = T_m\left[0.9771-0.2492\frac{\tau_m}{T_m}+3.4651\left(\frac{\tau_m}{T_m}\right)^2-7.4538\left(\frac{\tau_m}{T_m}\right)^3+8.2567\left(\frac{\tau_m}{T_m}\right)^4-4.7536\left(\frac{\tau_m}{T_m}\right)^5+1.1496\left(\frac{\tau_m}{T_m}\right)^6\right]$$

Rule	K_c	T_i	Comment
Minimum ISTSE - Zhuang and Atherton [20] *Model: Method 6*	$\frac{0.712}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.921}$	$\frac{T_m}{0.968-0.247\frac{\tau_m}{T_m}}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
	$\frac{0.786}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.559}$	$\frac{T_m}{0.883-0.158\frac{\tau_m}{T_m}}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$
Minimum ISTES – Zhuang and Atherton [20] *Model: Method 6*	$\frac{0.569}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.951}$	$\frac{T_m}{1.023-0.179\frac{\tau_m}{T_m}}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
	$\frac{0.628}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.583}$	$\frac{T_m}{1.007-0.167\frac{\tau_m}{T_m}}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$
Direct synthesis			
Haalman [23] *Model: Method 6*	$\frac{2T_m}{3K_m\tau_m}$	T_m	Closed loop sensitivity $M_s = 1.9$. (Astrom and Hagglund [3])
Chen and Yang [23a] *Model: Method 25*	$\frac{0.7T_m}{K_m\tau_m}$	T_m	$M_s = 1.26$; $A_m = 2.24$; $\phi_m = 50^0$
Minimum IAE – regulator - Pemberton [24], Smith and Corripio [25] – *page 343-346. Model: Method 6*	$\frac{T_m}{K_m\tau_m}$	T_m	$0.1 \le \frac{\tau_m}{T_m} \le 0.5$
Minimum IAE – servo - Smith and Corripio [25] – *page 343-346. Model: Method 6*	$\frac{3T_m}{5K_m\tau_m}$	T_m	$0.1 \le \frac{\tau_m}{T_m} \le 0.5$
5% overshoot – servo – Smith *et al*. [26] – *deduced from graph. Model: Method not stated*	$\frac{0.52T_m}{K_m\tau_m}$	T_m	$0.04 \le \frac{\tau_m}{T_m} \le 1.4$
1% overshoot – servo – Smith *et al*. [26] – *deduced from graph Model: Method not stated*	$\frac{0.44T_m}{K_m\tau_m}$	T_m	$0.04 \le \frac{\tau_m}{T_m} \le 1.4$
5% overshoot - servo - Smith and Corripio [25] – *page 343-346. Model: Method 6*	$\frac{T_m}{2K_m\tau_m}$	T_m	
5% overshoot - servo - Hang *et al*. [27] *Model: Method 1*	$\frac{13T_m}{25K_m\tau_m}$	T_m	
Miluse *et al*. [27a] *Model: Method not stated*	$\frac{0.368T_m}{K_m\tau_m}$	T_m	Closed loop response overshoot = 0% (Model: Method 26 – Miluse *et al*. [27b])
	$\frac{0.514T_m}{K_m\tau_m}$	T_m	Closed loop response overshoot = 5%
	$\frac{0.581T_m}{K_m\tau_m}$	T_m	Closed loop response overshoot = 10%

Rule	K_c	T_i	Comment
Miluse *et al*. [27a] - continued	$\frac{0.641 T_m}{K_m \tau_m}$	T_m	Closed loop response overshoot = 15%
	$\frac{0.696 T_m}{K_m \tau_m}$	T_m	Closed loop response overshoot = 20%
Model: Method not stated	$\frac{0.748 T_m}{K_m \tau_m}$	T_m	Closed loop response overshoot = 25%
	$\frac{0.801 T_m}{K_m \tau_m}$	T_m	Closed loop response overshoot = 30%
	$\frac{0.853 T_m}{K_m \tau_m}$	T_m	Closed loop response overshoot = 35%
	$\frac{0.906 T_m}{K_m \tau_m}$	T_m	Closed loop response overshoot = 40%
	$\frac{0.957 T_m}{K_m \tau_m}$	T_m	Closed loop response overshoot = 45%
	$\frac{1.008 T_m}{K_m \tau_m}$	T_m	Closed loop response overshoot = 50%
Regulator – Gorecki *et al*. [28] (considered as 2 rules) *Model: Method 6*	$K_c^{(3)}$	$T_i^{(3)}$	Pole is real and has max. attainable multiplicity
	$K_c^{(4)}$ [3]	$T_i^{(4)}$	Low freq. part of magnitude Bode diagram is flat
Chiu *et al*. [29] *Model: Method 6*	$\frac{\lambda T_m}{K_m(1+\lambda \tau_m)}$	T_m	λ variable; suggested values: 0.2, 0.4, 0.6, 1.0.
Astrom *et al*. [30] *Model: Method 22*	$\frac{3}{K_m\left(1+\frac{3\tau_m}{T_m}\right)}$	T_m	Honeywell UDC 6000 controller
Davydov *et al*. [31] *Model: Method 12*	$\frac{1}{K_m\left(1.905\frac{\tau_m}{T_m}+0.826\right)}$	$\left(0.153\frac{\tau_m}{T_m}+0.362\right)T_m$	Closed loop response damping factor = 0.9; $0.2 \le \tau_m/T_m \le 1$.

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[3] $$K_c^{(3)} = \frac{2}{K_m}\frac{T_m}{\tau_m}\left[\sqrt{2+\left(\frac{\tau_m}{2T_m}\right)^2}-1\right]e^{\sqrt{2+\left(\frac{\tau_m}{2T_m}\right)^2}-2-\frac{\tau_m}{2T_m}}$$

$$T_i^{(3)} = \tau_m \frac{1+\left(\frac{\tau_m}{2T_m}\right)^2}{3+\left(\frac{\tau_m}{2T_m}\right)+\left(\frac{\tau_m}{T_m}\right)^2+\left(\frac{\tau_m}{2T_m}\right)^3-\left[2+\left(\frac{\tau_m}{2T_m}\right)^2\right]\sqrt{2+\left(\frac{\tau_m}{2T_m}\right)^2}}$$

$$K_c^{(4)} = \frac{1}{K_m}\frac{1+3\frac{T_m}{\tau_m}+6\left(\frac{T_m}{\tau_m}\right)^2+6\left(\frac{T_m}{\tau_m}\right)^3}{4\left[1+3\frac{T_m}{\tau_m}+3\left(\frac{T_m}{\tau_m}\right)^2\right]},\quad T_i^{(4)} = \tau_m\frac{1+3\frac{T_m}{\tau_m}+6\left(\frac{T_m}{\tau_m}\right)^2+6\left(\frac{T_m}{\tau_m}\right)^3}{3\left[1+2\frac{T_m}{\tau_m}+2\left(\frac{T_m}{\tau_m}\right)^2\right]}$$

Rule	K_c	T_i	Comment
Schneider [32] *Model: Method 6*	$0.368\dfrac{T_m}{K_m\tau_m}$	T_m	Closed loop response damping factor = 1
	$0.403\dfrac{T_m}{K_m\tau_m}$	T_m	Closed loop response damping factor = 0.6
McAnany [33] *Model: Method 5*	$\dfrac{(1.44T_m + 0.72T_m\tau_m - 0.43\tau_m - 2.14)}{K_m(1.2\tau_m + 0.36\tau_m{}^2 + 2)}$	$\dfrac{K_m(5.56 + 2\tau_m + \tau_m{}^2)}{4T_m + 1.28\tau_m - 2.4}$	Closed loop time constant = $1.67T_m$.
Leva *et al*. [34] *Model: Method 16*	$\dfrac{\omega_{cn}T_m}{K_m}\sqrt{\dfrac{1+\omega_{cn}{}^2T_m{}^2}{1+\omega_{cn}{}^2T_i^2}}$	$\dfrac{\tan\left[\phi_m - \dfrac{\pi}{2} + \tau_m\omega_{cn} + \tan^{-1}(\omega_{cn}T_m)\right]}{\omega_{cn}}$	$\omega_{cn} = \dfrac{2.82 - \phi_m - \tan^{-1}\left[\dfrac{T_m}{\tau_m}\left(\dfrac{\pi}{2} - \phi_m\right)\right]}{\tau_m}$
Khan and Lehman [22] *Model: Method 6*	$\dfrac{T_m}{K_m}\left(\dfrac{0.3852}{\tau_m} + \dfrac{0.723}{T_m} - 0.404\dfrac{\tau_m}{T_m{}^2}\right)$	$\dfrac{K_cK_m}{\left(\dfrac{0.4104}{\tau_mT_m} - \dfrac{0.00024}{\tau_m{}^2} - \dfrac{0.525}{T_m{}^2}\right)}$	$0.01 \le \dfrac{\tau_m}{T_m} \le 0.2$
	$\dfrac{T_m}{K_m}\left(\dfrac{0.404}{\tau_m} + \dfrac{0.256}{T_m} - \dfrac{0.1275}{\sqrt{\tau_mT_m}}\right)$	$\dfrac{K_cK_m}{\left(\dfrac{0.719}{\tau_mT_m} + \dfrac{0.0808}{\tau_m{}^2} - \dfrac{0.324}{\tau_m\sqrt{\tau_mT_m}}\right)}$	$0.2 \le \dfrac{\tau_m}{T_m} \le 20$
Hang *et al*. [35, 36] *Model: Method 11* *Or* *Model: Method 14*	$\dfrac{1.048T_m}{K_m\tau_m}$	T_m	Gain Margin = 1.5 Phase Margin = 30^0
	$\dfrac{0.7854T_m}{K_m\tau_m}$	T_m	Gain Margin = 2 Phase Margin = 45^0
	$\dfrac{0.524T_m}{K_m\tau_m}$	T_m	Gain Margin = 3 Phase Margin = 60^0
	$\dfrac{0.393T_m}{K_m\tau_m}$	T_m	Gain Margin = 4 Phase Margin = 67.5^0
	$\dfrac{0.314T_m}{K_m\tau_m}$	T_m	Gain Margin = 5 Phase Margin = 72^0
Gain and phase margin – Ho *et al*. [37] (considered as 2 rules) Model: Method 6	$\dfrac{\omega_pT_m}{K_mA_m}$	$\dfrac{1}{2\omega_p - \dfrac{4\omega_p{}^2\tau_m}{\pi} + \dfrac{1}{T_m}}$	$\omega_p = \dfrac{A_m\phi_m + 0.5\pi A_m(A_m - 1)}{(A_m{}^2 - 1)\tau_m}$
	Given A_m, ISE is minimised when $\phi_m = 68.8884 - 34.3534A_m + 9.1606\dfrac{\tau_m}{T_m}$ for servo tuning (Ho *et al* [104]).		
	Given A_m, ISE is minimised when $\phi_m = 45.9848A_m{}^{0.2677}(\tau_m/T_m)^{0.2755}$ for regulator tuning (Ho *et al* [104]).		
Tan *et al*. [39] *Model: Method 10*	$\dfrac{\beta T_i\omega_\phi\sqrt{1+(\beta T_m\omega_\phi)^2}}{A_m\sqrt{1+(\beta T_i\omega_\phi)^2}}$	$\dfrac{1}{\beta\omega_\phi\tan\left[-\tan^{-1}\beta T_m\omega_\phi - \beta\tau_m\omega_\phi - \phi\right]}$ $\omega_\phi < \omega_u$	$\beta = 0.8, \dfrac{\tau_m}{T_m} < 0.5$; $\beta = 0.5, \dfrac{\tau_m}{T_m} > 0.5$
Symmetrical optimum principle - Voda and Landau [40] *Model: Method not relevant*	$\dfrac{1}{3.5\lvert G_p(j\omega_{135^0})\rvert}$	$\dfrac{4.6}{\omega_{135^0}}$	$\dfrac{\tau_m}{T_m} \le 0.1$
	$\dfrac{1}{2.828\lvert G_p(j\omega_{135^0})\rvert}$	$\dfrac{4}{\omega_{135^0}}$	$0.1 < \dfrac{\tau_m}{T_m} \le 0.15$
	$\dfrac{1}{4.6\lvert G_p(j\omega_{135^0})\rvert - 0.6K_m}$	$\dfrac{1.15\lvert G_p(j\omega_{135^0})\rvert + 0.75K_m}{\omega_{135^0}\left[2.3\lvert G_p(j\omega_{135^0})\rvert - 0.3K_m\right]}$	$0.15 < \dfrac{\tau_m}{T_m} \le 1$

Rule	K_c	T_i	Comment
Friman and Waller [41] *Model: Method 6*	$\frac{0.2333}{\lvert G_p(j\omega_{135^0})\rvert}$	$\frac{1}{\omega_{135^0}}$	$\tau_m > 2T_m$. Gain margin = 3; Phase margin = 45^0
Voda and Landau [40] *Model: Method 6*	$\frac{T_m}{2K_m\tau_m}$	T_m	Phase margin = 60^0 ; $0.25 \le \frac{\tau_m}{T_m} \le 1$
Smith [42] *Model: Method not specified*	$\frac{0.35}{K_m}$	$0.42\tau_m$	6 dB gain margin - dominant delay process
Modulus optimum principle - Cox *et al*. [43][4] *Model: Method 17*	$K_c^{(5)}$	$T_i^{(5)}$	$\frac{\tau_m}{T_m} \le 1$
	$\frac{0.5T_m}{K_m\tau_m}$	T_m	$\frac{\tau_m}{T_m} > 1$
Cluett and Wang [44] *Model: Method 6*	$\frac{0.019952\tau_m + 0.20042T_m}{K_m\tau_m}$	$\frac{0.099508\tau_m + 0.99956T_m}{0.99747\tau_m - 8.7425.10^{-5}T_m}\tau_m$	Closed loop time constant = $4\tau_m$
	$\frac{0.05548\tau_m + 0.33639T_m}{K_m\tau_m}$	$\frac{0.16440\tau_m + 0.99558T_m}{0.98607\tau_m - 1.5032.10^{-4}T_m}\tau_m$	Closed loop time constant = $2\tau_m$
	$\frac{0.092654\tau_m + 0.43620T_m}{K_m\tau_m}$	$\frac{0.20926\tau_m + 0.98518T_m}{0.96515\tau_m + 4.2550.10^{-3}T_m}\tau_m$	Closed loop time constant = $1.33\tau_m$
	$\frac{0.12786\tau_m + 0.51235T_m}{K_m\tau_m}$	$\frac{0.24145\tau_m + 0.96751T_m}{0.93566\tau_m + 2.2988.10^{-2}T_m}\tau_m$	Closed loop time constant = τ_m
	$\frac{0.16051\tau_m + 0.57109T_m}{K_m\tau_m}$	$\frac{0.26502\tau_m + 0.94291T_m}{0.89868\tau_m + 6.9355.10^{-2}T_m}\tau_m$	Closed loop time constant = $0.8\tau_m$
	$\frac{0.19067\tau_m + 0.61593T_m}{K_m\tau_m}$	$\frac{0.28242\tau_m + 0.91231T_m}{0.85491\tau_m + 0.15937T_m}\tau_m$	Closed loop time constant = $0.67\tau_m$
Abbas [45] *Model: Method 6*	$\frac{0.148 + 0.186\left(\frac{\tau_m}{T_m}\right)^{-1.045}}{K_m(0.497 - 0.464V^{0.590})}$	$T_m + 0.5\tau_m$	V = fractional overshoot, $0 \le V \le 0.2$ $0.1 \le \frac{\tau_m}{T_m} \le 5.0$
Bi *et al*. [46] *Model: Method 20*	$\frac{0.5064T_m}{K_m\tau_m}$	T_m	

[4] $K_c^{(5)} = \frac{0.5}{K_m}\left(\frac{T_m^3 + T_m^2\tau_m + 0.5T_m\tau_m^2 + 0.167\tau_m^3}{T_m^2\tau_m + T_m\tau_m^2 + 0.667\tau_m^3}\right)$, $T_i^{(5)} = \left(\frac{T_m^3 + T_m^2\tau_m + 0.5T_m\tau_m^2 + 0.167\tau_m^3}{T_m^2 + T_m\tau_m + 0.5\tau_m^2}\right)$

Rule	K_c	T_i	Comment
Wang and Shao [47] *Model: Method 6*	$K_c^{(5a)}$ [5]	$T_i^{(5a)}$	λ = inverse of the maximum of the absolute real part of the open loop transfer function; $\lambda = [1.5, 2.5]$
Robust			
Brambilla *et al*. [48] - *Model: Method 6*	$\dfrac{T_m + 0.5\tau_m}{K_m(\lambda + \tau_m)}$	$T_m + 0.5\tau_m$	
	Closed loop response has less than 5% overshoot with no model uncertainty: $\lambda = 1$, $0.1 \le \dfrac{\tau_m}{T_m} \le 1$; $\lambda = 1 - 0.5\log_{10}\dfrac{\tau_m}{T_m}$, $1 < \dfrac{\tau_m}{T_m} \le 10$		
Rivera *et al*. [49] *Model: Method 6*	$\dfrac{T_m}{\lambda K_m}$	T_m	$\lambda \ge 1.7\tau_m$, $\lambda > 0.1T_m$.
	$\dfrac{2T_m + \tau_m}{2\lambda K_m}$	$T_m + 0.5\tau_m$	$\lambda \ge 1.7\tau_m$, $\lambda > 0.1T_m$.
Chien [50] *Model: Method 6*	$\dfrac{T_m}{K_m(\tau_m + \lambda)}$	T_m	$\lambda = T_m$ [50]; $\lambda > T_m + \tau_m$, $\tau_m << T_m$ (Thomasson [51])
Thomasson [51] *Model: Method not defined*	$\dfrac{\tau_m}{2K_m(\tau_m + \lambda)}$	$0.5\tau_m$	$\tau_m >> T_m$; λ = desired closed loop time constant
Fruehauf *et al*. [52] *Model: Method 1*	$\dfrac{5T_m}{9\tau_m K_m}$	$5\tau_m$	$\dfrac{\tau_m}{T_m} < 0.33$
	$\dfrac{T_m}{2\tau_m K_m}$	T_m	$\dfrac{\tau_m}{T_m} \ge 0.33$
Chen *et al*. [53] *Model: Method 6*	$\dfrac{0.50T_m}{\tau_m K_m}$	T_m	$A_m = 3.14$, $\phi_m = 61.4^0$, $M_s = 1.00$
	$\dfrac{0.61T_m}{\tau_m K_m}$	T_m	$A_m = 2.58$, $\phi_m = 55.0^0$, $M_s = 1.10$
	$\dfrac{0.67T_m}{\tau_m K_m}$	T_m	$A_m = 2.34$, $\phi_m = 51.6^0$, $M_s = 1.20$
	$\dfrac{0.70T_m}{\tau_m K_m}$	T_m	$A_m = 2.24$, $\phi_m = 50.0^0$, $M_s = 1.26$
	$\dfrac{0.72T_m}{\tau_m K_m}$	T_m	$A_m = 2.18$, $\phi_m = 48.7^0$, $M_s = 1.30$

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[5] $K_c^{(5a)} = \dfrac{1}{\lambda f_1(\omega_{90^0})}\left[f_2(\omega_{90^0}) - \dfrac{1}{\omega_{90^0}}\right]$,

$$f_1(\omega_{90^0}) = \frac{K_m}{\left(1 + \omega_{90^0}{}^2 T_m{}^2\right)^{1.5}}\left[(T_m + \{1 + \omega_{90^0}{}^2 T_m{}^2\}\tau_m)\sin(-\omega_{90^0}\tau_m - \tan^{-1}\omega_{90^0}T_m) - \omega_{90^0}T_m{}^2\cos(-\omega_{90^0}\tau_m - \tan^{-1}\omega_{90^0}T_m)\right]$$

$$f_2(\omega_{90^0}) = -\frac{1}{1 + \omega_{90^0}{}^2 T_m{}^2}\left[(T_m + \{1 + \omega_{90^0}{}^2 T_m{}^2\}\tau_m)\cot(-\omega_{90^0}\tau_m - \tan^{-1}\omega_{90^0}T_m) + \omega_{90^0}T_m{}^2\right]$$

$$T_i^{(5a)} = \frac{\omega_{90^0}\left[T_m + (1 + \omega_{90^0}{}^2 T_m{}^2)\tau_m\right]\cos(-\omega_{90^0}\tau_m - \tan^{-1}\omega_{90^0}T_m) + (1 + 2\omega_{90^0}{}^2 T_m{}^2)\sin(-\omega_{90^0}\tau_m - \tan^{-1}\omega_{90^0}T_m)}{-\omega_{90^0}{}^3 T_m{}^2\cos(-\omega_{90^0}\tau_m - \tan^{-1}\omega_{90^0}T_m) + \omega_{90^0}{}^2[T_m + (1 + \omega_{90^0}{}^2 T_m{}^2)\tau_m]\sin(-\omega_{90^0}\tau_m - \tan^{-1}\omega_{90^0}T_m)}$$

<table>
<tr><th>Rule</th><th colspan="3">K_c</th><th colspan="3">T_i</th><th>Comment</th></tr>
<tr><td rowspan="2">Chen et al. [53]
- continued

Model: Method 6</td><td colspan="3">$\frac{0.76T_m}{\tau_m K_m}$</td><td colspan="3">$T_m$</td><td>$A_m = 2.07$, $\phi_m = 46.5^0$,
$M_s = 1.40$</td></tr>
<tr><td colspan="3">$\frac{0.80T_m}{\tau_m K_m}$</td><td colspan="3">$T_m$</td><td>$A_m = 1.96$, $\phi_m = 44.1^0$,
$M_s = 1.50$</td></tr>
<tr><td rowspan="10">Ogawa [54] – deduced from graphs

Model: Method 6</td><td colspan="3">$\frac{\alpha}{K_m}$</td><td colspan="3">βT_m</td><td></td></tr>
<tr><td>τ_m/T_m</td><td>α</td><td>β</td><td>τ_m/T_m</td><td>α</td><td>β</td><td></td></tr>
<tr><td>0.5</td><td>0.9</td><td>1.3</td><td>2.0</td><td>0.45</td><td>2.0</td><td rowspan="2">20% uncertainty in the process parameters</td></tr>
<tr><td>1.0</td><td>0.6</td><td>1.6</td><td>10.0</td><td>0.4</td><td>7.0</td></tr>
<tr><td>0.5</td><td>0.7</td><td>1.3</td><td>2.0</td><td>0.4</td><td>2.2</td><td rowspan="2">33% uncertainty in the process parameters</td></tr>
<tr><td>1.0</td><td>0.47</td><td>1.7</td><td>10.0</td><td>0.35</td><td>7.5</td></tr>
<tr><td>0.5</td><td>0.47</td><td>1.3</td><td>2.0</td><td>0.32</td><td>2.4</td><td rowspan="2">50% uncertainty in the process parameters</td></tr>
<tr><td>1.0</td><td>0.36</td><td>1.8</td><td>10.0</td><td>0.3</td><td>8.5</td></tr>
<tr><td>0.5</td><td>0.4</td><td>1.3</td><td>2.0</td><td>0.3</td><td>2.4</td><td rowspan="2">60% uncertainty in the process parameters</td></tr>
<tr><td>1.0</td><td>0.33</td><td>1.8</td><td>10.0</td><td>0.29</td><td>9.0</td></tr>
<tr><td>Lee et al. [55]

Model: Method 6</td><td colspan="3">$\frac{T_i}{K_m(\lambda + \tau_m)}$</td><td colspan="3">$T_m + \frac{\tau_m^{\ 2}}{2(\lambda + \tau_m)}$</td><td>Desired closed loop response = $\frac{e^{-\tau_m s}}{(\lambda s + 1)}$,
$\lambda = 0.333\tau_m$</td></tr>
<tr><td>Isaksson and Graebe [56]
Model: Method 6</td><td colspan="3">$\frac{T_m + 0.25\tau_m}{K_m \lambda}$</td><td colspan="3">$T_m + 0.25\tau_m$</td><td>$T_m > \tau_m$</td></tr>
<tr><td>Chun et al. [57]

Model: Method 21</td><td colspan="3">$\frac{T_m(\tau_m + 2\lambda) - \lambda^2}{K_m(\tau_m + \lambda)^2}$</td><td colspan="3">$\frac{T_m(\tau_m + 2\lambda) - \lambda^2}{\tau_m + T_m}$</td><td>$\lambda = 0.4T_m$</td></tr>
<tr><td>Ultimate cycle</td><td colspan="7"></td></tr>
<tr><td>McMillan [58]

Model: Method 1 or Method 18</td><td colspan="3">$\frac{1.881}{K_m}\frac{T_m}{\tau_m}\left\{\frac{1}{1+\left(\frac{T_m}{T_m+\tau_m}\right)^{0.65}}\right\}$</td><td colspan="3">$1.66\tau_m\left\{1+\left(\frac{T_m}{T_m+\tau_m}\right)^{0.65}\right\}$</td><td>Tuning rules developed from K_u, T_u</td></tr>
<tr><td>Regulator – minimum IAE – Shinskey [59] – page 167.
Model: Method not specified</td><td colspan="3">$\frac{K_u}{3.05 - 0.35\frac{T_u}{\tau_m}}$</td><td colspan="3">$T_u\left(0.87 - 0.855\frac{T_u}{\tau_m} + 0.172\left[\frac{T_u}{\tau_m}\right]^2\right)$</td><td></td></tr>
<tr><td rowspan="2">Regulator – minimum IAE – Shinskey [17] – page 121.
Model: Method not specified</td><td colspan="3">$0.55K_u$</td><td colspan="3">$0.78T_u$</td><td>$\tau_m/T_m = 0.2$</td></tr>
<tr><td colspan="3">$0.48K_u$</td><td colspan="3">$0.47T_u$</td><td>$\tau_m/T_m = 1$</td></tr>
<tr><td rowspan="4">Regulator – minimum IAE – Shinskey [16] – page 148
Model: Method 6</td><td colspan="3">$0.5848K_u$</td><td colspan="3">$0.81T_u$</td><td>$\tau_m/T_m = 0.2$</td></tr>
<tr><td colspan="3">$0.5405K_u$</td><td colspan="3">$0.66T_u$</td><td>$\tau_m/T_m = 0.5$</td></tr>
<tr><td colspan="3">$0.4762K_u$</td><td colspan="3">$0.47T_u$</td><td>$\tau_m/T_m = 1$</td></tr>
<tr><td colspan="3">$0.4608K_u$</td><td colspan="3">$0.37T_u$</td><td>$\tau_m/T_m = 2$</td></tr>
<tr><td>Regulator – nearly minimum IAE, ISE, ITAE – Hwang [60]
Model: Method 8</td><td colspan="3">$(1-\mu_1)K_u$,
$\mu_1 = \frac{1.14\left(1 - 0.482\omega_u\tau_m + 0.068\omega_u^2\tau_m^2\right)}{K_u K_m / 1 + K_u K_m}$</td><td colspan="3">$K_c/\mu_2 K_u \omega_u$,
$m_2 = \frac{0.0694\left(-1 + 2.1w_u t_m - 0.367w_u^2 t_m^2\right)}{K_u K_m / 1 + K_u K_m}$</td><td>Decay ratio = 0.15 -
$0.1 \le \frac{\tau_m}{T_m} \le 2.0$</td></tr>
</table>

Rule	K_c	T_i	Comment
Regulator – nearly minimum IAE, ISE, ITAE – Hwang [60] (continued) *Model: Method 8*	$(1-\mu_1)K_u$, $\mu_1 = \frac{1.09(1 - 0.497\omega_u\tau_m + 0.0724\omega_u^2\tau_m^2)}{K_uK_m/1 + K_uK_m}$	$K_c/\mu_2K_u\omega_u$, $m_2 = \frac{0.054(-1+2.54w_ut_m - 0.457w_u^2t_m^2)}{K_uK_m/1+K_uK_m}$	Decay ratio = 0.2 - $0.1 \le \frac{\tau_m}{T_m} \le 2.0$
	$(1-\mu_1)K_u$, $m_1 = \frac{1.03(1-0.51w_ut_m + 0.0759w_u^2t_m^2)}{K_uK_m/1+K_uK_m}$	$K_c/\mu_2K_u\omega_u$, $m_2 = \frac{0.0386(-1+3.26w_ut_m - 0.6w_u^2t_m^2)}{K_uK_m/1+K_uK_m}$	Decay ratio = 0.25 - $0.1 \le \frac{\tau_m}{T_m} \le 2.0$
Servo – small IAE – Hwang [60] *Model: Method 8* r = parameter related to the position of the dominant real pole.	$(1-\mu_1)K_u$, $m_1 = \frac{1.07(1-0.466w_ut_m + 0.0667w_u^2t_m^2)}{K_uK_m/1+K_uK_m}$	$K_c/\mu_2K_u\omega_u$, $m_2 = \frac{0.0328(-1+2.21w_ut_m - 0.338w_u^2t_m^2)}{K_uK_m/1+K_uK_m}$	Decay ratio = 0.1, r = 0.5 - $0.1 \le \frac{\tau_m}{T_m} \le 2.0$
	$(1-\mu_1)K_u$, $\mu_1 = \frac{1.11(1-0.467\omega_u\tau_m + 0.0657\omega_u^2\tau_m^2)}{K_uK_m/1+K_uK_m}$	$K_c/\mu_2K_u\omega_u$, $\mu_2 = \frac{0.0477(-1+2.07\omega_u\tau_m - 0.333\omega_u^2\tau_m^2)}{K_uK_m/1+K_uK_m}$	Decay ratio = 0.1, r = 0.75 - $0.1 \le \frac{\tau_m}{T_m} \le 2.0$
	$(1-\mu_1)K_u$, $m_1 = \frac{1.14(1-0.466w_ut_m + 0.0647w_u^2t_m^2)}{K_uK_m/1+K_uK_m}$	$K_c/\mu_2K_u\omega_u$, $m_2 = \frac{0.0609(-1+1.97w_ut_m - 0.323w_u^2t_m^2)}{K_uK_m/1+K_uK_m}$	Decay ratio = 0.1, r = 1.0 - $0.1 \le \frac{\tau_m}{T_m} \le 2.0$
Hang *et al*. [65] *Model: Method not specified*	$\frac{5}{6}\left(\frac{12+2\left[\frac{11\frac{\tau_m}{T_m}+13}{37\frac{\tau_m}{T_m}-4}\right]}{15+14\left[\frac{11\frac{\tau_m}{T_m}+13}{37\frac{\tau_m}{T_m}-4}\right]}\right)K_u$	$0.2\left(\frac{4}{15}\left[\frac{11\frac{\tau_m}{T_m}+13}{37\frac{\tau_m}{T_m}-4}\right]+1\right)T_u$	$0.16 \le \frac{\tau_m}{T_m} < 0.96$; Servo response: 10% overshoot, 3% undershoot
Servo – minimum ISTSE – Zhuang and Atherton [20] *Model: Method not relevant*	$0.361K_u$	$0.083(1.935K_mK_u + 1)T_u$	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$
Regulator – minimum ISTSE – Zhuang and Atherton [20] *Model: Method not relevant*	$\left(\frac{1.892K_mK_u + 0.244}{3.249K_mK_u + 2.097}\right)K_u$	$\left(\frac{0.706K_mK_u - 0.227}{0.7229K_mK_u + 1.2736}\right)K_u$	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$
Servo – nearly minimum IAE and ITAE – Hwang and Fang [61] *Model: Method 9*	$\left(0.438 - 0.110\frac{\tau_m}{T_m} + 0.0376\left[\frac{\tau_m}{T_m}\right]^2\right)K_u$	$\frac{(K_c / K_u\omega_u)}{\left(0.0388 + 0.108\frac{\tau_m}{T_m} - 0.0154\left[\frac{\tau_m}{T_m}\right]^2\right)}$	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$; decay ratio = 0.03
Regulator – nearly minimum IAE and ITAE – Hwang and Fang [61] *Model: Method 9*	$\left(0.515 - 0.0521\frac{\tau_m}{T_m} + 0.0254\left[\frac{\tau_m}{T_m}\right]^2\right)K_u$	$\frac{(K_c / K_u\omega_u)}{\left(0.0877 + 0.0918\frac{\tau_m}{T_m} - 0.0141\left[\frac{\tau_m}{T_m}\right]^2\right)}$	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$; decay ratio = 0.12
Simultaneous Servo/regulator – Hwang and Fang [61] *Model: Method 9*	$\left(0.46 - 0.0835\frac{\tau_m}{T_m} + 0.0305\left[\frac{\tau_m}{T_m}\right]^2\right)K_u$	$\frac{(K_c / K_u\omega_u)}{\left(0.0644 + 0.0759\frac{\tau_m}{T_m} - 0.0111\left[\frac{\tau_m}{T_m}\right]^2\right)}$	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$

Table 2: PI tuning rules - FOLPD model - $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{1 + sT_m}$ – Controller $G_c(s) = K_c\left(b + \dfrac{1}{T_i s}\right)$. *2 tuning rules*

Rule	K_c	T_i	Comment
Direct synthesis	**(Maximum sensitivity)**		
Astrom and Hagglund [3] - dominant pole design – *page 205-208* *Model: Method 3 or 4*	$\dfrac{0.29e^{-2.7\tau+3.7\tau^2}T_m}{K_m\tau_m}$, $\tau = \tau_m/(\tau_m + T_m)$	$8.9\tau_m e^{-6.6\tau+3.0\tau^2}$ or $0.79T_m e^{-1.4\tau+2.4\tau^2}$	$b = 0.81e^{0.73\tau+1.9\tau^2}$ $M_s = 1.4$, $0.14 \le \dfrac{\tau_m}{T_m} \le 5.5$
	$\dfrac{0.78e^{-4.1\tau+5.7\tau^2}T_m}{K_m\tau_m}$	$8.9\tau_m e^{-6.6\tau+3.0\tau^2}$ or $0.79T_m e^{-1.4\tau+2.4\tau^2}$	$b = 0.44e^{0.78\tau-0.45\tau^2}$; $M_s = 2.0$, $0.14 \le \dfrac{\tau_m}{T_m} \le 5.5$
Astrom and Hagglund [3] - modified Ziegler-Nichols – *page 208* *Model: Method 3 or 4*	$\dfrac{0.4T_m}{K_m\tau_m}$	$0.7T_m$	$b = 0.5$; $0.1 \le \dfrac{\tau_m}{T_m} \le 2$

Table 3: PI tuning rules - FOLPD model - $G_m(s) = \frac{K_m e^{-s\tau_m}}{1+sT_m}$ – Two degree of freedom controller:

$U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - \alpha K_c R(s)$. *1 tuning rule*

Rule	K_c	T_i	Comment
Minimum servo/regulator	**Performance index minimisation**		
Taguchi and Araki [61a] *Model: ideal process*	$\frac{1}{K_m}\left(0.1098+\frac{0.7382}{\frac{\tau_m}{T_m}-0.002434}\right)$	$T_i^{(5b)}$ [6]	$\frac{\tau_m}{T_m} \le 1.0$. Overshoot (servo step) $\le 20\%$; settling time $\le$ settling time of tuning rules of Chien *et al*. [10]
	$\alpha = 0.6830 - 0.4242\frac{\tau_m}{T_m} + 0.06568\left(\frac{\tau_m}{T_m}\right)^2$		

[6] $T_i^{(5b)} = T_m\left(0.06216 + 3.171\frac{\tau_m}{T_m} - 3.058\left[\frac{\tau_m}{T_m}\right]^2 + 1.205\left[\frac{\tau_m}{T_m}\right]^3\right)$

Table 4: PI tuning rules - non-model specific – controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)$. *20 tuning rules*

Rule	K_c	T_i	Comment
Ultimate cycle			
Ziegler and Nichols [8]	$0.45K_u$	$0.83T_u$	Quarter decay ratio
Hwang and Chang [62]	$0.45K_u$	$\frac{1}{p_1}\left(5.22 - \frac{5.22}{T_1}\right)$	p_1, T_1 = decay rate, period measured under proportional control when $K_c = 0.5K_u$
** Hang *et al*. [36]	$0.25K_u$	$0.2546T_u$	dominant time delay process
McMillan [14] – *page 90*	$0.3571K_u$	T_u	
Pessen [63]	$0.25K_u$	$0.042K_uT_u$	dominant time delay process
Astrom and Hagglund [3] – *page 142*	$0.4698K_u$	$0.4373T_u$	Gain margin = 2, phase margin = 20^0
	$0.1988K_u$	$0.0882T_u$	Gain margin = 2.44, phase margin = 61^0
	$0.2015K_u$	$0.1537T_u$	Gain margin = 3.45, phase margin = 46^0
Parr [64] – *page 191*	$0.5K_u$	$0.43T_u$	Quarter decay ratio
Yu [122] – *page 11*	$0.33K_u$	$2T_u$	
Other tuning rules			
Parr [64] – *page 191*	$0.667K_{25\%}$	$T_{25\%}$	Quarter decay ratio
McMillan [14] – *pages 42-43*	$0.42K_{25\%}$	$T_{25\%}$	'Fast' tuning
	$0.33K_{25\%}$	$T_{25\%}$	'Slow' tuning
Parr [64] – *page 192*	$\frac{0.333}{\lvert G_p(j\omega_u)\rvert}$	$2T_u$	Bang-bang oscillation test
Hagglund and Astrom [66]	$\frac{0.5}{\lvert G_p(j\omega_{135^0})\rvert}$	$\frac{4}{\omega_{135^0}}$	Alfa-Laval Automation ECA400 controller
	$\frac{0.25}{\lvert G_p(j\omega_{135^0})\rvert}$	$\frac{1.6}{\omega_{135^0}}$	Alfa-Laval Automation ECA400 controller - process has a long delay

Rule	K_c	T_i	Comment
Leva [67]	$\dfrac{\tan\left(\phi_m - \phi_{p\omega} - 0.5\pi\right)}{\left\lvert G_p(j\omega)\right\rvert \sqrt{1+\tan^2\left(\phi_m - \phi_{p\omega} - 0.5\pi\right)}}$	$\dfrac{\tan\left(\phi_m - \phi_{p\omega} - 0.5\pi\right)}{\omega}$	$\phi_m > \phi_{p\omega} + 0.5\pi$, $\phi_{p\omega}$ = process phase at frequency ω
Astrom [68]	$\dfrac{\sin\phi_m}{\left\lvert G_p(j\omega_{90^0})\right\rvert}$	$\dfrac{\tan\phi_m}{\omega_{90^0}}$	
Calcev and Gorez [69]	$\dfrac{1}{2\sqrt{2}\left\lvert G_p(j\omega_u)\right\rvert}$	$\dfrac{1}{\omega_u}$	$\phi_m = 45^0$, 'small' τ_m $\phi_m = 15^0$, 'large' τ_m
Cox *et al*. [70]	$\dfrac{0.20 V T_u \sin\phi_m}{A}$	$0.16 T_u \tan\phi_m$	V = relay amplitude, A = limit cycle amplitude.
Direct synthesis			
Vrancic *et al*. [71]	$\dfrac{0.5A_3}{A_1A_2 - K_mA_3}$ [1]	$\dfrac{A_3}{A_2}$	$A_m \ge 2, \phi_m \ge 60^0$
Vrancic [72]	$\dfrac{0.5A_3}{A_1A_2 - K_mA_3}$	$3.33\tau_m$	Modified Ziegler-Nichols process reaction method
Friman and Waller [41]	$\dfrac{0.4830}{\left\lvert G_p(j\omega_{150^0})\right\rvert}$	$\dfrac{3.7321}{\omega_{150^0}}$	Gain margin = 2, Phase margin = 30^0
Kristiansson and Lennartson [158a]	$\dfrac{1.18K_uK_m - 1.72}{K_uK_m^{\ 2}}$	$\dfrac{1.18K_uK_m - 1.72}{(0.33K_uK_m - 0.17)\omega_u}$	$0.1 \le K_uK_m \le 0.5$
	$\dfrac{0.50K_uK_m - 0.36}{K_uK_m^{\ 2}}$	$\dfrac{0.50K_uK_m - 0.36}{(0.33K_uK_m - 0.17)\omega_u}$	$K_uK_m > 0.5$
	$\dfrac{20K_{135^0}K_m - 160}{K_{135^0}K_m^{\ 2}}$	$\dfrac{20K_{135^0}K_m - 160}{(0.315K_{135^0}K_m - 0.175)\omega_u}$	$K_uK_m < 0.1$; $K_{135^0}K_m \le 0.1$
	$\dfrac{5.4K_{135^0}K_m - 13.6}{K_{135^0}K_m^{\ 2}}$	$\dfrac{5.4K_{135^0}K_m - 13.6}{(1.32K_{135^0}K_m - 3.2)\omega_u}$	$K_uK_m < 0.1$; $K_{135^0}K_m > 0.1$

[1] $A_1 = y_1(\infty)$, $A_2 = y_2(\infty)$, $A_3 = y_3(\infty)$

Alternatively, if the process model is $G_m(s) = K_m \dfrac{1 + b_1s + b_2s^2 + b_3s^3}{1 + a_1s + a_2s^3 + a_3s^3} e^{-s\tau_m}$, then

$A_1 = K_m\left(a_1 - b_1 + \tau_m\right)$, $A_2 = K_m\left(b_2 - a_2 + A_1a_1 - b_1\tau_m + 0.5\tau_m^{\ 2}\right)$,

$A_3 = K_m\left(a_3 - b_3 + A_2a_1 - A_1a_2 + b_2\tau_m - 0.5b_1\tau_m^{\ 2} + 0.167\tau_m^{\ 3}\right)$

Table 5: PI tuning rules - non-model specific – controller $G_c(s) = K_c\left(b + \frac{1}{T_i s}\right)$. *1 tuning rule*

Rule	K_c	T_i	Comment
Direct synthesis	**(Maximum sensitivity)**		
Astrom and Hagglund [3] - $M_s = 1.4$ – *page 215*	$0.053K_u e^{2.9\kappa - 2.6\kappa^2}$, $\kappa = 1/K_m K_u$	$0.90T_u e^{-4.4\kappa + 2.7\kappa^2}$	$b = 1.1e^{-0.0061\kappa + 1.8\kappa^2}$; $0 < K_m K_u < \infty$. maximum sensitivity M_s =1.4
	$0.13K_u e^{1.9\kappa - 1.3\kappa^2}$	$0.90T_u e^{-4.4\kappa + 2.7\kappa^2}$	$b = 0.48e^{0.40\kappa - 0.17\kappa^2}$; $0 < K_m K_u < \infty$. $M_s = 2.0$

Table 6: PI tuning rules - non-model specific – controller $U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) + K_c(b-1)R(s)$. *1 tuning rule*

Rule	K_c	T_i	Comment
Direct synthesis			
Vrancic [72]	[2] $K_c^{(6)}$	$\dfrac{A_1}{K_m + \dfrac{1}{2K_c^{(6)}} + \dfrac{K_c^{(6)} K_m^{\,2}}{2}\left(1-b^2\right)}$	$b = [0.5, 0.8]$ - good servo and regulator response

[2] $K_c^{(6)} = \dfrac{A_1A_2 - K_mA_3 - \sqrt{\left(K_mA_3 - A_1A_2\right)^2 - \left(1-b^2\right)A_3\left(K_m^{\,2}A_3 + A_1^{\,3} - 2K_mA_1A_2\right)}}{\left(1-b^2\right)\left(K_m^{\,2}A_3 + A_1^{\,3} - 2K_mA_1A_2\right)}$, $K_mA_3 - A_1A_2 < 0$

$$K_c^{(6)} = \frac{A_1A_2 - K_mA_3 + \sqrt{\left(K_mA_3 - A_1A_2\right)^2 - \left(1-b^2\right)A_3\left(K_m^{\,2}A_3 + A_1^{\,3} - 2K_mA_1A_2\right)}}{\left(1-b^2\right)\left(K_m^{\,2}A_3 + A_1^{\,3} - 2K_mA_1A_2\right)}, \; K_mA_3 - A_1A_2 > 0$$

$$y_1(t) = \int_0^t \left(K_m - \frac{y(\tau)}{\Delta u}\right)d\tau, \; y_2(t) = \int_0^t \left(A_1 - y_1(\tau)\right)d\tau, \; y_3(t) = \int_0^t \left(A_2 - y_2(\tau)\right)d\tau$$

Table 7: PI tuning rules - non-model specific – controller $U(s) = K_c Y(s) - \frac{K_c}{T_i s} E(s)$. *1 tuning rule*

Rule	K_c	T_i	Comment
Direct synthesis			
Chien *et al*. [74]	$\frac{-T_{CL}^2 + 1.414 T_{CL} T_m + \tau_m T_m}{K_m \left(T_{CL}^2 + 1.414 T_{CL} \tau_m + \tau_m^2\right)}$	$\frac{-T_{CL}^2 + 1.414 T_{CL} T_m + T_m \tau_m}{T_m + \tau_m}$	Underdamped system response - $\xi = 0.707$. $\tau_m > 0.2 T_m$

Table 8: PI tuning rules - IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$ - controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)$. *21 tuning rules*

Rule	K_c	T_i	Comment
Process reaction			
Ziegler and Nichols [8] *Model: Method 1.*	$\frac{0.9}{K_m\tau_m}$	$3.33\tau_m$	Quarter decay ratio
Two constraints method – Wolfe [12] *Model: Method 1*	$\frac{0.6}{K_m\tau_m}$	$2.78\tau_m$	Decay ratio = 0.4; minimum error integral (regulator mode).
	$\frac{0.87}{K_m\tau_m}$	$4.35\tau_m$	Decay ratio is as small as possible; minimum error integral (regulator mode).
Tyreus and Luyben [75] *Model: Method 2 or 3*	$\frac{0.487}{K_m\tau_m}$	$8.75\tau_m$	Maximum closed loop log modulus = 2dB ; closed loop time constant = $\tau_m\sqrt{10}$
Astrom and Hagglund [3] – *page 138* *Model: Not relevant*	$\frac{0.63}{K_m\tau_m}$	$3.2\tau_m$	Ultimate cycle Ziegler-Nichols equivalent
Regulator tuning	**Performance index minimisation**		
Minimum ISE – Hazebroek and Van der Waerden [9] *Model: Method 1*	$\frac{1.5}{K_m\tau_m}$	$5.56\tau_m$	
Shinskey [59] – minimum IAE regulator – *page 74.* *Model: Method not specified*	$\frac{0.9259}{K_m\tau_m}$	$4\tau_m$	
Poulin and Pomerleau [82] – minimum ITAE (process output step load disturbance) *Model: Method 2*	$\frac{0.5264}{K_m\tau_m}$	$4.5804\tau_m$	
Poulin and Pomerleau [82] – minimum ITAE (process input step load disturbance) *Model: Method 2*	$\frac{0.5327}{K_m\tau_m}$	$3.8853\tau_m$	
Ultimate cycle			
Tyreus and Luyben [75] *Model: Method 2 or 3.*	$0.31K_u$	$2.2T_u$	Maximum closed loop log modulus = 2dB ; closed loop time constant = $\tau_m\sqrt{10}$
Regulator – minimum IAE – Shinskey [17] – *page 121.* *Model: method not specified*	$0.61K_u$	T_u	
Robust			
Fruehauf *et al*. [52] *Model: Method 5*	$\frac{0.5}{K_m\tau_m}$	$5\tau_m$	

<table>
<tr><th>Rule</th><th colspan="3">K_c</th><th colspan="2">T_i</th><th>Comment</th></tr>
<tr><td rowspan="6">Chien [50]

Model: Method 2</td><td colspan="3">$\frac{1}{K_m}\left(\frac{2\lambda+\tau_m}{[\lambda+\tau_m]^2}\right)$</td><td colspan="2">$2\lambda+\tau_m$</td><td>$\lambda=\left[\frac{1}{K_m},\tau_m\right]$ [50];
$\lambda>\tau_m+T_m$ (Thomasson [51])</td></tr>
<tr><td>λ</td><td>Overshoot</td><td>TS</td><td>PP</td><td>RT</td><td rowspan="5">Zhang *et al.* [135]
$\lambda=[1.5\tau_m, 4.5\tau_m]$
- values deduced from graphs</td></tr>
<tr><td>$1.5\tau_m$</td><td>58%</td><td>$6\tau_m$</td><td>$\frac{1.7K_m}{\tau_m}$</td><td>$\frac{7K_m}{\tau_m}$</td></tr>
<tr><td>$2.5\tau_m$</td><td>35%</td><td>$11\tau_m$</td><td>$\frac{2.0K_m}{\tau_m}$</td><td>$\frac{16K_m}{\tau_m}$</td></tr>
<tr><td>$3.5\tau_m$</td><td>26%</td><td>$16\tau_m$</td><td>$\frac{2.2K_m}{\tau_m}$</td><td>$\frac{23K_m}{\tau_m}$</td></tr>
<tr><td>$4.5\tau_m$</td><td>22%</td><td>$20\tau_m$</td><td>$\frac{2.5K_m}{\tau_m}$</td><td>$\frac{30K_m}{\tau_m}$</td></tr>
<tr><td rowspan="5">Ogawa [54] – *deduced from graph*

Model: Method 5</td><td colspan="3">$\frac{0.45}{K_m\tau_m}$</td><td colspan="2">$11\tau_m$</td><td>20% uncertainty in the process parameters</td></tr>
<tr><td colspan="3">$\frac{0.39}{K_m\tau_m}$</td><td colspan="2">$12\tau_m$</td><td>30% uncertainty in the process parameters</td></tr>
<tr><td colspan="3">$\frac{0.34}{K_m\tau_m}$</td><td colspan="2">$13\tau_m$</td><td>40% uncertainty in the process parameters</td></tr>
<tr><td colspan="3">$\frac{0.30}{K_m\tau_m}$</td><td colspan="2">$14\tau_m$</td><td>50% uncertainty in the process parameters</td></tr>
<tr><td colspan="3">$\frac{0.27}{K_m\tau_m}$</td><td colspan="2">$15\tau_m$</td><td>60% uncertainty in the process parameters</td></tr>
</table>

Rule	K_c				T_i				Comment			
Direct synthesis												
Wang and Cluett [76] – *deduced from graph* *Model: Method 2*	$\frac{\alpha}{K_m \tau_m}$				$\beta \tau_m$							
	Closed loop time const.	Damp. Factor ξ	Gain margin A_m	Phase margin ϕ_m [deg.]	α	β	Closed loop time const.	Damp. Factor ξ	Gain margin A_m	Phase margin ϕ_m [deg.]	α	β
	τ_m	0.707	1.3	11	0.9056	2.6096	τ_m	1.0	1.3	14	0.8859	3.212
	$2\tau_m$	0.707	2.5	33	0.5501	4.0116	$2\tau_m$	1.0	2.3	37	0.6109	5.2005
	$3\tau_m$	0.707	3.6	42	0.3950	5.4136	$3\tau_m$	1.0	3.0	46	0.4662	7.1890
	$4\tau_m$	0.707	4.7	47	0.3081	6.8156	$4\tau_m$	1.0	4.0	52	0.3770	9.1775
	$5\tau_m$	0.707	5.9	50	0.2526	8.2176	$5\tau_m$	1.0	4.8	56	0.3164	11.166
	$6\tau_m$	0.707	7.1	52	0.2140	9.6196	$6\tau_m$	1.0	5.6	59	0.2726	13.155
	$7\tau_m$	0.707	8.2	54	0.1856	11.022	$7\tau_m$	1.0	6.3	61	0.2394	15.143
	$8\tau_m$	0.707	9.2	55	0.1639	12.424	$8\tau_m$	1.0	7.2	62	0.2135	17.132
	$9\tau_m$	0.707	10.4	56	0.1467	13.826	$9\tau_m$	1.0	8.0	64	0.1926	19.120
	$10\tau_m$	0.707	11.5	57	0.1328	15.228	$10\tau_m$	1.0	8.7	65	0.1754	21.109
	$11\tau_m$	0.707	12.7	58	0.1213	16.630	$11\tau_m$	1.0	9.6	66	0.1611	23.097
	$12\tau_m$	0.707	13.8	59	0.1117	18.032	$12\tau_m$	1.0	10.4	67	0.1489	25.086
	$13\tau_m$	0.707	14.9	59	0.1034	19.434	$13\tau_m$	1.0	11.2	67	0.1384	27.074
	$14\tau_m$	0.707	16.0	60	0.0963	20.836	$14\tau_m$	1.0	12.0	68	0.1293	29.063
	$15\tau_m$	0.707	17.0	60	0.0901	22.238	$15\tau_m$	1.0	12.7	68	0.1213	31.051
	$16\tau_m$	0.707	18.2	60	0.0847	23.640	$16\tau_m$	1.0	13.6	69	0.1143	33.040
Cluett and Wang [44] *Model: Method 2*	$\frac{0.9588}{K_m \tau_m}$				$3.0425\tau_m$				Closed loop time constant = τ_m			
	$\frac{0.6232}{K_m \tau_m}$				$5.2586\tau_m$				Closed loop time constant = $2\tau_m$			
	$\frac{0.4668}{K_m \tau_m}$				$7.2291\tau_m$				Closed loop time constant = $3\tau_m$			
	$\frac{0.3752}{K_m \tau_m}$				$9.1925\tau_m$				Closed loop time constant = $4\tau_m$			
	$\frac{0.3144}{K_m \tau_m}$				$11.1637\tau_m$				Closed loop time constant = $5\tau_m$			
	$\frac{0.2709}{K_m \tau_m}$				$13.1416\tau_m$				Closed loop time constant = $6\tau_m$			
Rotach [77] *Model: Method 4*	$\frac{0.75}{K_m \tau_m}$				$2.41\tau_m$				Damping factor for oscillations to a disturbance input = 0.75.			
Poulin and Pomerleau [78] *Model: Method 2*	$0.34K_u$ or $\frac{2.13}{K_m T_u}$				$1.04T_u$				Maximum sensitivity = 5 dB			

Rule	K_c	T_i	Comment
Gain and phase margin – Kookos *et al*. [38] *Model: Method 2*	$\dfrac{\omega_p}{A_m K_m}$	$\dfrac{1}{\omega_p\left(0.5\pi - \omega_p \tau_m\right)}$	$\omega_p = \dfrac{A_m \phi_m + 0.5\pi A_m (A_m - 1)}{(A_m{}^2 - 1)\tau_m}$
Representative results	$\dfrac{0.942}{K_m \tau_m}$	$4.510\tau_m$	Gain Margin = 1.5 Phase Margin = 22.5^0
	$\dfrac{0.698}{K_m \tau_m}$	$4.098\tau_m$	Gain Margin = 2 Phase Margin = 30^0
	$\dfrac{0.491}{K_m \tau_m}$	$6.942\tau_m$	Gain Margin = 3 Phase Margin = 45^0
	$\dfrac{0.384}{K_m \tau_m}$	$18.710\tau_m$	Gain Margin = 4 Phase Margin = 60^0
Other methods			
Penner [79] *Model: Method 2*	$\dfrac{0.58}{K_m \tau_m}$	$10\tau_m$	Maximum closed loop gain = 1.26
	$\dfrac{0.8}{K_m \tau_m}$	$5.9\tau_m$	Maximum closed loop gain = 2.0
Srividya and Chidambaram [80] *Model: Method 5*	$\dfrac{0.67075}{K_m \tau_m}$	$3.6547\tau_m$	

Table 9: PI tuning rules - IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$ - controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)\frac{1}{1 + T_f s}$. *1 tuning rule*

Rule	K_c	T_i	Comment
Robust			
Tan *et al*. [81] *Model: Method 2*	$\frac{0.463\lambda + 0.277}{K_m \tau_m}$	$\frac{\tau_m}{0.238\lambda + 0.123}$	$T_f = \frac{\tau_m}{5.750\lambda + 0.590}$, $\lambda = 0.5$

Table 10: PI tuning rules - IPD model $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{s}$ - controller $U(s) = K_c Y(s) - \dfrac{K_c}{T_i s} E(s)$. *1 tuning rule*

Rule	K_c	T_i	Comment
Direct synthesis			
Chien *et al*. [74] *Model: Method 1*	$\dfrac{1.414T_{CL} + \tau_m}{K_m\left(T_{CL}^2 + 1.414T_{CL}\tau_m + \tau_m\right.}$	$1.414T_{CL} + \tau_m$	Underdamped system response - $\xi = 0.707$. $\tau_m \le 0.2T_m$

Table 11: PI tuning rules - IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$ - Two degree of freedom controller:

$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - \alpha K_c R(s)$. *1 tuning rule*

Rule	K_c	T_i	Comment
Servo/regulator tuning	**Performance index minimisation**		
Taguchi and Araki [61a] *Model: ideal process*	$\frac{0.7662}{K_m\tau_m}$ $\alpha = 0.6810$	$4.091\tau_m$	$\frac{\tau_m}{T_m} \le 1.0$. Overshoot (servo step) $\le 20\%$; settling time $\le$ settling time of tuning rules of Chien *et al*. [10]
Minimum ITAE - Pecharroman and Pagola [134b] $K_m = 1$ *Model: Method 6*	$0.049K_u$	$2.826T_u$	$\alpha = 0.506$, $\phi_c = -164^0$
	$0.066K_u$	$2.402T_u$	$\alpha = 0.512$, $\phi_c = -160^0$
	$0.099K_u$	$1.962T_u$	$\alpha = 0.522$, $\phi_c = -155^0$
	$0.129K_u$	$1.716T_u$	$\alpha = 0.532$, $\phi_c = -150^0$
	$0.159K_u$	$1.506T_u$	$\alpha = 0.544$, $\phi_c = -145^0$
	$0.189K_u$	$1.392T_u$	$\alpha = 0.555$, $\phi_c = -140^0$
	$0.218K_u$	$1.279T_u$	$\alpha = 0.564$, $\phi_c = -135^0$
	$0.250K_u$	$1.216T_u$	$\alpha = 0.573$, $\phi_c = -130^0$
	$0.286K_u$	$1.127T_u$	$\alpha = 0.578$, $\phi_c = -125^0$
	$0.330K_u$	$1.114T_u$	$\alpha = 0.579$, $\phi_c = -120^0$
	$0.351K_u$	$1.093T_u$	$\alpha = 0.577$, $\phi_c = -118^0$

Table 12: PI tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$ - controller $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)$. *6 tuning rules*

Rule	K_c	T_i	Comment
Ultimate cycle			
McMillan [58] *Model: Method not relevant*	$\frac{1.477}{K_m}\frac{T_m}{\tau_m^2}\left\{\frac{1}{1+\left(\frac{T_m}{\tau_m}\right)^{0.65}}\right\}^2$	$3.32\tau_m\left\{1+\left(\frac{T_m}{\tau_m}\right)^{0.65}\right\}$	Tuning rules developed from K_u, T_u
Regulator tuning	**Minimum performance index**		
Minimum IAE – Shinskey [59] – *page 75.* *Model: Method not specified*	$\frac{0.556}{K_m(\tau_m + T_m)}$	$3.7(\tau_m + T_m)$	
Minimum IAE – Shinskey [59] – *page 158* *Model: Open loop method not specified*	$\frac{0.952}{K_m(T_m + \tau_m)}$	$4(T_m + \tau_m)$	
Minimum ITAE – Poulin and Pomerleau [82] – *deduced from graph* *Model: Method 2* (2 tuning rules)	$\frac{b}{K_m(\tau_m + T_m)}\sqrt{\frac{T_m^2}{a(\tau_m + T_m)^2}+1}$	$a(\tau_m + T_m)$	

Output step load disturbance:

τ_m/T_m	a	b	τ_m/T_m	a	b	τ_m/T_m	a	b
0.2	5.0728	0.5231	1.0	4.7839	0.5249	1.8	4.6837	0.5256
0.4	4.9688	0.5237	1.2	4.7565	0.5250	2.0	4.6669	0.5257
0.6	4.8983	0.5241	1.4	4.7293	0.5252			
0.8	4.8218	0.5245	1.6	4.7107	0.5254			

Input step load disturbance:

τ_m/T_m	a	b	τ_m/T_m	a	b	τ_m/T_m	a	b
0.2	3.9465	0.5320	1.0	4.0397	0.5311	1.8	4.0218	0.5313
0.4	3.9981	0.5315	1.2	4.0337	0.5312	2.0	4.0099	0.5314
0.6	4.0397	0.5311	1.4	4.0278	0.5312			
0.8	4.0397	0.5311	1.6	4.0278	0.5312			

Rule	K_c	T_i	Comment
Direct synthesis			
Poulin and Pomerleau [78] *Model: Method 2*	$0.34K_u$ or $\frac{2.13}{K_m T_u}$	$1.04T_u$	Maximum sensitivity = 5 dB

Table 13: PI tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$ - controller $G_c(s) = K_c\left(b + \frac{1}{T_i s}\right)$. *1 tuning rule*

Rule	K_c	T_i	Comment
Direct synthesis			
Astrom and Hagglund [3] - maximum sensitivity – *pages 210-212* *Model: Method 1.*	$\frac{0.41e^{-0.23\tau+0.019\tau^2}}{K_m(T_m+\tau_m)}$, $\tau = \tau_m/(\tau_m+T_m)$	$5.7\tau_m e^{1.7\tau-0.69\tau^2}$	$b = 0.33e^{2.5\tau-1.9\tau^2}$. $M_s = 1.4$; $0.14 \le \frac{\tau_m}{T_m} \le 5.5$
	$\frac{0.81e^{-1.1\tau+0.76\tau^2}}{K_m(T_m+\tau_m)}$	$3.4\tau_m e^{0.28\tau-0.0089\tau^2}$	$b = 0.78e^{-1.9\tau+1.2\tau^2}$. $M_s = 2.0$; $0.14 \le \frac{\tau_m}{T_m} \le 5.5$

Table 14: PI tuning rules - FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$ - Two degree of freedom controller:

$$U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - \alpha K_c R(s)$$. *1 tuning rule*.

Rule	K_c	T_i	Comment
Servo/regulator tuning	**Minimum performance index**		
Taguchi and Araki [61a] *Model: ideal process*	$\frac{1}{K_m}\left(0.1787+\frac{0.2839}{\frac{\tau_m}{T_m}+0.001723}\right)$	$4.296+3.794\frac{\tau_m}{T_m}+0.2591\left(\frac{\tau_m}{T_m}\right)^2$	$\frac{\tau_m}{T_m}\le 1.0$. Overshoot (servo step) $\le 20\%$; settling time $\le$ settling time of tuning rules of Chien *et al*. [10]
	$\alpha = 0.6551+0.01877\frac{\tau_m}{T_m}$		
Minimum ITAE - Pecharroman and Pagola [134b] $K_m = 1$; $T_m = 1$ *Model: Method 4*	$0.049 K_u$	$2.826 T_u$	$\alpha = 0.506$, $\phi_c = -164^0$
	$0.066 K_u$	$2.402 T_u$	$\alpha = 0.512$, $\phi_c = -160^0$
	$0.099 K_u$	$1.962 T_u$	$\alpha = 0.522$, $\phi_c = -155^0$
	$0.129 K_u$	$1.716 T_u$	$\alpha = 0.532$, $\phi_c = -150^0$
	$0.159 K_u$	$1.506 T_u$	$\alpha = 0.544$, $\phi_c = -145^0$
	$0.189 K_u$	$1.392 T_u$	$\alpha = 0.555$, $\phi_c = -140^0$
	$0.218 K_u$	$1.279 T_u$	$\alpha = 0.564$, $\phi_c = -135^0$
	$0.250 K_u$	$1.216 T_u$	$\alpha = 0.573$, $\phi_c = -130^0$
	$0.286 K_u$	$1.127 T_u$	$\alpha = 0.578$, $\phi_c = -125^0$
	$0.330 K_u$	$1.114 T_u$	$\alpha = 0.579$, $\phi_c = -120^0$
	$0.351 K_u$	$1.093 T_u$	$\alpha = 0.577$, $\phi_c = -118^0$

<u>Table 15</u>: PI tuning rules - SOSPD model $\frac{K_m e^{-s\tau_m}}{T_{m1}{}^2 s^2 + 2\xi_m T_{m1} s + 1}$ or $\frac{K_m e^{-s\tau_m}}{(1+T_{m1}s)(1+T_{m2}s)}$ - controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)$. *11 tuning rules.*

Rule	K_c	T_i	Comment
Robust			
Brambilla *et al*. [48] *Model: Method 1*	$\frac{T_{m1} + T_{m2} + 0.5\tau_m}{K_m \tau_m (2\lambda + 1)}$	$T_{m1} + T_{m2} + 0.5\tau_m$	λ varies graphically with $\tau_m/(T_{m1} + T_{m2})$ - $0.1 \le \tau_m/(T_{m1} + T_{m2}) \le 10$

Brambilla *et al*. [48]	$\tau_m/(T_{m1} + T_{m2})$	λ	$\tau_m/(T_{m1} + T_{m2})$	λ	$\tau_m/(T_{m1} + T_{m2})$	λ
	0.1	3.0	1.0	0.6	10.0	0.2
	0.2	1.8	2.0	0.4		
	0.5	1.0	5.0	0.2		

Rule	K_c	T_i	Comment
Direct synthesis			
Gain and phase margin - Tan *et al*. [39] – repeated pole *Model: Method 11*	$\frac{\beta T_i \omega_\phi \sqrt{1 + (\beta T_m \omega_\phi)^2}}{A_m \sqrt{1 + (\beta T_i \omega_\phi)^2}}$	$\frac{1}{\beta\omega_\phi \tan\left[-2\tan^{-1}\beta T_m \omega_\phi - \beta\tau_m\omega_\phi - \phi_m\right]}$ $\omega_\phi < \omega_u$	$\beta = 0.8, \frac{\tau_m}{T_m} < 0.33$; $\beta = 0.5, \frac{\tau_m}{T_m} > 0.33$
Regulator tuning	**Minimum performance index**		
Minimum IAE - Shinskey [59] – *page 158* *Model: Open loop method not specified*	$\frac{100 T_{m1}}{K_m(\tau_m + T_{m2})\left(50 + 55\left[1 - e^{-\frac{T_{m1}}{\tau_m + T_{m2}}}\right]\right)}$	$\tau_m\left(0.5 + 35\left[1 - e^{-\frac{3T_{m1}}{(\tau_m + T_{m2})}}\right]\right)$	
Minimum ISE – McAvoy and Johnson [83] – *deduced from graph* *Model: Method 1*	$\frac{\alpha}{K_m}$	$\beta\tau_m$	

Minimum ISE – McAvoy and Johnson [83]	ξ_m	τ_m/T_{m1}	α	β	ξ_m	τ_m/T_{m1}	α	β	ξ_m	τ_m/T_{m1}	α	β
	1	0.5	0.8	1.82	4	0.5	4.3	3.45	7	0.5	7.8	3.85
	1	4.0	5.7	12.5	4	4.0	27.1	6.67	7	4.0	51.2	5.88
	1	10.0	13.6	25.0								

Rule	K_c	T_i	Comment
Minimum ITAE – Lopez *et al*. [84] – *deduced from graph* *Model: Method 7*	$\frac{\alpha}{K_m}$	βT_m	

Minimum ITAE – Lopez *et al*. [84]	ξ_m	τ_m/T_{m1}	α	β	ξ_m	τ_m/T_{m1}	α	β	ξ_m	τ_m/T_{m1}	α	β
	0.5	0.1	3.0	2.86	1	0.1	7.0	2.00	4	0.1	40.0	0.83
	0.5	1.0	0.2	0.83	1	1.0	0.95	2.22	4	1.0	6.0	3.33
	0.5	10.0	0.3	4.0	1	10.0	0.35	5.00	4	4.0	0.75	10.0

Rule	K_c	T_i	Comment
Minimum IAE – Shinskey [17] – *page 48. Model: method not specified.*	$\frac{0.77 T_{m1}}{K_m \tau_m}$	$2.83(\tau_m + T_{m2})$	$\frac{\tau_m}{T_{m1}} = 0.2$, $\frac{T_{m2}}{T_{m1}} = 0.1$
	$\frac{0.70 T_{m1}}{K_m \tau_m}$	$2.65(\tau_m + T_{m2})$	$\frac{\tau_m}{T_{m1}} = 0.2$, $\frac{T_{m2}}{T_{m1}} = 0.2$
	$\frac{0.80 T_{m1}}{K_m \tau_m}$	$2.29(\tau_m + T_{m2})$	$\frac{\tau_m}{T_{m1}} = 0.2$, $\frac{T_{m2}}{T_{m1}} = 0.5$
	$\frac{0.80 T_{m1}}{K_m \tau_m}$	$1.67(\tau_m + T_{m2})$	$\frac{\tau_m}{T_{m1}} = 0.2$, $\frac{T_{m2}}{T_{m1}} = 1.0$

Rule	K_c	T_i	Comment
Minimum IAE - Huang *et al*. [18] *Model: Method 1*	$K_c^{(31)}$ [1]	$T_i^{(31)}$	$0 < \frac{T_{m2}}{T_{m1}} \le 1$; $0.1 \le \frac{\tau_m}{T_{m1}} \le 1$

[1] $$K_c^{(31)} = \frac{1}{K_m}\left[6.4884 + 4.6198\frac{\tau_m}{T_{m1}} - 3.491\frac{T_{m2}}{T_{m1}} - 25.3143\frac{\tau_m T_{m2}}{{T_{m1}}^2} + 0.8196\left(\frac{\tau_m}{T_{m1}}\right)^{-0.9077} - 5.2132\left(\frac{\tau_m}{T_{m1}}\right)^{-0.063}\right]$$

$$+\frac{1}{K_m}\left[-7.2712\left(\frac{\tau_m}{T_{m1}}\right)^{0.5961} - 18.0448\left(\frac{T_{m2}}{T_{m1}}\right)^{0.7204} + 5.3263\left(\frac{T_{m2}}{T_{m1}}\right)^{1.0049} + 13.9108\left(\frac{T_{m2}}{T_{m1}}\right)^{1.005} + 0.4937\frac{T_{m2}}{\tau_m}\right]$$

$$+\frac{1}{K_m}\left[19.1783\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^{0.8529} + 12.2494\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^{0.5613} + 8.4355\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^{0.557} - 17.6781\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^{1.1818}\right]$$

$$+\frac{1}{K_m}\left[-0.7241e^{\frac{\tau_m}{T_{m1}}} - 2.2525e^{\frac{T_{m2}}{T_{m1}}} + 5.4959e^{\frac{\tau_m T_{m2}}{{T_{m1}}^2}}\right]$$

$$T_i^{(31)} = T_{m1}\left[0.0064 + 3.9574\frac{\tau_m}{T_{m1}} + 4.4087\frac{T_{m2}}{T_{m1}} - 6.4789\left(\frac{\tau_m}{T_{m1}}\right)^2 - 12.8702\frac{\tau_m T_{m2}}{{T_{m1}}^2} - 1.5083\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 9.4348\left(\frac{\tau_m}{T_{m1}}\right)^3\right]$$

$$+T_{m1}\left[17.0736\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^2 + 15.9816\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^2 - 3.909\left(\frac{T_{m2}}{T_{m1}}\right)^2 - 10.7619\left(\frac{\tau_m}{T_{m1}}\right)^4\right]$$

$$+T_{m1}\left[-10.684\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^3 - 22.3194\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^2 - 6.6602\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^3 + 6.8122\left(\frac{T_{m2}}{T_{m1}}\right)^4\right]$$

$$+T_{m1}\left[7.5146\left(\frac{\tau_m}{T_{m1}}\right)^5 + 2.8724\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^4 + 11.4666\left(\frac{T_{m2}}{T_{m1}}\right)^2\left(\frac{\tau_m}{T_{m1}}\right)^3 + 11.1207\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^3\right]$$

$$+T_{m1}\left[-1.2174\left(\frac{\tau_m}{T_{m1}}\right)\left(\frac{T_{m2}}{T_{m1}}\right)^4 - 4.3675\left(\frac{T_{m2}}{T_{m1}}\right)^5 - 2.2236\left(\frac{\tau_m}{T_{m1}}\right)^6 - 0.112\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^5 + 1.0308\left(\frac{T_{m2}}{T_{m1}}\right)^6\right]$$

$$+T_{m1}\left[-1.9136\left(\frac{\tau_m}{T_{m1}}\right)^4\left(\frac{T_{m2}}{T_{m1}}\right)^2 - 3.4994\left(\frac{\tau_m}{T_{m1}}\right)^3\left(\frac{T_{m2}}{T_{m1}}\right)^3 - 1.5777\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^4 + 1.1408\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^5\right]$$

Rule	K_c	T_i	Comment
Minimum IAE - Huang *et al.* [18] *Model: Method 1*	$K_c^{(32)}$ [2]	$T_i^{(32)}$	$0.4 \le \xi_m \le 1$; $0.05 \le \frac{\tau_m}{T_{m1}} \le 1$

[2] $$K_c^{(32)} = \frac{1}{K_m}\left[-10.4183 - 20.9497\frac{\tau_m}{T_{m1}} - 5.5175\xi_m - 26.5149\xi_m\frac{\tau_m}{T_{m1}} + 42.7745\left(\frac{\tau_m}{T_{m1}}\right)^{1.4439} + 10.5069\left(\frac{\tau_m}{T_{m1}}\right)^{0.1456}\right]$$
$$+\frac{1}{K_m}\left[15.4103\left(\frac{\tau_m}{T_{m1}}\right)^{0.3157} + 34.3236\xi_m^{3.7057} - 17.8860\xi_m^{4.5359} - 54.0584\xi_m^{1.9593} + 22.4263\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^{-0.0541}\right]$$
$$+\frac{1}{K_m}\left[2.7497\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^{4.7426} + 50.2197\xi_m^{1.8288}\left(\frac{\tau_m}{T_{m1}}\right) - 17.1968\frac{\tau_m}{T_{m1}}\xi_m^{2.7227} + 1.0293\xi_m\frac{T_{m1}}{\tau_m}\right]$$
$$+\frac{1}{K_m}\left[-16.7667e^{\frac{\tau_m}{T_{m1}}} + 14.5737e^{\xi_m} - 7.3025e^{\xi_m\frac{\tau_m}{T_{m1}}}\right]$$

$$T_i^{(32)} = T_{m1}\left[11.447 + 4.5128\frac{\tau_m}{T_{m1}} - 75.2486\xi_m - 110.807\left(\frac{\tau_m}{T_{m1}}\right)^2 - 12.282\xi_m\frac{\tau_m}{T_{m1}} + 345.3228\xi_m^2 + 191.9539\left(\frac{\tau_m}{T_{m1}}\right)^3\right]$$
$$+T_{m1}\left[359.3345\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^2 - 158.7611\frac{\tau_m}{T_{m1}}\xi_m^2 - 770.2897\xi_m^2 - 153.633\left(\frac{\tau_m}{T_{m1}}\right)^4\right]$$
$$+T_{m1}\left[-412.5409\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^3 - 414.7786\xi_m^2\left(\frac{\tau_m}{T_{m1}}\right)^2 + 485.0976\frac{\tau_m}{T_{m1}}\xi_m^3 + 864.5195\xi_m^4\right]$$
$$+T_{m1}\left[55.4366\left(\frac{\tau_m}{T_{m1}}\right)^5 + 222.2865\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^4 + 275.166\xi_m^2\left(\frac{\tau_m}{T_{m1}}\right)^3 + 205.2493\left(\frac{\tau_m}{T_{m1}}\right)^2\xi_m^3\right]$$
$$+T_{m1}\left[-479.5627\left(\frac{\tau_m}{T_{m1}}\right)\xi_m^4 - 473.1346\xi_m^5 - 6.547\left(\frac{\tau_m}{T_{m1}}\right)^6 - 43.2822\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^5 + 99.8717\xi_m^6\right]$$
$$+T_{m1}\left[-73.5666\left(\frac{\tau_m}{T_{m1}}\right)^4\xi_m^2 - 56.4418\left(\frac{\tau_m}{T_{m1}}\right)^3\xi_m^3 - 37.497\left(\frac{\tau_m}{T_{m1}}\right)^2\xi_m^4 + 160.7714\frac{\tau_m}{T_{m1}}\xi_m^5\right]$$

Rule	K_c	T_i	Comment
Servo tuning	**Minimum performance index**		
Minimum IAE - Huang *et al*. [18] *Model: Method 1*	$K_c^{(33)}$ [3]	$T_i^{(33)}$	$0 < \frac{T_{m2}}{T_{m1}} \le 1$; $0.1 \le \frac{\tau_m}{T_{m1}} \le 1$

[3] $$K_c^{(33)} = \frac{1}{K_m}\left[-13.0454 - 9.0916\frac{\tau_m}{T_{m1}} + 2.6647\frac{T_{m2}}{T_{m1}} + 9.162\frac{\tau_m T_{m2}}{T_{m1}^2} + 0.3053\left(\frac{\tau_m}{T_{m1}}\right)^{-1.0169} + 1.1075\left(\frac{\tau_m}{T_{m1}}\right)^{3.5959}\right]$$
$$+\frac{1}{K_m}\left[-2.2927\left(\frac{\tau_m}{T_{m1}}\right)^{3.6843} - 31.0306\left(\frac{T_{m2}}{T_{m1}}\right)^{0.8476} - 13.0155\left(\frac{T_{m2}}{T_{m1}}\right)^{2.6083} + 9.6899\left(\frac{T_{m2}}{T_{m1}}\right)^{2.9049} - 0.6418\frac{T_{m2}}{\tau_m}\right]$$
$$+\frac{1}{K_m}\left[18.9643\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^{-0.2016} - 39.7340\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^{1.3293} + 28.155\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^{0.801}\right]$$
$$+\frac{1}{K_m}\left[-2.0067\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^{3.956} + 4.8259e^{\frac{\tau_m}{T_{m1}}} + 2.1137e^{\frac{T_{m2}}{T_{m1}}} + 8.4511e^{\frac{\tau_m T_{m2}}{T_{m1}^2}}\right]$$

$$T_i^{(33)} = T_{m1}\left[0.9771 - 0.2492\frac{\tau_m}{T_{m1}} + 0.8753\frac{T_{m2}}{T_{m1}} + 3.4651\left(\frac{\tau_m}{T_{m1}}\right)^2 - 3.8516\frac{\tau_m T_{m2}}{T_{m1}^2} + 7.5106\left(\frac{T_{m2}}{T_{m1}}\right)^2 - 7.4538\left(\frac{\tau_m}{T_{m1}}\right)^3\right]$$
$$+T_{m1}\left[11.6768\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^2 - 10.9909\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^2 - 16.1461\left(\frac{T_{m2}}{T_{m1}}\right)^3 + 8.2567\left(\frac{\tau_m}{T_{m1}}\right)^4\right]$$
$$+T_{m1}\left[-18.1011\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^3 + 6.2208\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 21.9893\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^3 + 15.8538\left(\frac{T_{m2}}{T_{m1}}\right)^4\right]$$
$$+T_{m1}\left[-4.7536\left(\frac{\tau_m}{T_{m1}}\right)^5 + 14.5405\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^4 - 2.2691\left(\frac{T_{m2}}{T_{m1}}\right)^2\left(\frac{\tau_m}{T_{m1}}\right)^3 - 8.387\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^3\right]$$
$$+T_{m1}\left[-16.651\left(\frac{\tau_m}{T_{m1}}\right)\left(\frac{T_{m2}}{T_{m1}}\right)^4 - 7.1990\left(\frac{T_{m2}}{T_{m1}}\right)^5 + 1.1496\left(\frac{\tau_m}{T_{m1}}\right)^6 - 4.728\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^5 + 1.1395\left(\frac{T_{m2}}{T_{m1}}\right)^6\right]$$
$$+T_{m1}\left[0.6385\left(\frac{\tau_m}{T_{m1}}\right)^4\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 1.0885\left(\frac{\tau_m}{T_{m1}}\right)^3\left(\frac{T_{m2}}{T_{m1}}\right)^3 + 3.1615\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^4 + 4.5398\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^5\right]$$

Rule	K_c	T_i	Comment
Minimum IAE - Huang *et al.* [18] *Model: Method 1*	$K_c^{(34)}$ [4]	$T_i^{(34)}$	$0.4 \le \xi_m \le 1$; $0.05 \le \frac{\tau_m}{T_{m1}} \le 1$

[4]
$$
\begin{aligned}
K_c^{(34)} = {} & \frac{1}{K_m}\left[-10.95 - 1.8845\frac{\tau_m}{T_{m1}} - 3.4123\xi_m + 4.5954\xi_m\frac{\tau_m}{T_{m1}} - 1.7002\left(\frac{\tau_m}{T_{m1}}\right)^3 - 2.1324\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^2\right] \\
& + \frac{1}{K_m}\left[-14.4149\xi_m^{\,2}\left(\frac{\tau_m}{T_{m1}}\right) - 0.7683\xi_m^{\,3} + 7.5142\left(\frac{\tau_m}{T_{m1}}\right)^{0.421} + 3.7291\left(\frac{\tau_m}{T_{m1}}\right)^{0.1984} + 5.3444\left(\frac{\tau_m}{T_{m1}}\right)^{1.8033}\right] \\
& + \frac{1}{K_m}\left[-0.0819\xi_m^{\,19.5419} - 3.603\xi_m^{\,1.0749} + 7.1163\xi_m^{\,1.1006} + 3.206\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^{-0.6753} - 7.8480\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^{-0.1642}\right] \\
& + \frac{1}{K_m}\left[11.3222\xi_m^{\,1.9948}\left(\frac{\tau_m}{T_{m1}}\right) + 2.4239e^{\frac{\tau_m}{T_{m1}}} + 3.4137e^{\xi_m} + 1.0251e^{\frac{\xi_m \tau_m}{T_{m1}}} - 0.5593\xi_m\frac{T_{m1}}{\tau_m}\right]
\end{aligned}
$$

$$
\begin{aligned}
T_i^{(34)} = {} & T_{m1}\left[2.4866 - 23.3234\frac{\tau_m}{T_{m1}} + 5.3662\xi_m + 65.6053\left(\frac{\tau_m}{T_{m1}}\right)^2 + 29.0062\xi_m\frac{\tau_m}{T_{m1}} - 24.1648\xi_m^{\,2} - 83.6796\left(\frac{\tau_m}{T_{m1}}\right)^3\right] \\
& + T_{m1}\left[-135.9699\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^2 + 43.1477\frac{\tau_m}{T_{m1}}\xi_m^{\,2} + 51.9749\xi_m^{\,3} + 86.0228\left(\frac{\tau_m}{T_{m1}}\right)^4\right] \\
& + T_{m1}\left[70.4553\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^3 + 153.4877\xi_m^{\,2}\left(\frac{\tau_m}{T_{m1}}\right)^2 - 125.0112\frac{\tau_m}{T_{m1}}\xi_m^{\,3} - 68.5893\xi_m^{\,4}\right] \\
& + T_{m1}\left[-62.7517\left(\frac{\tau_m}{T_{m1}}\right)^5 + 27.6178\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^4 - 152.7422\xi_m^{\,2}\left(\frac{\tau_m}{T_{m1}}\right)^3 + 20.8705\left(\frac{\tau_m}{T_{m1}}\right)^2\xi_m^{\,3}\right] \\
& + T_{m1}\left[54.0012\left(\frac{\tau_m}{T_{m1}}\right)\xi_m^{\,4} + 58.7376\xi_m^{\,5} + 13.1193\left(\frac{\tau_m}{T_{m1}}\right)^6 + 20.2645\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^5 - 23.2064\xi_m^{\,6}\right] \\
& + T_{m1}\left[-61.6742\left(\frac{\tau_m}{T_{m1}}\right)^4\xi_m^{\,2} + 136.2439\left(\frac{\tau_m}{T_{m1}}\right)^3\xi_m^{\,3} - 95.4092\left(\frac{\tau_m}{T_{m1}}\right)^2\xi_m^{\,4} + 20.4168\frac{\tau_m}{T_{m1}}\xi_m^{\,5}\right]
\end{aligned}
$$

Rule	K_c	T_i	Comment
Ultimate cycle			
Regulator - nearly minimum IAE, ISE, ITAE – Hwang [60] *Model: Method 3*	$K_c^{(7)}$	$T_i^{(7)}$	Decay ratio = 0.15 - $\varepsilon < 2.4$, $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$, $0.6 \le \xi_m \le 4.2$
	$K_c^{(8)}$	$T_i^{(8)}$	Decay ratio = 0.15 - $2.4 \le \varepsilon < 3$, $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$, $0.6 \le \xi_m \le 4.2$
	$K_c^{(9)}$ [5]	$T_i^{(9)}$	Decay ratio = 0.15 - $3 \le \varepsilon < 20$, $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$, $0.6 \le \xi_m \le 4.2$

[5] $\varepsilon = \dfrac{6T_{m1}^2 + 4\xi_m T_{m1}\tau_m + K_H K_m \tau_m^2}{2T_{m1}^2 \tau_m \omega_H}$, $K_H = \dfrac{9}{2\tau_m^2 K_m}\left[\dfrac{\tau_m^2}{18} - T_{m1}^2 - \dfrac{\xi_m T_{m1}\tau_m^2}{18}\right]$,

$$\omega_H = \sqrt{\dfrac{1 + K_H K_m}{T_{m1}^2 + \dfrac{2T_{m1}\tau_m \xi_m}{3} + \dfrac{K_H K_m \tau_m^2}{6}}}$$

$$K_c^{(7)} = \left(1 - \dfrac{0.674\left[1 - 0.447\omega_H \tau_m + 0.0607(\omega_H \tau_m)^2\right]}{K_H K_m/(1 + K_H K_m)}\right) K_H \, , \; T_i^{(7)} = \dfrac{K_c(1 + K_H K_m)}{0.0607\omega_H K_m\left(1 + 1.05\omega_H \tau_m - 0.233\omega_H^2 \tau_m^2\right)}$$

$$K_c^{(8)} = \left(1 - \dfrac{0.778\left[1 - 0.467\omega_H \tau_m + 0.0609(\omega_H \tau_m)^2\right]}{K_H K_m/(1 + K_H K_m)}\right) K_H \, , \; T_i^{(8)} = \dfrac{K_c(1 + K_H K_m)}{0.0309\omega_H K_m\left(1 + 2.84\omega_H \tau_m - 0.532\omega_H^2 \tau_m^2\right)}$$

$$K_c^{(9)} = \left(1 - \dfrac{1.31(0.519)^{\omega_H \tau_m}\left[1 - 1.03/\varepsilon + 0.514/\varepsilon^2\right]}{K_H K_m/(1 + K_H K_m)}\right) K_H \, , \; T_i^{(9)} = \dfrac{K_c(1 + K_H K_m)}{0.0603\left(1 + 0.929\ln[\omega_H \tau_m]\right)\left(1 + 2.01/\varepsilon - 1.2/\varepsilon^2\right)}$$

Rule	K_c	T_i	Comment
Regulator – nearly minimum IAE, ISE, ITAE - Hwang [60] *Model: Method 3*	$K_c^{(10)}$ [6]	$T_i^{(10)}$	Decay ratio = 0.15 - $\varepsilon > 20$, $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$, $0.6 \le \xi_m \le 4.2$
	$K_c^{(11)}$	$T_i^{(11)}$	Decay ratio = 0.2 - $\varepsilon < 2.4$, $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$, $0.6 \le \xi_m \le 4.2$
	$K_c^{(12)}$	$T_i^{(12)}$	Decay ratio = 0.2 - $2.4 \le \varepsilon < 3$, $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$, $0.6 \le \xi_m \le 4.2$
	$K_c^{(13)}$	$T_i^{(13)}$	Decay ratio = 0.2 - $3 \le \varepsilon < 20$, $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$, $0.6 \le \xi_m \le 4.2$
	$K_c^{(14)}$	$T_i^{(14)}$	Decay ratio = 0.2 - $\varepsilon > 20$, $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$, $0.6 \le \xi_m \le 4.2$
	$K_c^{(15)}$	$T_i^{(15)}$	Decay ratio = 0.25 - $\varepsilon < 2.4$, $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$, $0.6 \le \xi_m \le 4.2$
	$K_c^{(16)}$	$T_i^{(16)}$	Decay ratio = 0.25 - $2.4 \le \varepsilon < 3$, $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$, $0.6 \le \xi_m \le 4.2$

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[6] $$K_c^{(10)} = \left(1 - \frac{1.14\left[1 - 0.482\omega_H\tau_m + 0.068(\omega_H\tau_m)^2\right]}{K_H K_m/(1 + K_H K_m)}\right)K_H,\ T_i^{(10)} = \frac{K_c(1 + K_H K_m)}{0.0694\omega_H K_m\left(-1 + 2.1\omega_H\tau_m - 0.367\omega_H{}^2\tau_m{}^2\right)}$$

$$K_c^{(11)} = \left(1 - \frac{0.622\left[1 - 0.435\omega_H\tau_m + 0.052(\omega_H\tau_m)^2\right]}{K_H K_m/(1 + K_H K_m)}\right)K_H,\ T_i^{(11)} = \frac{K_c(1 + K_H K_m)}{0.0697\omega_H K_m\left(1 + 0.752\omega_H\tau_m - 0.145\omega_H{}^2\tau_m{}^2\right)}$$

$$K_c^{(12)} = \left(1 - \frac{0.724\left[1 - 0.469\omega_H\tau_m + 0.0609(\omega_H\tau_m)^2\right]}{K_H K_m/(1 + K_H K_m)}\right)K_H,\ T_i^{(12)} = \frac{K_c(1 + K_H K_m)}{0.0405\omega_H K_m\left(1 + 1.93\omega_H\tau_m - 0.363\omega_H{}^2\tau_m{}^2\right)}$$

$$K_c^{(13)} = \left(1 - \frac{1.26(0.506)^{\omega_H\tau_m}\left[1 - 1.07/\varepsilon + 0.616/\varepsilon^2\right]}{K_H K_m/(1 + K_H K_m)}\right)K_H,\ T_i^{(13)} = \frac{K_c(1 + K_H K_m)}{0.0661\left(1 + 0.824\ln[\omega_H\tau_m]\right)\left(1 + 1.71/\varepsilon - 1.17/\varepsilon^2\right)}$$

$$K_c^{(14)} = \left(1 - \frac{1.09\left[1 - 0.497\omega_H\tau_m + 0.0724(\omega_H\tau_m)^2\right]}{K_H K_m/(1 + K_H K_m)}\right)K_H,\ T_i^{(14)} = \frac{K_c(1 + K_H K_m)}{0.054\omega_H K_m\left(-1 + 2.54\omega_H\tau_m - 0.457\omega_H{}^2\tau_m{}^2\right)}$$

$$K_c^{(15)} = \left(1 - \frac{0.584\left[1 - 0.439\omega_H\tau_m + 0.0514(\omega_H\tau_m)^2\right]}{K_H K_m/(1 + K_H K_m)}\right)K_H,\ T_i^{(15)} = \frac{K_c(1 + K_H K_m)}{0.0714\omega_H K_m\left(1 + 0.685\omega_H\tau_m - 0.131\omega_H{}^2\tau_m{}^2\right)}$$

$$K_c^{(16)} = \left(1 - \frac{0.675\left[1 - 0.472\omega_H\tau_m + 0.061(\omega_H\tau_m)^2\right]}{K_H K_m/(1 + K_H K_m)}\right)K_H,\ T_i^{(16)} = \frac{K_c(1 + K_H K_m)}{0.0484\omega_H K_m\left(1 + 1.43\omega_H\tau_m - 0.273\omega_H{}^2\tau_m{}^2\right)}$$

Rule	K_c	T_i	Comment
Regulator - nearly minimum IAE, ISE, ITAE - Hwang [60] *Model: Method 3*	$K_c^{(17)}$ [7]	$T_i^{(17)}$	Decay ratio = 0.25 - $3 \le \varepsilon < 20$, $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$, $0.6 \le \xi_m \le 4.2$
	$K_c^{(18)}$	$T_i^{(18)}$	Decay ratio = 0.25 - $\varepsilon > 20$, $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$, $0.6 \le \xi_m \le 4.2$
Servo - nearly minimum IAE, ISE, ITAE - Hwang [60] *Model: Method 3*	$K_c^{(19)}$	$T_i^{(19)}$	Decay ratio = 0.1 - $\varepsilon < 2.4$, $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$, $0.6 \le \xi_m \le 4.2$, $\xi_m \le 0.776 + 0.0568\frac{\tau_m}{T_{m1}} + 0.18\left(\frac{\tau_m}{T_{m1}}\right)^2$
	$K_c^{(20)}$	$T_i^{(20)}$	Decay ratio = 0.1 - $2.4 \le \varepsilon < 3$, $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$, $0.6 \le \xi_m \le 4.2$, $\xi_m \le 0.776 + 0.0568\frac{\tau_m}{T_{m1}} + 0.18\left(\frac{\tau_m}{T_{m1}}\right)^2$
	$K_c^{(21)}$	$T_i^{(21)}$	Decay ratio = 0.1 - $3 \le \varepsilon < 20$, $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$, $0.6 \le \xi_m \le 4.2$, $\xi_m \le 0.776 + 0.0568\frac{\tau_m}{T_{m1}} + 0.18\left(\frac{\tau_m}{T_{m1}}\right)^2$
	$K_c^{(22)}$	$T_i^{(22)}$	Decay ratio = 0.1 - $\varepsilon > 20$, $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$, $0.6 \le \xi_m \le 4.2$, $\xi_m \le 0.776 + 0.0568\frac{\tau_m}{T_{m1}} + 0.18\left(\frac{\tau_m}{T_{m1}}\right)^2$

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[7] $$K_c^{(17)} = \left(1 - \frac{1.2(0.495)^{\omega_H\tau_m}\left[1 - 1.1/\varepsilon + 0.698/\varepsilon^2\right]}{K_H K_m/(1 + K_H K_m)}\right)K_H,\ T_i^{(17)} = \frac{K_c(1 + K_H K_m)}{0.0702\left(1 + 0.734\ln[\omega_H\tau_m]\right)\left(1 + 1.48/\varepsilon - 1.1/\varepsilon^2\right)}$$

$$K_c^{(18)} = \left(1 - \frac{1.03\left[1 - 0.51\omega_H\tau_m + 0.0759(\omega_H\tau_m)^2\right]}{K_H K_m/(1 + K_H K_m)}\right)K_H,\ T_i^{(18)} = \frac{K_c(1 + K_H K_m)}{0.0386\omega_H K_m\left(-1 + 3.26\omega_H\tau_m - 0.6\omega_H{}^2\tau_m{}^2\right)}$$

$$K_c^{(19)} = \left(1 - \frac{0.822\left[1 - 0.549\omega_H\tau_m + 0.112(\omega_H\tau_m)^2\right]}{K_H K_m/(1 + K_H K_m)}\right)K_H,\ T_i^{(19)} = \frac{K_c(1 + K_H K_m)}{0.0142\omega_H K_m\left(1 + 6.96\omega_H\tau_m - 1.77\omega_H{}^2\tau_m{}^2\right)}$$

$$K_c^{(20)} = \left(1 - \frac{0.786\left[1 - 0.441\omega_H\tau_m + 0.0569(\omega_H\tau_m)^2\right]}{K_H K_m/(1 + K_H K_m)}\right)K_H,\ T_i^{(20)} = \frac{K_c(1 + K_H K_m)}{0.0172\omega_H K_m\left(1 + 4.62\omega_H\tau_m - 0.823\omega_H{}^2\tau_m{}^2\right)}$$

$$K_c^{(21)} = \left(1 - \frac{1.28(0.542)^{\omega_H\tau_m}\left[1 - 0.986/\varepsilon + 0.558/\varepsilon^2\right]}{K_H K_m/(1 + K_H K_m)}\right)K_H,\ T_i^{(21)} = \frac{K_c(1 + K_H K_m)}{0.0476\left(1 + 0.996\ln[\omega_H\tau_m]\right)\left(1 + 2.13/\varepsilon - 1.13/\varepsilon^2\right)}$$

$$K_c^{(22)} = \left(1 - \frac{1.14\left[1 - 0.466\omega_H\tau_m + 0.0647(\omega_H\tau_m)^2\right]}{K_H K_m/(1 + K_H K_m)}\right)K_H,\ T_i^{(22)} = \frac{K_c(1 + K_H K_m)}{0.0609\omega_H K_m\left(-1 + 1.97\omega_H\tau_m - 0.323\omega_H{}^2\tau_m{}^2\right)}$$

Rule	K_c	T_i	Comment
Servo - nearly minimum IAE, ISE, ITAE - Hwang [60] *Model: Method 3*	$K_c^{(23)}$ [8]	$T_i^{(23)}$	Decay ratio = 0.1 - $\varepsilon < 2.4$, $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$, $0.6 \le \xi_m \le 4.2$ $\xi > 0.889 + 0.496\frac{\tau_m}{T_{m1}} + 0.26\left(\frac{\tau_m}{T_{m1}}\right)^2$
	$K_c^{(24)}$	$T_i^{(24)}$	Decay ratio = 0.1 - $2.4 \le \varepsilon < 3$, $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$, $0.6 \le \xi_m \le 4.2$ $\xi > 0.889 + 0.496\frac{\tau_m}{T_{m1}} + 0.26\left(\frac{\tau_m}{T_{m1}}\right)^2$
	$K_c^{(25)}$	$T_i^{(25)}$	Decay ratio = 0.1 - $3 \le \varepsilon < 20$, $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$, $0.6 \le \xi_m \le 4.2$ $\xi > 0.889 + 0.496\frac{\tau_m}{T_{m1}} + 0.26\left(\frac{\tau_m}{T_{m1}}\right)^2$
	$K_c^{(26)}$	$T_i^{(26)}$	Decay ratio = 0.1 - $\varepsilon > 20$, $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$, $0.6 \le \xi_m \le 4.2$ $\xi > 0.889 + 0.496\frac{\tau_m}{T_{m1}} + 0.26\left(\frac{\tau_m}{T_{m1}}\right)^2$
	$K_c^{(27)}$	$T_i^{(27)}$	Decay ratio = 0.1 - $\varepsilon < 2.4$, $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$, $0.6 \le \xi_m \le 4.2$ $\xi_m > 0.776 + 0.0568\frac{\tau_m}{T_{m1}} + 0.18\left(\frac{\tau_m}{T_{m1}}\right)^2$ $\xi_m \le 0.889 + 0.496\frac{\tau_m}{T_{m1}} + 0.26\left(\frac{\tau_m}{T_{m1}}\right)^2$

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[8] $$K_c^{(23)} = \left(1 - \frac{0.794\left[1 - 0.541\omega_H\tau_m + 0.126(\omega_H\tau_m)^2\right]}{K_H K_m/(1 + K_H K_m)}\right)K_H,\ T_i^{(23)} = \frac{K_c(1 + K_H K_m)}{0.0078\omega_H K_m\left(1 + 8.38\omega_H\tau_m - 1.97\omega_H^2\tau_m^2\right)}$$

$$K_c^{(24)} = \left(1 - \frac{0.738\left[1 - 0.415\omega_H\tau_m + 0.0575(\omega_H\tau_m)^2\right]}{K_H K_m/(1 + K_H K_m)}\right)K_H,\ T_i^{(24)} = \frac{K_c(1 + K_H K_m)}{0.0124\omega_H K_m\left(1 + 4.05\omega_H\tau_m - 0.63\omega_H^2\tau_m^2\right)}$$

$$K_c^{(25)} = \left(1 - \frac{1.15(0.564)^{\omega_H\tau_m}\left[1 - 0.959/\varepsilon + 0.773/\varepsilon^2\right]}{K_H K_m/(1 + K_H K_m)}\right)K_H,\ T_i^{(25)} = \frac{K_c(1 + K_H K_m)}{0.0355\left(1 + 0.947\ln[\omega_H\tau_m]\right)\left(1 + 1.9/\varepsilon - 1.07/\varepsilon^2\right)}$$

$$K_c^{(26)} = \left(1 - \frac{1.07\left[1 - 0.466\omega_H\tau_m + 0.0667(\omega_H\tau_m)^2\right]}{K_H K_m/(1 + K_H K_m)}\right)K_H,\ T_i^{(26)} = \frac{K_c(1 + K_H K_m)}{0.0328\omega_H K_m\left(-1 + 2.21\omega_H\tau_m - 0.338\omega_H^2\tau_m^2\right)}$$

$$K_c^{(27)} = \left(1 - \frac{0.789\left[1 - 0.527\omega_H\tau_m + 0.11(\omega_H\tau_m)^2\right]}{K_H K_m/(1 + K_H K_m)}\right)K_H,\ T_i^{(27)} = \frac{K_c(1 + K_H K_m)}{0.009\omega_H K_m\left(1 + 9.7\omega_H\tau_m - 2.4\omega_H^2\tau_m^2\right)}$$

Rule	K_c	T_i	Comment
Servo - nearly minimum IAE, ISE, ITAE - Hwang [60] *Model: Method 3*	$K_c^{(28)}$ [9]	$T_i^{(28)}$	Decay ratio = 0.1 - $2.4 \le \varepsilon < 3$, $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$, $0.6 \le \xi_m \le 4.2$ $\xi_m > 0.776 + 0.0568\frac{\tau_m}{T_{m1}} + 0.18\left(\frac{\tau_m}{T_{m1}}\right)^2$ $\xi_m \le 0.889 + 0.496\frac{\tau_m}{T_{m1}} + 0.26\left(\frac{\tau_m}{T_{m1}}\right)^2$
	$K_c^{(29)}$	$T_i^{(29)}$	Decay ratio = 0.1 - $3 \le \varepsilon < 20$, $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$, $0.6 \le \xi_m \le 4.2$ $\xi_m > 0.776 + 0.0568\frac{\tau_m}{T_{m1}} + 0.18\left(\frac{\tau_m}{T_{m1}}\right)^2$ $\xi_m \le 0.889 + 0.496\frac{\tau_m}{T_{m1}} + 0.26\left(\frac{\tau_m}{T_{m1}}\right)^2$
	$K_c^{(30)}$	$T_i^{(30)}$	Decay ratio = 0.1 - $\varepsilon > 20$, $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$, $0.6 \le \xi_m \le 4.2$ $\xi_m > 0.776 + 0.0568\frac{\tau_m}{T_{m1}} + 0.18\left(\frac{\tau_m}{T_{m1}}\right)^2$ $\xi_m \le 0.889 + 0.496\frac{\tau_m}{T_{m1}} + 0.26\left(\frac{\tau_m}{T_{m1}}\right)^2$
Regulator - minimum IAE - Shinskey [17] – *page 121*. *Model: method not specified*	$0.48K_u$	$0.83T_u$	$\frac{\tau_m}{T_{m1}} = 0.2$, $\frac{T_{m2}}{T_{m1}} = 0.2$

[9] $$K_c^{(28)} = \left(1 - \frac{0.76\left[1 - 0.426\omega_H\tau_m + 0.0551(\omega_H\tau_m)^2\right]}{K_H K_m/(1+K_H K_m)}\right)K_H,\ T_i^{(28)} = \frac{K_c(1+K_H K_m)}{0.0153\omega_H K_m\left(1+4.37\omega_H\tau_m - 0.743\omega_H^2\tau_m^2\right)}$$

$$K_c^{(29)} = \left(1 - \frac{1.22(0.55)^{\omega_H\tau_m}\left[1 - 0.978/\varepsilon + 0.659/\varepsilon^2\right]}{K_H K_m/(1+K_H K_m)}\right)K_H,\ T_i^{(29)} = \frac{K_c(1+K_H K_m)}{0.0421\left(1+0.969\ln[\omega_H\tau_m]\right)\left(1+2.02/\varepsilon - 1.11/\varepsilon^2\right)}$$

$$K_c^{(30)} = \left(1 - \frac{1.11\left[1 - 0.467\omega_H\tau_m + 0.0657(\omega_H\tau_m)^2\right]}{K_H K_m/(1+K_H K_m)}\right)K_H,\ T_i^{(30)} = \frac{K_c(1+K_H K_m)}{0.0477\omega_H K_m\left(-1+2.07\omega_H\tau_m - 0.333\omega_H^2\tau_m^2\right)}$$

Table 16: PI tuning rules - SOSPD model - $\dfrac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$

- Two degree of freedom controller: $U(s) = K_c\left(1 + \dfrac{1}{T_i s}\right)E(s) - \alpha K_c R(s)$. *3 tuning rules.*

Rule	K_c	T_i	Comment
Servo/regulator tuning	**Minimum performance index**		
Taguchi and Araki [61a] *Model: ideal process*	$\dfrac{1}{K_m}\left(0.3717 + \dfrac{0.5613}{\dfrac{\tau_m}{T_m} + 0.0003414}\right)$	$T_i^{(30a)}$ [10]	$\dfrac{\tau_m}{T_m} \le 1.0$; $\xi_m = 1$ Overshoot (servo step) $\le 20\%$; settling time $\le$ settling time of tuning rules of Chien *et al*. [10]
	$\alpha = 0.6438 - 0.5056\dfrac{\tau_m}{T_m} + 0.3087\left(\dfrac{\tau_m}{T_m}\right)^2 - 0.1201\left(\dfrac{\tau_m}{T_m}\right)^3$		
	$\dfrac{1}{K_m}\left(0.1000 + \dfrac{0.05627}{\left[\dfrac{\tau_m}{T_m} + 0.06041\right]^2}\right)$	$T_i^{(30b)}$	$\dfrac{\tau_m}{T_m} \le 1.0$; $\xi_m = 0.5$ Overshoot (servo step) $\le 20\%$; settling time $\le$ settling time of tuning rules of Chien *et al*. [10]
	$\alpha = 0.6178 - 0.4439\dfrac{\tau_m}{T_m} - 7.575\left(\dfrac{\tau_m}{T_m}\right)^2 + 9.317\left(\dfrac{\tau_m}{T_m}\right)^3 - 3.182\left(\dfrac{\tau_m}{T_m}\right)^4$		
Minimum ITAE - Pecharroman and Pagola [134a] *Model: Method 15*	$0.1713 K_u$	$1.0059 T_u$	$\alpha = 0.4002$, $\phi_c = -139.65^0$ $K_m = 1;\ T_m = 1;\ \xi_m = 1$
Minimum ITAE - Pecharroman and Pagola [134b] $K_m = 1;\ T_m = 1;\ \xi_m = 1$ *Model: Method 15*	$0.147 K_u$	$1.150 T_u$	$\alpha = 0.411$, $\phi_c = -146^0$
	$0.170 K_u$	$1.013 T_u$	$\alpha = 0.401$, $\phi_c = -140^0$
	$0.195 K_u$	$0.880 T_u$	$\alpha = 0.386$, $\phi_c = -133^0$
	$0.210 K_u$	$0.720 T_u$	$\alpha = 0.342$, $\phi_c = -125^0$
	$0.234 K_u$	$0.672 T_u$	$\alpha = 0.345$, $\phi_c = -115^0$
	$0.249 K_u$	$0.610 T_u$	$\alpha = 0.323$, $\phi_c = -105^0$
	$0.262 K_u$	$0.568 T_u$	$\alpha = 0.308$, $\phi_c = -94^0$
	$0.274 K_u$	$0.545 T_u$	$\alpha = 0.291$, $\phi_c = -84^0$
	$0.280 K_u$	$0.512 T_u$	$\alpha = 0.281$, $\phi_c = -73^0$
	$0.291 K_u$	$0.503 T_u$	$\alpha = 0.270$, $\phi_c = -63^0$
	$0.297 K_u$	$0.483 T_u$	$\alpha = 0.260$, $\phi_c = -52^0$
	$0.303 K_u$	$0.462 T_u$	$\alpha = 0.246$, $\phi_c = -41^0$
	$0.307 K_u$	$0.431 T_u$	$\alpha = 0.229$, $\phi_c = -30^0$

[10] $T_i^{(30a)} = T_m\left(2.069 - 0.3692\dfrac{\tau_m}{T_m} + 1.081\left[\dfrac{\tau_m}{T_m}\right]^2 - 0.5524\left[\dfrac{\tau_m}{T_m}\right]^3\right)$

$T_i^{(30b)} = T_m\left(4.340 - 16.39\dfrac{\tau_m}{T_m} + 30.04\left[\dfrac{\tau_m}{T_m}\right]^2 - 25.85\left[\dfrac{\tau_m}{T_m}\right]^3 + 8.567\left[\dfrac{\tau_m}{T_m}\right]^4\right)$

Rule	K_c	T_i	Comment
Minimum ITAE - Pecharroman and Pagola [134b] - continued	$0.317\,K_u$	$0.386\,T_u$	$\alpha = 0.171$, $\phi_c = -19^0$
	$0.324\,K_u$	$0.302\,T_u$	$\alpha = 0.004$, $\phi_c = -10^0$
	$0.320\,K_u$	$0.223\,T_u$	$\alpha = -0.204$, $\phi_c = -6^0$

Table 17: PI tuning rules - SOSIPD model (repeated pole) - $\frac{K_m e^{-s\tau_m}}{s(1+T_{m1}s)^2}$

- Two degree of freedom controller: $U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - \alpha K_c R(s)$. *1 tuning rule*.

Rule	K_c	T_i	Comment
Servo/regulator tuning	**Minimum performance index**		
Taguchi and Araki [61a] *Model: ideal process*	$\frac{1}{K_m}\left(0.07368+\frac{0.3840}{\frac{\tau_m}{T_m}+0.7640}\right)$	$8.549+4.029\frac{\tau_m}{T_m}$	$\frac{\tau_m}{T_m}\le 1.0$; Overshoot (servo step) $\le 20\%$; settling time $\le$ settling time of tuning rules of Chien *et al*. [10]
	$\alpha = 0.6691 + 0.006606\frac{\tau_m}{T_m}$		
Minimum ITAE - Pecharroman and Pagola [134b] $K_m = 1$; $T_m = 1$ *Model: Method 1*	$0.049K_u$	$2.826T_u$	$\alpha = 0.506$, $\phi_c = -164^0$
	$0.066K_u$	$2.402T_u$	$\alpha = 0.512$, $\phi_c = -160^0$
	$0.099K_u$	$1.962T_u$	$\alpha = 0.522$, $\phi_c = -155^0$
	$0.129K_u$	$1.716T_u$	$\alpha = 0.532$, $\phi_c = -150^0$
	$0.159K_u$	$1.506T_u$	$\alpha = 0.544$, $\phi_c = -145^0$
	$0.189K_u$	$1.392T_u$	$\alpha = 0.555$, $\phi_c = -140^0$
	$0.218K_u$	$1.279T_u$	$\alpha = 0.564$, $\phi_c = -135^0$
	$0.250K_u$	$1.216T_u$	$\alpha = 0.573$, $\phi_c = -130^0$
	$0.286K_u$	$1.127T_u$	$\alpha = 0.578$, $\phi_c = -125^0$
	$0.330K_u$	$1.114T_u$	$\alpha = 0.579$, $\phi_c = -120^0$
	$0.351K_u$	$1.093T_u$	$\alpha = 0.577$, $\phi_c = -118^0$

<u>Table 19</u>: PI tuning rules - TOLPD model $\frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})(1+sT_{m3})}$ - controller $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)$. *1 tuning rule*

Rule	K_c	T_i	Comment
Hougen [85] *Model: Method 1*	$\frac{0.7}{K_m}\left(\frac{T_{m1}}{\tau_m}\right)^{0.333}$	$1.5\tau_m^{0.08}\sqrt{T_{m1}(T_{m2}+T_{m3})}$	$\frac{\tau_m}{T_{m1}} > 0.04$; $T_{m1} \ge T_{m2} \ge T_{m3}$
	$\frac{1}{2K_m}\left[0.7\left(\frac{T_{m1}}{\tau_m}\right)^{0.333} + 0.8\frac{T_{m1}+T_{m2}+T_{m3}}{(T_{m1}T_{m2}T_{m3})^{0.333}}\right]$	$1.5\tau_m^{0.08}\sqrt{T_{m1}(T_{m2}+T_{m3})}$	$\frac{\tau_m}{T_{m1}} \le 0.04$; $T_{m1} \ge T_{m2} \ge T_{m3}$

Table 20: PI tuning rules - TOLPD model (repeated pole) - $\frac{K_m e^{-s\tau_m}}{(1+T_{m1}s)^3}$

- Two degree of freedom controller: $U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - \alpha K_c R(s)$. *1 tuning rule*.

Rule	K_c	T_i	Comment
Servo/regulator tuning	**Minimum performance index**		
Taguchi and Araki [61a] *Model: ideal process*	$\frac{1}{K_m}\left(0.2713+\frac{0.7399}{\frac{\tau_m}{T_m}+0.5009}\right)$	$T_i^{(30c)}$ [1]	$\frac{\tau_m}{T_m} \le 1.0$; Overshoot (servo step) $\le 20\%$; settling time $\le$ settling time of tuning rules of Chien *et al*. [10]
	$\alpha = 0.4908 - 0.2648\frac{\tau_m}{T_m} + 0.05159\left(\frac{\tau_m}{T_m}\right)^2$		

[1] $T_i^{(30c)} = T_m\left(2.759 - 0.003899\frac{\tau_m}{T_m} + 0.1354\left[\frac{\tau_m}{T_m}\right]^2\right)$

Table 21: PI tuning rules - unstable FOLPD model $\frac{K_m e^{-s\tau_m}}{1 - sT_m}$ - controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)$. *6 tuning rules*

Rule	K_c	T_i	Comment
Direct synthesis			
De Paor and O'Malley [86] *Model: Method 1*	$K_c^{(35)}$	$\frac{1}{T_m\left[\sqrt{\frac{1 - T_m\tau_m}{T_m\tau_m}}\right]\tan(0.5\phi_m)}$	gain margin = 2; $\frac{\tau_m}{T_m} < 1$ $\phi_m = \tan^{-1}\sqrt{\frac{1 - T_m\tau_m}{T_m\tau_m}} - \sqrt{T_m\tau_m(1 - T_m\tau_m)}$
Venkatashankar and Chidambaram [87] *Model: Method 1*	$K_c^{(36)}$	$25(T_m - \tau_m)$	$\frac{\tau_m}{T_m} < 0.67$
Chidambaram [88] *Model: Method 1*	$\frac{1}{K_m}\left(1 + 0.26\frac{T_m}{\tau_m}\right)$	$25T_m - 27\tau_m$	$\frac{\tau_m}{T_m} < 0.6$
Ho and Xu [90] *Model: Method 1*	$\frac{\omega_p T_m}{A_m K_m}$	$\frac{1}{1.57\omega_p - \omega_p^2\tau_m - \frac{1}{T_m}}$	$\omega_p = \frac{A_m\phi_m + 1.57A_m(A_m - 1)}{(A_m^2 - 1)\tau_m}$, $\frac{\tau_m}{T_m} < 0.62$
Robust			
Rotstein and Lewin [89]	$\frac{T_m\lambda\left(\frac{\lambda}{T_m} + 2\right)}{\lambda^2 K_m}$	$\lambda\left(\frac{\lambda}{T_m} + 2\right)$	λ obtained graphically – sample values below
Model: Method 1	K_m uncertainty = 50%	$\tau_m/T_m = 0.2$	$\lambda = [0.6T_m, 1.9T_m]$
	K_m uncertainty = 30%	$\tau_m/T_m = 0.2$	$\lambda = [0.5T_m, 4.5T_m]$
		$\tau_m/T_m = 0.4$	$\lambda = [1.5T_m, 4.5T_m]$
		$\tau_m/T_m = 0.6$	$\lambda = [3.9T_m, 4.1T_m]$
Ultimate cycle			
Luyben [91] *Model: Method not relevant*	$0.31K_u$	$2.2T_u$	Maximum closed loop log modulus = 2 dB; closed loop time constant = $3.16\tau_m$

[2]

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[2] $$K_c^{(35)} = \frac{T_m}{K_m}\left[\cos\sqrt{(1 - T_m\tau_m)T_m\tau_m} + \sqrt{\frac{1 - T_m\tau_m}{T_m\tau_m}}\sin\sqrt{(1 - T_m\tau_m)T_m\tau_m}\right]$$

$$K_c^{(36)} = \frac{1}{K_m}\sqrt{\left(0.98\sqrt{1 + \frac{0.04T_m^2}{(T_m - \tau_m)^2}}\right)\left(\frac{25}{\tau_m}\right)\beta(T_m - \tau_m)\sqrt{\frac{1 + \frac{\beta^2 T_m^2(T_m - \tau_m)^2}{(T_m - \tau_m)^2\tau_m^2}}{1 + \beta^2\frac{625}{\tau_m^2}(T_m - \tau_m)^2}}} : \beta = 1.373,\ \frac{\tau_m}{T_m} < 0.25$$

$$\beta = 0.953,\ 0.25 \le \frac{\tau_m}{T_m} < 0.67$$

Table 22: PI tuning rules - unstable SOSPD model $\frac{K_m e^{-s\tau_m}}{(1-sT_{m1})(1+sT_{m2})}$ - controller $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)$. *3 tuning rules.*

Rule	K_c			T_i			Comment		
Ultimate cycle									
McMillan [58] *Model: Method not relevant*	$K_c^{(37)}$ [3]			$T_i^{(37)}$			Tuning rules developed from K_u, T_u		
Regulator tuning	**Minimum performance index**								
Minimum ITAE – Poulin and Pomerleau [82] – *deduced from graph*	$\frac{bT_{m1}\sqrt{1+\frac{aT_{m2}^2}{4(\tau_m+T_{m2})^2}}}{K_m(aT_{m1}-4[\tau_m+T_{m2}])}$			$\frac{4T_{m1}(\tau_m+T_{m2})}{aT_{m1}-4(\tau_m+T_{m2})}$					
Model: Method 1	τ_m/T_m	a	b	τ_m/T_m	a	b	τ_m/T_m	a	b
Output step load disturbance	0.05	0.9479	2.3546	0.25	1.4905	2.5992	0.45	2.0658	2.9004
	0.10	1.0799	2.4111	0.30	1.6163	2.6612	0.50	2.2080	2.9826
	0.15	1.2013	2.4646	0.35	1.7650	2.7368			
(considered as 2 tuning rules)	0.20	1.3485	2.5318	0.40	1.9139	2.8161			
	0.05	1.1075	2.4230	0.25	1.5698	2.6381	0.45	2.1022	2.9210
	0.10	1.2013	2.4646	0.30	1.6943	2.7007	0.50	2.2379	3.0003
Input step load disturbance	0.15	1.3132	2.5154	0.35	1.8161	2.7637			
	0.20	1.4384	2.5742	0.40	1.9658	2.8445			

[3] $K_c^{(37)} = \frac{1.477}{K_m}\frac{T_{m1}T_{m2}}{\tau_m^2}\left\{\frac{1}{1+\left[\frac{(T_{m1}+T_{m2})T_{m1}T_{m2}}{(T_{m1}-T_{m2})(T_{m1}-\tau_m)\tau_m}\right]^{0.65}}\right\}^2$, $T_i^{(37)} = 3.32\tau_m\left\{1+\left[\frac{(T_{m1}+T_{m2})T_{m1}T_{m2}}{(T_{m1}-T_{m2})(T_{m1}-\tau_m)\tau_m}\right]^{0.65}\right\}$

$$K_c^{(38)} = \frac{T_i A_{max}}{K_m}\sqrt{\frac{T_{m1}^2 T_{m2}^2\omega_{max}^6 + (T_{m1}^2+T_{m2}^2)\omega_{max}^4 + \omega_{max}^2}{1+T_i^2\omega_{max}^2}}$$

Table 23: PI tuning rules – delay model $e^{-s\tau_m}$ - controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)$. *2 tuning rules*

Rule	K_c	T_i	Comment
Direct synthesis			
Hansen [91a] *Model: Method not specified*	0.2	$0.3\tau_m$	
Regulator tuning	**Minimum performance index**		
Shinskey [57] – minimum IAE – regulator - *page 67*. *Model: method not specified*	$\frac{0.4}{K_m}$	$0.5\tau_m$	

Table 25: PID tuning rules - FOLPD model $\frac{K_m e^{-s\tau_m}}{1+sT_m}$ - ideal controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$. **56.**

Rule	K_c	T_i	T_d	Comment
Process reaction				
Ziegler and Nichols [8] *Model: Method 1*	$[\frac{1.2T_m}{K_m\tau_m}, \frac{2T_m}{K_m\tau_m}]$	$2\tau_m$	$0.5\tau_m$	Quarter decay ratio
Astrom and Hagglund [3] – *page 139* *Model: Method 6*	$\frac{0.94T_m}{K_m\tau_m}$	$2\tau_m$	$0.5\tau_m$	Ultimate cycle Ziegler-Nichols equivalent
Parr [64] – *page 194* *Model: Method 1*	$\frac{1.25T_m}{K_m\tau_m}$	$2.5\tau_m$	$0.4\tau_m$	
Chien *et al.* [10] – regulator *Model: Method 1*	$\frac{0.95T_m}{K_m\tau_m}$	$2.38\tau_m$	$0.42t_m$	0% overshoot; $0.11 < \frac{\tau_m}{T_m} < 1$
	$\frac{1.2T_m}{K_m\tau_m}$	$2\tau_m$	$0.42\tau_m$	20% overshoot; $0.11 < \frac{\tau_m}{T_m} < 1$
Chien *et al.* [10] - servo *Model: Method 1*	$\frac{0.6T_m}{K_m\tau_m}$	T_m	$0.5\tau_m$	0% overshoot; $0.11 < \frac{\tau_m}{T_m} < 1$
	$\frac{0.95T_m}{K_m\tau_m}$	$1.36T_m$	$0.47\tau_m$	20% overshoot; $0.11 < \frac{\tau_m}{T_m} < 1$
Three constraints method - Murrill [13]- *page 356* *Model: Method 3*	$\frac{1.370}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.950}$	$\frac{T_m}{1.351}\left(\frac{\tau_m}{T_m}\right)^{0.738}$	$0.365T_m\left(\frac{\tau_m}{T_m}\right)^{0.950}$	Quarter decay ratio; minimum integral error (servo mode); $\frac{K_c K_m T_d}{T_m} = 0.5$; $0.1 \le \frac{\tau_m}{T_m} \le 1$
Cohen and Coon [11] *Model: Method 1*	$\frac{1}{K_m}\left(1.35\frac{T_m}{\tau_m} + 0.25\right)$	$T_m\left(\frac{2.5\frac{\tau_m}{T_m} + 0.46\left(\frac{\tau_m}{T_m}\right)^2}{1 + 0.61\frac{\tau_m}{T_m}}\right)$	$\frac{0.37\tau_m}{1 + 0.2\frac{\tau_m}{T_m}}$	Quarter decay ratio
Astrom and Hagglund [93]- *pages 120-126* *Model: Method 19*	$\frac{3}{K_m}$	$T_{90\%}$	$0.5\tau_m$	$T_{90\%}$ = 90% closed loop step response time. Leeds and Northrup Electromax V.
Sain and Ozgen [94] *Model: Method 5*	$\frac{1}{K_m}\left(0.6939\frac{T_m}{\tau_m} + 0.1814\right)$	$\frac{0.8647T_m + 0.226\tau_m}{\frac{T_m}{\tau_m} + 0.8647}$	$\frac{0.0565T_m}{0.8647\frac{T_m}{\tau_m} + 0.226}$	
Regulator tuning	**Minimum performance index**			
Minimum IAE – Murrill [13] – *pages 358-363* *Model: Method 3*	$\frac{1.435}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.921}$	$\frac{T_m}{0.878}\left(\frac{\tau_m}{T_m}\right)^{0.749}$	$0.482T_m\left(\frac{\tau_m}{T_m}\right)^{1.137}$	$0.1 < \frac{\tau_m}{T_m} \le 1$

Rule	K_c	T_i	T_d	Comment
Modified minimum IAE - Cheng and Hung [95] *Model: Method 7*	$\frac{3}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.921}$	$\frac{T_m}{0.878}\left(\frac{\tau_m}{T_m}\right)^{0.749}$	$0.482T_m\left(\frac{\tau_m}{T_m}\right)^{1.137}$	
Minimum ISE - Murrill [13] – *pages 358-363* *Model: Method 3*	$\frac{1.495}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.945}$	$\frac{T_m}{1.101}\left(\frac{\tau_m}{T_m}\right)^{0.771}$	$0.56T_m\left(\frac{\tau_m}{T_m}\right)^{1.006}$	$0.1 < \frac{\tau_m}{T_m} \le 1$
Minimum ISE - Zhuang and Atherton [20]	$\frac{1.473}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.970}$	$\frac{T_m}{1.115}\left(\frac{\tau_m}{T_m}\right)^{0.753}$	$0.55T_m\left(\frac{\tau_m}{T_m}\right)^{0.948}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
Model: Method 6	$\frac{1.524}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.735}$	$\frac{T_m}{1.130}\left(\frac{\tau_m}{T_m}\right)^{0.641}$	$0.552T_m\left(\frac{\tau_m}{T_m}\right)^{0.851}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$
Minimum ITAE - Murrill [13] – *pages 358-363* *Model: Method 3*	$\frac{1.357}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.947}$	$\frac{T_m}{0.842}\left(\frac{\tau_m}{T_m}\right)^{0.738}$	$0.381T_m\left(\frac{\tau_m}{T_m}\right)^{0.995}$	$0.1 < \frac{\tau_m}{T_m} \le 1$
Minimum ISTSE - Zhuang and Atherton [20]	$\frac{1.468}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.970}$	$\frac{T_m}{0.942}\left(\frac{\tau_m}{T_m}\right)^{0.725}$	$0.443T_m\left(\frac{\tau_m}{T_m}\right)^{0.939}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
Model: Method 6	$\frac{1.515}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.730}$	$\frac{T_m}{0.957}\left(\frac{\tau_m}{T_m}\right)^{0.598}$	$0.444T_m\left(\frac{\tau_m}{T_m}\right)^{0.847}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$
Minimum ISTES - Zhuang and Atherton [20]	$\frac{1.531}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.960}$	$\frac{T_m}{0.971}\left(\frac{\tau_m}{T_m}\right)^{0.746}$	$0.413T_m\left(\frac{\tau_m}{T_m}\right)^{0.933}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
Model: Method 6	$\frac{1.592}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.705}$	$\frac{T_m}{0.957}\left(\frac{\tau_m}{T_m}\right)^{0.597}$	$0.414T_m\left(\frac{\tau_m}{T_m}\right)^{0.850}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$
Minimum error - step load change - Gerry [96] *Model: Method 6*	$\frac{0.3}{K_m}$	$0.5\tau_m$	T_m	$\frac{\tau_m}{T_m} > 5$
Servo tuning	**Minimum performance index**			
Minimum IAE - Rovira *et al*. [21] *Model: Method 3*	$\frac{1.086}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.869}$	$\frac{T_m}{0.740 - 0.13\frac{\tau_m}{T_m}}$	$0.348T_m\left(\frac{\tau_m}{T_m}\right)^{0.914}$	$0.1 < \frac{\tau_m}{T_m} \le 1$
Minimum IAE - Wang *et al*. [97] *Model: Method 6*	$\frac{\left(0.7645 + \frac{0.6032}{\tau_m/T_m}\right)(T_m + 0.5\tau_m)}{K_m(T_m + \tau_m)}$	$T_m + 0.5\tau_m$	$\frac{0.5T_m\tau_m}{T_m + 0.5\tau_m}$	$0.05 < \frac{\tau_m}{T_m} < 6$
Minimum ISE - Wang *et al*. [97] *Model: Method 6*	$\frac{\left(0.9155 + \frac{0.7524}{\tau_m/T_m}\right)(T_m + 0.5\tau_m)}{K_m(T_m + \tau_m)}$	$T_m + 0.5\tau_m$	$\frac{0.5T_m\tau_m}{T_m + 0.5\tau_m}$	$0.05 < \frac{\tau_m}{T_m} < 6$
Minimum ISE - Zhuang and Atherton [20]	$\frac{1.048}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.897}$	$\frac{T_m}{1.195 - 0.368\frac{\tau_m}{T_m}}$	$0.489T_m\left(\frac{\tau_m}{T_m}\right)^{0.888}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
Model: Method 6	$\frac{1.154}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.567}$	$\frac{T_m}{1.047 - 0.220\frac{\tau_m}{T_m}}$	$0.490T_m\left(\frac{\tau_m}{T_m}\right)^{0.708}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$

Rule	K_c	T_i	T_d	Comment
Minimum ITAE - Rovira *et al*. [21] *Model: Method 3*	$\frac{0.965}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.85}$	$\frac{T_m}{0.796 - 0.1465\frac{\tau_m}{T_m}}$	$0.308T_m\left(\frac{\tau_m}{T_m}\right)^{0.929}$	$0.1 \le \frac{\tau_m}{T_m} \le 1$
Modified minimum ITAE - Cheng and Hung [95] *Model: Method 7*	$\frac{1.2}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.855}$	$\frac{T_m}{0.796 - 0.147\frac{\tau_m}{T_m}}$	$0.308T_m\left(\frac{\tau_m}{T_m}\right)^{0.929}$	Damping factor of closed loop system = 0.707.
Minimum ITAE – Wang *et al*. [97] *Model: Method 6*	$\frac{\left(0.7303 + \frac{0.5307}{\tau_m/T_m}\right)\left(T_m + 0.5\tau_m\right)}{K_m\left(T_m + \tau_m\right)}$	$T_m + 0.5\tau_m$	$\frac{0.5T_m\tau_m}{T_m + 0.5\tau_m}$	$0.05 < \frac{\tau_m}{T_m} < 6$
Minimum ISTSE – Zhuang and Atherton [20] *Model: Method 6*	$\frac{1.042}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.897}$	$\frac{T_m}{0.987 - 0.238\frac{\tau_m}{T_m}}$	$0.385T_m\left(\frac{\tau_m}{T_m}\right)^{0.906}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
	$\frac{1.142}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.579}$	$\frac{T_m}{0.919 - 0.172\frac{\tau_m}{T_m}}$	$0.384T_m\left(\frac{\tau_m}{T_m}\right)^{0.839}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$
Minimum ISTES – Zhuang and Atherton [20] *Model: Method 6*	$\frac{0.968}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.904}$	$\frac{T_m}{0.977 - 0.253\frac{\tau_m}{T_m}}$	$0.316T_m\left(\frac{\tau_m}{T_m}\right)^{0.892}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
	$\frac{1.061}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.583}$	$\frac{T_m}{0.892 - 0.165\frac{\tau_m}{T_m}}$	$0.315T_m\left(\frac{\tau_m}{T_m}\right)^{0.832}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$
Ultimate cycle				
Regulator – minimum IAE – Pessen [63] *Model: Method 6*	$0.7K_u$	$0.4T_u$	$0.149T_u$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
Servo – minimum ISTSE – Zhuang and Atherton [20] *Model: Method not relevant*	$0.509K_u$	$0.051(3.302K_mK_u + 1)T_u$	$0.125T_u$	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$
Servo – minimum ISTSE – Pi-Mira *et al*. [97a] *Model: Method 27*	$0.604K_u$	$0.04(4.972K_mK_u + 1)T$	$0.130T_u$	
Regulator - minimum ISTSE - Zhuang and Atherton [20] *Model: Method not relevant*	$\frac{4.434K_mK_u - 0.966}{5.12K_mK_u + 1.734}K_u$	$\frac{1.751K_mK_u - 0.612}{3.776K_mK_u + 1.388}T_u$	$0.144T_u$	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$

Rule	K_c	T_i	T_d	Comment
Regulator - nearly minimum IAE, ISE, ITAE - Hwang [60] *Model: Method 8*	[1] $K_c^{(38)}$	$T_i^{(38)}$	$\frac{0.471K_u}{K_m\omega_u}$	Model: Method 8 Decay ratio = 0.15 - $0.1 \le \frac{\tau_m}{T_m} \le 2.0$; $\varepsilon < 2.4$
	$K_c^{(39)}$	$T_i^{(39)}$	$\frac{0.471K_u}{K_m\omega_u}$	Decay ratio = 0.15 - $0.1 \le \frac{\tau_m}{T_m} \le 2.0$; $2.4 \le \varepsilon < 3$
	$K_c^{(40)}$	$T_i^{(40)}$	$\frac{0.471K_u}{K_m\omega_u}$	Decay ratio = 0.15 - $0.1 \le \frac{\tau_m}{T_m} \le 2.0$; $3 \le \varepsilon < 20$
	$K_c^{(41)}$	$T_i^{(41)}$	$\frac{0.471K_u}{K_m\omega_u}$	Decay ratio = 0.15 - $0.1 \le \frac{\tau_m}{T_m} \le 2.0$; $\varepsilon \ge 20$
	$K_c^{(42)}$	$T_i^{(42)}$	$\frac{0.471K_u}{K_m\omega_u}$	Decay ratio = 0.2 - $0.1 \le \frac{\tau_m}{T_m} \le 2.0$; $\varepsilon < 2.4$
	$K_c^{(43)}$	$T_i^{(43)}$	$\frac{0.471K_u}{K_m\omega_u}$	Decay ratio = 0.2 - $0.1 \le \frac{\tau_m}{T_m} \le 2.0$; $2.4 \le \varepsilon < 3$

[2]

Rule	K_c	T_i	T_d	Comment

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[1] $$K_H = \frac{9}{2K_m\tau_m^2}\left[\frac{\tau_m^2}{18} - \frac{\tau_m T_m}{18} + \frac{1.884K_mK_cK_u\tau_m}{9\omega_u} + \sqrt{\frac{\tau_m^4}{324} + \frac{49\tau_m^2T_m^2}{324} + \frac{7T_m\tau_m^3}{162} - \frac{0.471K_uK_c}{\omega_u}\left[\frac{10\tau_m^3}{81} + \frac{16T_m\tau_m^2}{81}\right] - \frac{1.775K_c^2K_u^2\tau_m^2}{81\omega_u^2}}\right]$$

$$\omega_H = \sqrt{\frac{1 + K_HK_m}{\frac{\tau_mT_m}{3} + \frac{K_HK_m\tau_m^2}{6} - \frac{0.942K_cK_u\tau_m}{3\omega_u}}}, \ \varepsilon = \frac{2T_m\omega_u + K_HK_m\tau_m\omega_u - 1.884K_uK_c}{0.471K_cK_u\omega_H\tau_m}$$

$$K_c^{(38)} = \left(K_H - \frac{0.674\left[1 - 0.447\omega_H\tau_m + 0.0607\omega_H^2\tau_m^2\right]}{K_m/(1 + K_HK_m)}\right), \ T_i^{(38)} = \frac{K_c^{(38)}(1 + K_HK_m)}{\omega_HK_m0.0607(1 + 1.05\omega_H\tau_m - 0.233\omega_H^2\tau_m^2)}$$

$$K_c^{(39)} = \left(K_H - \frac{0.778\left[1 - 0.467\omega_H\tau_m + 0.0609\omega_H^2\tau_m^2\right]}{K_m/(1 + K_HK_m)}\right), \ T_i^{(39)} = \frac{K_c^{(39)}(1 + K_HK_m)}{\omega_HK_m0.0309(1 + 2.84\omega_H\tau_m - 0.532\omega_H^2\tau_m^2)}$$

[2] $$K_c^{(40)} = \left(K_H - \frac{1.31(0.519)^{\omega_H\tau_m}\left[1 - 1.03/\varepsilon + 0.514/\varepsilon^2\right]}{K_m/(1 + K_HK_m)}\right), T_i^{(40)} = \frac{K_c^{(40)}(1 + K_HK_m)}{\omega_HK_m0.0603(1 + 0.929\ln[\omega_H\tau_m])(1 + 2.01/\varepsilon - 1.2/\varepsilon^2)}$$

$$K_c^{(41)} = \left(K_H - \frac{1.14\left[1 - 0.482\omega_H\tau_m + 0.068\omega_H^2\tau_m^2\right]}{K_m/(1 + K_HK_m)}\right), \ T_i^{(41)} = \frac{K_c^{(41)}(1 + K_HK_m)}{\omega_HK_m0.0694(-1 + 2.1\omega_H\tau_m - 0.367\omega_H^2\tau_m^2)}$$

$$K_c^{(42)} = \left(K_H - \frac{0.622\left[1 - 0.435\omega_H\tau_m + 0.052\omega_H^2\tau_m^2\right]}{K_m/(1 + K_HK_m)}\right), \ T_i^{(42)} = \frac{K_c^{(42)}(1 + K_HK_m)}{\omega_HK_m0.0697(1 + 0.752\omega_H\tau_m - 0.145\omega_H^2\tau_m^2)}$$

$$K_c^{(43)} = \left(K_H - \frac{0.724\left[1 - 0.469\omega_H\tau_m + 0.0609\omega_H^2\tau_m^2\right]}{K_m/(1 + K_HK_m)}\right), \ T_i^{(43)} = \frac{K_c^{(43)}(1 + K_HK_m)}{\omega_HK_m0.0405(1 + 1.93\omega_H\tau_m - 0.363\omega_H^2\tau_m^2)}$$

Regulator – nearly minimum IAE, ISE, ITAE - Hwang [60] (continued) *Model: Method 8*	$K_c^{(44)}$ [3]	$T_i^{(44)}$	$\frac{0.471K_u}{K_m\omega_u}$	Decay ratio = 0.2 - $0.1 \le \frac{\tau_m}{T_m} \le 2.0$; $3 \le \varepsilon < 20$
	$K_c^{(45)}$	$T_i^{(45)}$	$\frac{0.471K_u}{K_m\omega_u}$	Decay ratio = 0.2 - $0.1 \le \frac{\tau_m}{T_m} \le 2.0$; $\varepsilon \ge 20$
	$K_c^{(46)}$	$T_i^{(46)}$	$\frac{0.471K_u}{K_m\omega_u}$	Decay ratio = 0.25 - $0.1 \le \frac{\tau_m}{T_m} \le 2.0$; $\varepsilon < 2.4$
	$K_c^{(47)}$	$T_i^{(47)}$	$\frac{0.471K_u}{K_m\omega_u}$	Decay ratio = 0.25 - $0.1 \le \frac{\tau_m}{T_m} \le 2.0$; $2.4 \le \varepsilon < 3$
	$K_c^{(48)}$	$T_i^{(48)}$	$\frac{0.471K_u}{K_m\omega_u}$	Decay ratio = 0.25 - $0.1 \le \frac{\tau_m}{T_m} \le 2.0$; $3 \le \varepsilon < 20$
	$K_c^{(49)}$	$T_i^{(49)}$	$\frac{0.471K_u}{K_m\omega_u}$	Decay ratio = 0.25 - $0.1 \le \frac{\tau_m}{T_m} \le 2.0$; $\varepsilon \ge 20$
Servo - nearly min. IAE, ISE, ITAE - Hwang [60] *Model: Method 8*	$K_c^{(50)}$	$T_i^{(50)}$	$\frac{0.471K_u}{K_m\omega_u}$	Decay ratio = 0.1 - $0.1 \le \frac{\tau_m}{T_m} \le 2.0$; $\varepsilon < 2.4$

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[3] $$K_c^{(44)} = \left(K_H - \frac{1.26(0.506)^{\omega_H \tau_m}\left[1 - 1.07/\varepsilon + 0.616/\varepsilon^2\right]}{K_m/(1+K_H K_m)} \right),\ T_i^{(44)} = \frac{K_c^{(44)}(1+K_H K_m)}{\omega_H K_m 0.0661\left(1+0.824\ln[\omega_H \tau_m]\right)\left(1+1.71/\varepsilon - 1.17/\varepsilon^2\right)}$$

$$K_c^{(45)} = \left(K_H - \frac{1.09\left[1 - 0.497\omega_H \tau_m + 0.0724\omega_H^2 \tau_m^2\right]}{K_m/(1+K_H K_m)} \right),\ T_i^{(45)} = \frac{K_c^{(45)}(1+K_H K_m)}{\omega_H K_m 0.054\left(-1 + 2.54\omega_H \tau_m - 0.457\omega_H^2 \tau_m^2\right)}$$

$$K_c^{(46)} = \left(K_H - \frac{0.584\left[1 - 0.439\omega_H \tau_m + 0.0514\omega_H^2 \tau_m^2\right]}{K_m/(1+K_H K_m)} \right),\ T_i^{(46)} = \frac{K_c^{(46)}(1+K_H K_m)}{\omega_H K_m 0.0714\left(1 + 0.685\omega_H \tau_m - 0.131\omega_H^2 \tau_m^2\right)}$$

$$K_c^{(47)} = \left(K_H - \frac{0.675\left[1 - 0.472\omega_H \tau_m + 0.061\omega_H^2 \tau_m^2\right]}{K_m/(1+K_H K_m)} \right),\ T_i^{(47)} = \frac{K_c^{(47)}(1+K_H K_m)}{\omega_H K_m 0.0484\left(1 + 1.43\omega_H \tau_m - 0.273\omega_H^2 \tau_m^2\right)}$$

$$K_c^{(48)} = \left(K_H - \frac{1.2(0.495)^{\omega_H \tau_m}\left[1 - 1.1/\varepsilon + 0.698/\varepsilon^2\right]}{K_m/(1+K_H K_m)} \right),\ T_i^{(48)} = \frac{K_c^{(48)}(1+K_H K_m)}{\omega_H K_m 0.0702\left(1+0.734\ln[\omega_H \tau_m]\right)\left(1+1.48/\varepsilon - 1.1/\varepsilon^2\right)}$$

$$K_c^{(49)} = \left(K_H - \frac{1.03\left[1 - 0.51\omega_H \tau_m + 0.0759\omega_H^2 \tau_m^2\right]}{K_m/(1+K_H K_m)} \right),\ T_i^{(49)} = \frac{K_c^{(49)}(1+K_H K_m)}{\omega_H K_m 0.0386\left(-1 + 3.26\omega_H \tau_m - 0.6\omega_H^2 \tau_m^2\right)}$$

$$K_c^{(50)} = \left(K_H - \frac{0.822\left[1 - 0.549\omega_H \tau_m + 0.112\omega_H^2 \tau_m^2\right]}{K_m/(1+K_H K_m)} \right),\ T_i^{(50)} = \frac{K_c^{(50)}(1+K_H K_m)}{\omega_H K_m 0.0142\left(1 + 6.96\omega_H \tau_m - 1.77\omega_H^2 \tau_m^2\right)}$$

Rule	K_c	T_i	T_d	Comment
Servo - nearly min. IAE, ISE, ITAE - Hwang [60] (continued) Model: Method 8	$K_c^{(51)}$ [4]	$T_i^{(51)}$	$\frac{0.471K_u}{K_m\omega_u}$	Decay ratio = 0.1 - $0.1 \le \frac{\tau_m}{T_m} \le 2.0$; $2.4 \le \varepsilon < 3$
	$K_c^{(52)}$	$T_i^{(52)}$	$\frac{0.471K_u}{K_m\omega_u}$	Decay ratio = 0.1 - $0.1 \le \frac{\tau_m}{T_m} \le 2.0$; $3 \le \varepsilon < 20$
	$K_c^{(53)}$	$T_i^{(53)}$	$\frac{0.471K_u}{K_m\omega_u}$	Decay ratio = 0.1 - $0.1 \le \frac{\tau_m}{T_m} \le 2.0$; $\varepsilon \ge 20$
Servo – nearly minimum IAE, ITAE – Hwang and Fang [61] *Model: Method 9*	$\left[c_1 + c_2\frac{\tau_m}{T_m} + c_3\left(\frac{\tau_m}{T_m}\right)^2\right]K_u$ $c_1 = 0.537, c_2 = -0.0165$ $c_3 = 0.00173$	$\frac{K_c}{K_u\omega_u\left[c_4 + c_5\frac{\tau_m}{T_m} + c_6\left(\frac{\tau_m}{T_m}\right)^2\right]}$ $c_4 = 0.0503, c_5 = 0.163$ $c_6 = -0.0389$	$\left[c_7 + c_8\frac{\tau_m}{T_m} + c_9\left(\frac{\tau_m}{T_m}\right)^2\right]\frac{K_u}{\omega_u K_c}$ $c_7 = 0.350, c_8 = -0.0344$ $c_9 = 0.00644$	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$ - decay ratio = 0.03
Regulator – nearly minimum IAE, ITAE – Hwang and Fang [61] *Model: Method 9*	$\left[c_1 + c_2\frac{\tau_m}{T_m} + c_3\left(\frac{\tau_m}{T_m}\right)^2\right]K$ $c_1 = 0.802, c_2 = -0.154$ $c_3 = 0.0460$	$\frac{K_c}{K_u\omega_u\left[c_4 + c_5\frac{\tau_m}{T_m} + c_6\left(\frac{\tau_m}{T_m}\right)^2\right]}$ $c_4 = 0.190, c_5 = 0.0532$ $c_6 = -0.00509$	$\left[c_7 + c_8\frac{\tau_m}{T_m} + c_9\left(\frac{\tau_m}{T_m}\right)^2\right]\frac{K_u}{\omega_u K_c}$ $c_7 = 0.421, c_8 = 0.00915$ $c_9 = -0.00152$	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$ - decay ratio = 0.12
Simultaneous Servo/regulator - nearly minimum IAE, ITAE - Hwang and Fang [61] *Model: Method 9*	$\left[c_1 + c_2\frac{\tau_m}{T_m} + c_3\left(\frac{\tau_m}{T_m}\right)^2\right]K_u$ $c_1 = 0.713, c_2 = -0.176$ $c_3 = 0.0513$	$\frac{K_c}{K_u\omega_u\left[c_4 + c_5\frac{\tau_m}{T_m} + c_6\left(\frac{\tau_m}{T_m}\right)^2\right]}$ $c_4 = 0.149, c_5 = 0.0556$ $c_6 = -0.00566$	$\left[c_7 + c_8\frac{\tau_m}{T_m} + c_9\left(\frac{\tau_m}{T_m}\right)^2\right]\frac{K_u}{\omega_u K_c}$ $c_7 = 0.371, c_8 = -0.0274$ $c_9 = 0.00557$	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$
McMillan [58] *Model: Method 1 or Method 18*	$\frac{1.415}{K_m}\frac{T_m}{\tau_m}\left\{\frac{1}{1+\left(\frac{T_m}{T_m+\tau_m}\right)^{0.65}}\right\}$	$\tau_m\left\{1+\left(\frac{T_m}{T_m+\tau_m}\right)^{0.65}\right\}$	$0.25\tau_m\left\{1+\left(\frac{T_m}{T_m+\tau_m}\right)^{0.65}\right\}$	Tuning rules developed from K_u, T_u
Astrom and Hagglund [98] *Model: Method not relevant*	$\frac{K_u}{A_m}$	arbitrary	$\frac{T_u}{4\pi^2 T_i}$	Specify gain margin A_m
	$K_u \cos\phi_m$	αT_d	$T_u\frac{\tan\phi_m + \sqrt{\frac{4}{\alpha} + \tan^2\phi_m}}{4\pi}$	Specify phase margin ϕ_m ; $\alpha = 4$ (Astrom *et al.* [30])

[4] $K_c^{(51)} = \left(K_H - \frac{0.786\left[1 - 0.441\omega_H\tau_m + 0.0569\omega_H^2\tau_m^2\right]}{K_m/(1+K_H K_m)}\right)$, $T_i^{(51)} = \frac{K_c^{(51)}(1+K_H K_m)}{\omega_H K_m 0.0172\left(1+4.62\omega_H\tau_m - 0.823\omega_H^2\tau_m^2\right)}$

$K_c^{(52)} = \left(K_H - \frac{1.28(0.542)^{\omega_H\tau_m}\left[1 - 0.986/\varepsilon + 0.558/\varepsilon^2\right]}{K_m/(1+K_H K_m)}\right)$, $T_i^{(52)} = \frac{K_c^{(52)}(1+K_H K_m)}{\omega_H K_m 0.0476\left(1+0.996\ln[\omega_H\tau_m]\right)\left(1+2.131/\varepsilon - 1.13/\varepsilon^2\right)}$

$K_c^{(53)} = \left(K_H - \frac{1.14\left[1 - 0.466\omega_H\tau_m + 0.0647\omega_H^2\tau_m^2\right]}{K_m/(1+K_H K_m)}\right)$, $T_i^{(53)} = \frac{K_c^{(53)}(1+K_H K_m)}{\omega_H K_m 0.0609\left(-1+1.97\omega_H\tau_m - 0.323\omega_H^2\tau_m^2\right)}$

<table>
<tr><th>Rule</th><th colspan="3">K_c</th><th colspan="3">T_i</th><th colspan="3">T_d</th><th colspan="3">Comment</th></tr>
<tr><td rowspan="6">Li *et al*. [99]

Model: Method not relevant</td><td colspan="3">$K_c^{(54)}$ [5]</td><td colspan="3">εT_d</td><td colspan="3">$\frac{T}{4\pi}\left[\eta+\sqrt{\eta^2+\frac{4}{\eta}}\right]$</td><td colspan="3"></td></tr>
<tr><td>ϕ_m</td><td>A_m</td><td>η</td><td>ϕ_m</td><td>A_m</td><td>η</td><td>ϕ_m</td><td>A_m</td><td>η</td><td>ϕ_m</td><td>A_m</td><td>η</td></tr>
<tr><td>15^0</td><td>2</td><td>2.8</td><td>30^0</td><td>1.67</td><td>3.8</td><td>45^0</td><td>1.25</td><td>4.6</td><td>60^0</td><td>1.11</td><td>5.4</td></tr>
<tr><td>20^0</td><td>1.67</td><td>3.2</td><td>35^0</td><td>1.43</td><td>4.0</td><td>50^0</td><td>1.25</td><td>4.9</td><td>65^0</td><td>1.11</td><td>5.5</td></tr>
<tr><td>25^0</td><td>1.67</td><td>3.5</td><td>40^0</td><td>1.43</td><td>4.2</td><td>55^0</td><td>1.25</td><td>5.2</td><td></td><td></td><td></td></tr>
<tr><td colspan="3">$\frac{4h}{\pi A A_m}$</td><td colspan="3">$0.3183T$</td><td colspan="3">$0.0796T$</td><td colspan="3">simplified algorithm</td></tr>
<tr><td rowspan="2">Tan *et al*. [39]

Model: Method 10</td><td colspan="3">$\frac{K_u}{A_m}\cos\phi_m$</td><td colspan="3">$\alpha T_d$</td><td colspan="3">$T_u \frac{\tan\phi_m+\sqrt{\frac{4}{\alpha}+\tan^2\phi_m}}{4\pi}$</td><td colspan="3">$A_m = 2$, $\phi_m = 45^0$; α chosen arbitrarily</td></tr>
<tr><td colspan="3">$\frac{K_\phi}{A_m}$</td><td colspan="3">$\frac{rK_\phi\left(\omega_u^2-\omega_\phi^2\right)}{\omega_u\omega_\phi^2\sqrt{K_u^2-r^2K_\phi^2}}$</td><td colspan="3">$\frac{1}{\omega_\phi^2 T_i}$</td><td colspan="3">Arbitrary A_m , ϕ_m at ω_ϕ ; $r = 0.1 + 0.9(K_u/K_\phi)$</td></tr>
<tr><td rowspan="2">Friman and Waller [41]

Model: Method not relevant</td><td colspan="3">$\frac{0.25}{\lvert G_p(j\omega_u)\rvert}$</td><td colspan="3">$\frac{0.5774}{\omega_u}$</td><td colspan="3">$\frac{0.1443}{\omega_u}$</td><td colspan="3">$\frac{\tau_m}{T_m} < 0.25$, $A_m = 2$, $\phi_m = 60^0$</td></tr>
<tr><td colspan="3">$\frac{0.4830}{\lvert G_p(j\omega_{150^0})\rvert}$</td><td colspan="3">$\frac{3.7321}{\omega_{150^0}}$</td><td colspan="3">$\frac{0.9330}{\omega_{150^0}}$</td><td colspan="3">$0.25 \le \frac{\tau_m}{T_m} \le 2.0$
$A_m = 2$, $\phi_m = 45^0$</td></tr>
</table>

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[5] $K_c^{(54)} = \frac{4h\cos\phi_m}{\pi A_m\left[\sqrt{A^2-b^2}+\frac{2\pi b}{T}(T_i-T_d)\right]}$, $\eta = \frac{\left[\tan\phi_m-\left(b/\sqrt{A^2-b^2}\right)\right]}{\left[1+\left(b/\sqrt{A^2-b^2}\right)\right]}$ with $\pm h$ = amplitude of relay, $\pm b$ = deadband of relay, A, T = limit cycle amplitude and period, respectively.

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Regulator - Gorecki *et al*. [28] *Model: Method 6* (2 tuning rules)	$K_c^{(55)}$	$T_i^{(55)}$	$T_d^{(55)}$	Pole is real and has maximum attainable multiplicity; $\frac{\tau_m}{T_m} < 2$
	$K_c^{(56)}$ [6]	$T_i^{(56)}$	$T_d^{(56)}$	Low frequency part of magnitude Bode diagram is flat.
Regulator - minimum IAE - Smith and Corripio [25] – *page 343-346* *Model: Method 6*	$\frac{T_m}{K_m \tau_m}$	T_m	$0.5\tau_m$	$0.1 \le \frac{\tau_m}{T_m} \le 1.5$
Servo – minimum IAE – Smith and Corripio [25] – *page 343-346* *Model: Method 6*	$\frac{5T_m}{6K_m \tau_m}$	T_m	$0.5\tau_m$	$0.1 \le \frac{\tau_m}{T_m} \le 1.5$
Servo – 5% overshoot – Smith and Corripio [25] – *page 343-346* *Model: Method 6*	$\frac{T_m}{2K_m \tau_m}$	T_m	$0.5\tau_m$	

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6 $$K_c^{(55)} = \frac{2}{K_m}\frac{T_m}{\tau_m}\left[6+\frac{\tau_m}{2T_m}\sqrt{3+\left(\frac{\tau_m}{2T_m}\right)^2} - 9 - \left(\frac{\tau_m}{2T_m}\right) - \left(\frac{\tau_m}{2T_m}\right)^2\right] e^{\sqrt{3+\left(\frac{\tau_m}{2T_m}\right)^2} - 3 - \frac{\tau_m}{2T_m}}$$

$$T_i^{(55)} = \tau_m \frac{\left[6+\frac{\tau_m}{2T_m}\right]\sqrt{3+\left(\frac{\tau_m}{2T_m}\right)^2} - 9 - \frac{\tau_m}{2T_m} - \left(\frac{\tau_m}{2T_m}\right)^2}{\left[21+3\frac{\tau_m}{T_m}+\left(\frac{\tau_m}{2T_m}\right)^2\right]\sqrt{3+\left(\frac{\tau_m}{2T_m}\right)^2} - 36 - 4.5\frac{\tau_m}{T_m} - 6\left(\frac{\tau_m}{2T_m}\right)^2 - \left(\frac{\tau_m}{2T_m}\right)^3},$$

$$T_d^{(55)} = 0.5\tau_m \frac{\sqrt{3+\left(\frac{\tau_m}{2T_m}\right)^2} - 1}{\left[6+\frac{\tau_m}{2T_m}\right]\sqrt{3+\left(\frac{\tau_m}{2T_m}\right)^2} - 9 - \frac{\tau_m}{2T_m} - \left(\frac{\tau_m}{2T_m}\right)^2}$$

$$K_c^{(56)} = \frac{1}{K_m}\frac{1}{2\frac{\tau_m}{T_i^{(56)}}\left(\frac{T_m}{\tau_m}+1\right) - 2}, \quad T_i^{(56)} = \tau_m \frac{7+42\frac{T_m}{\tau_m}+135\left(\frac{T_m}{\tau_m}\right)^2+240\left(\frac{T_m}{\tau_m}\right)^3+180\left(\frac{T_m}{\tau_m}\right)^4}{15\left[2\frac{T_m}{\tau_m}+1\right]\left[1+3\frac{T_m}{\tau_m}+6\left(\frac{T_m}{\tau_m}\right)^2\right]}$$

$$T_d^{(56)} = \tau_m \frac{1+7\frac{T_m}{\tau_m}+27\left(\frac{T_m}{\tau_m}\right)^2+60\left(\frac{T_m}{\tau_m}\right)^3+60\left(\frac{T_m}{\tau_m}\right)^4}{7+42\frac{T_m}{\tau_m}+135\left(\frac{T_m}{\tau_m}\right)^2+240\left(\frac{T_m}{\tau_m}\right)^3+180\left(\frac{T_m}{\tau_m}\right)^4}$$

Suyama [100] *Model: Method 6*	$\frac{1}{K_m}\left[0.7236\frac{T_m}{\tau_m}+0.2236\right]$	$T_m + 0.309\tau_m$	$\frac{2.236T_m\tau_m}{7.236T_m + 2.236\tau_m}$	

Rule	K_c	T_i	T_d	Comment
Juang and Wang [101] *Model: Method 6*	$\dfrac{\alpha + \dfrac{\tau_m}{T_m} + 0.5\left(\dfrac{\tau_m}{T_m}\right)^2}{K_m\left(\alpha + \dfrac{\tau_m}{T_m}\right)^2}$	$T_m \dfrac{\alpha + \dfrac{\tau_m}{T_m} + 0.5\left(\dfrac{\tau_m}{T_m}\right)^2}{\left(\alpha + \dfrac{\tau_m}{T_m}\right)}$	$\dfrac{0.5\left(\dfrac{\tau_m}{T_m}\right)^2 T_m\left(\alpha + \dfrac{\tau_m}{T_m} - 0.5\alpha\dfrac{\tau_m}{T_m}\right)}{\left(\alpha + \dfrac{\tau_m}{T_m}\right)\left(\alpha + \dfrac{\tau_m}{T_m} + 0.5\left[\dfrac{\tau_m}{T_m}\right]^2\right)}$	Closed loop time constant = αT_m, $0 < \alpha < 1$
Cluett and Wang [44] *Model: Method 6*	$\dfrac{0.019952\tau_m + 0.20042T_m}{K_m\tau_m}$	$\dfrac{0.099508\tau_m + 0.99956T_m}{0.99747\tau_m - 8.7425.10^{-5}T_m}\tau_m$	$\dfrac{-0.0069905\tau_m + 0.029480T_m}{0.029773\tau_m + 0.29907T_m}\tau_m$	Closed loop time constant = $4\tau_m$
	$\dfrac{0.055548\tau_m + 0.33639T_m}{K_m\tau_m}$	$\dfrac{0.16440\tau_m + 0.99558T_m}{0.98607\tau_m - 1.5032.10^{-4}T_m}\tau_m$	$\dfrac{-0.016651\tau_m + 0.093641T_m}{0.093905\tau_m + 0.56867T_m}\tau_m$	Closed loop time constant = $2\tau_m$
	$\dfrac{0.092654\tau_m + 0.43620T_m}{K_m\tau_m}$	$\dfrac{0.20926\tau_m + 0.98518T_m}{0.96515\tau_m + 4.2550.10^{-3}T_m}\tau_m$	$\dfrac{-0.024442\tau_m + 0.17669T_m}{0.17150\tau_m + 0.80740T_m}\tau_m$	Closed loop time constant = $1.33\tau_m$
	$\dfrac{0.12786\tau_m + 0.51235T_m}{K_m\tau_m}$	$\dfrac{0.24145\tau_m + 0.96751T_m}{0.93566\tau_m + 2.2988.10^{-2}T_m}\tau_m$	$\dfrac{-0.030407\tau_m + 0.27480T_m}{0.25285\tau_m + 1.0132T_m}\tau_m$	Closed loop time constant = τ_m
	$\dfrac{0.16051\tau_m + 0.57109T_m}{K_m\tau_m}$	$\dfrac{0.26502\tau_m + 0.94291T_m}{0.89868\tau_m + 6.9355.10^{-2}T_m}\tau_m$	$\dfrac{-0.035204\tau_m + 0.38823T_m}{0.33303\tau_m + 1.1849T_m}\tau_m$	Closed loop time constant = $0.8\tau_m$
	$\dfrac{0.19067\tau_m + 0.61593T_m}{K_m\tau_m}$	$\dfrac{0.28242\tau_m + 0.91231T_m}{0.85491\tau_m + 0.15937T_m}\tau_m$	$\dfrac{-0.039589\tau_m + 0.51941T_m}{0.40950\tau_m + 1.3228T_m}\tau_m$	Closed loop time constant = $0.67\tau_m$
Gain and phase margin - Zhuang and Atherton [20] *Model: Method not relevant*	$mK_u\cos(\phi_m)$, $m = 0.614(1 - 0.233e^{-0.347K_mK_u})$ $\phi_m = 33.8^0\left(1 - 0.97e^{-0.45K_mK_u}\right)$	$\alpha\dfrac{\tan(\phi_m) + \sqrt{\dfrac{4}{\alpha} + \tan^2(\phi_m)}}{2\omega_u}$, $\alpha = 0.413(3.302K_mK_u + 1)$	$\dfrac{\tan(\phi_m) + \sqrt{\dfrac{4}{\alpha} + \tan^2(\phi_m)}}{2\omega_u}$	Gain margin = 2, phase margin = 60^0 $0.1 \le \dfrac{\tau_m}{T_m} \le 2.0$
Abbas [45] *Model: Method 6*	$\dfrac{0.177 + 0.348\left(\dfrac{\tau_m}{T_m}\right)^{-1.002}}{K_m(0.531 - 0.359V^{0.713})}$	$T_m + 0.5\tau_m$	$\dfrac{T_m\tau_m}{2T_m + \tau_m}$	V = fractional overshoot $0 \le V \le 0.2$ $0.1 \le \dfrac{\tau_m}{T_m} \le 5.0$
Camacho *et al*.[102] *Model: Method 6*	$\dfrac{1}{K_m}\dfrac{T_m + \tau_m}{T_m\tau_m}$	$\dfrac{4T_m\tau_m}{T_m + \tau_m}$	$\dfrac{T_m\tau_m}{T_m + \tau_m}$	
Servo – minimum ISE - Ho *et al*. [103] *Model: Method 6*	$\dfrac{1.8578}{K_m}\dfrac{\phi_m^{\ 0.0821}}{A_m^{\ 0.9087}}\left(\dfrac{\tau_m}{T_m}\right)^{-0.9471}$	$T_i^{(57)}$ [7]	$\dfrac{0.4899T_m\phi_m^{\ 0.1457}}{A_m^{\ 0.0845}}\left(\dfrac{\tau_m}{T_m}\right)^{1.0264}$	$A_m \in [2,5]$, $\phi_m \in \left[30^0, 60^0\right]$, $0.1 \le \dfrac{\tau_m}{T_m} \le 1.0$.
	Given A_m, ISE is minimised when $\phi_m = 29.7985 + \dfrac{62.1189}{A_m} + \dfrac{40.3182\tau_m}{T_m} - \dfrac{76.2833\tau_m}{A_mT_m}$ (Ho *et al* [104])			
Regulator - minimum ISE – Ho *et al*. [103] *Model: Method 6*	$\dfrac{1.0722}{K_m}\dfrac{\phi_m^{\ -0.116}}{A_m^{\ 0.8432}}\left(\dfrac{\tau_m}{T_m}\right)^{-0.908}$	$\dfrac{1.2497T_m\phi_m^{\ 1.0082}}{A_m^{\ 0.2099}}\left(\dfrac{\tau_m}{T_m}\right)^{0.3678}$	$\dfrac{0.4763T_m\phi_m^{\ -0.328}}{A_m^{\ 0.0961}}\left(\dfrac{\tau_m}{T_m}\right)^{1.0317}$	$A_m \in [2,5]$, $\phi_m \in \left[30^0, 60^0\right]$, $0.1 \le \dfrac{\tau_m}{T_m} \le 1.0$
	Given A_m, ISE is minimised when $\phi_m = 46.5489A_m^{\ 0.2035}(\tau_m/T_m)^{0.3693}$ (Ho *et al* [104])			

Rule	K_c	T_i	T_d	Comment

[7] $T_i^{(57)} = \dfrac{0.0211T_m\left[1 + 0.3289A_m + 6.4572\phi_m + 25.1914(\tau_m/T_m)\right]}{1 + 0.0625A_m - 0.8079\phi_m + 0.347(\tau_m/T_m)}$

Morilla *et al*. [104a] *Model: Method 24*	$K_c^{(57a)}$ [8]	$\frac{\tau_m}{1+\sqrt{1-4\alpha}}$	$\alpha T_i^{(57a)}$	$\alpha = 0.1; \delta_0 = [0.2, 0.5]$
Robust				
Robust - Brambilla *et al*. [48] *Model: Method 6*	$\frac{1}{K_m}\left(\frac{T_m + 0.5\tau_m}{\lambda\tau_m}\right)$	$T_m + 0.5\tau_m$	$\frac{T_m\tau_m}{2T_m + \tau_m}$	$0.1 \le \frac{\tau_m}{T_m} \le 10$ and no model uncertainty - $l \approx 0.35$
Rivera *et al*. [49] *Model: Method 6*	$\frac{1}{K_m}\left(\frac{T_m + 0.5\tau_m}{\lambda + 0.5\tau_m}\right)$	$T_m + 0.5\tau_m$	$\frac{T_m\tau_m}{2T_m + \tau_m}$	$\lambda > 0.1T_m$, $\lambda \ge 0.8\tau_m$.
Fruehauf *et al*. [52] *Model: Method 1*	$\frac{5T_m}{9\tau_m K_m}$	$5\tau_m$	$\le 0.5\tau_m$	$\frac{\tau_m}{T_m} < 0.33$
	$\frac{T_m}{2\tau_m K_m}$	T_m	$\le 0.5\tau_m$	$\frac{\tau_m}{T_m} \ge 0.33$
Lee *et al*. [55] *Model: Method 6*	$\frac{T_i}{K_m(\lambda + \tau_m)}$	$T_m + \frac{\tau_m^2}{2(\lambda + \tau_m)}$	$\frac{\tau_m^2}{2(\lambda + \tau_m)}\left(1 - \frac{\tau_m}{3T_i}\right)$	$\lambda = \frac{\tau_m}{3}$
	$\frac{T_i}{K_m(2\lambda + \tau_m - \alpha)}$	$T_m + \alpha - \frac{\lambda^2 + \alpha\tau_m - 0.5\tau_m^2}{2\lambda + \tau_m - \alpha}$	$\frac{T_m\alpha - \frac{\frac{\tau_m^3}{6} - \frac{\alpha\tau_m^2}{2}}{2\lambda + \tau_m - \alpha}}{T_i}$ - $\frac{\lambda^2 + \alpha\tau_m - 0.5\tau_m^2}{2\lambda + \tau_m - \alpha}$	Two degrees of freedom controller; $\alpha = T_m - T_m\left(1 - \frac{\lambda}{T_m}\right)^2 e^{-\frac{\tau_m}{T_m}}$; Desired response = $\frac{e^{-\tau_m s}}{1 + \lambda s}$

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[8] $K_c^{(57a)} = \frac{T_i^{(57a)}\omega_{n0}}{K_m[2\delta_0 + \omega_{n0}(\tau_m - T_i^{(57a)})]}$, $\omega_{n0} = \frac{-\delta_0 T_m + \sqrt{\delta_0^2 T_m^2 + T_m(\tau_m - T_i^{(57a)}) - \alpha T_i^{(57a)2}}}{T_m(\tau_m - T_i^{(57a)}) - \alpha T_i^{(57a)2}}$

with $\delta_0 = \frac{1}{\sqrt{1 + \left(\frac{2\pi}{\log_e[b/a]}\right)^2}}$, $\frac{b}{a}$ = desired closed loop response decay ratio

Table 26: PID tuning rules - FOLPD model $\frac{K_m e^{-s\tau_m}}{1+sT_m}$ - ideal controller with first order filter $G_c(s) = K_c\left(1+\frac{1}{T_i s}+T_d s\right)\frac{1}{T_f s+1}$. *3 tuning rules.*

Rule	K_c	T_i	T_d	Comment
Robust				
** Morari and Zafiriou [105]	$\frac{1}{K_m}\left(\frac{T_m+0.5\tau_m}{\lambda+\tau_m}\right)$	$T_m+0.5\tau_m$	$\frac{T_m\tau_m}{2T_m+\tau_m}$	$T_F=\frac{\lambda\tau_m}{2(T_m+\tau_m)}$, $\lambda>0.25\tau_m$, $\lambda>0.2T_m$.
Horn *et al*. [106] *Model: Method 6*	$\frac{2T_m+\tau_m}{2(\lambda+\tau_m)K_m}$	$T_m+0.5\tau_m$	$\frac{\tau_m T_m}{\tau_m+T_m}$	$T_f=\frac{\lambda\tau_m}{2(\lambda+\tau_m)}$; $\lambda>\tau_m$, $\lambda<T_m$.
H_∞ optimal – Tan *et al*. [81] *Model: Method 6*	$\frac{0.265\lambda+0.307}{K_m}\left(\frac{T_m}{\tau_m}+0.5\right)$	$T_m+0.5\tau_m$	$\frac{\tau_m T_m}{\tau_m+2T_m}$	$T_f=\frac{\tau_m}{5.314\lambda+0.951}$ $\lambda=2$ - 'fast' response $\lambda=1$ - 'robust' tuning $\lambda=1.5$ - recommended

Table 27: PID tuning rules - FOLPD model $\frac{K_m e^{-s\tau_m}}{1+sT_m}$ - ideal controller with second order filter $G_c(s) = K_c\left(1+\frac{1}{T_i s}+T_d s\right)\frac{1+b_1 s}{1+a_1 s+a_2 s^2}$. *1 tuning rule.*

Rule	K_c	T_i	T_d	Comment
Robust				
Horn *et al*. [106] *Model: Method 6*	$\frac{2T_m+\tau_m}{2(2\lambda+\tau_m-b_1)K_m}$, $b_1 = \frac{\lambda^2\tau_m + 2T_m[\tau_m(\tau_m-\lambda)]}{T_m(\tau_m+2\lambda)}$ $+ \frac{2[\lambda(2T_m-\lambda)]}{(\tau_m+2\lambda)}$	$T_m + 0.5\tau_m$	$\frac{\tau_m T_m}{\tau_m + 2T_m}$	Filter $\frac{1+b_1 s}{1+\frac{2\lambda\tau_m+2\lambda^2+b_1\tau_m}{2(2\lambda+\tau_m-b_1)}s+a_2 s^2}$ $a_2 = \frac{\lambda^2\tau_m}{2(2\lambda+\tau_m-b_1)}$; $\lambda > \tau_m$, $\lambda < T_m$.

Table 28: PID tuning rules - FOLPD model $\frac{K_m e^{-s\tau_m}}{1+sT_m}$ - ideal controller with set-point weighting

$$G_c(s) = K_c\left(b + \frac{1}{T_i s} + T_d s\right)$$. *1 tuning rule*.

Rule	K_c	T_i	T_d	Comment
Direct synthesis	**(Maximum sensitivity)**			
Astrom and Hagglund [3] – *pages 208-210* *Model: Method 3*	$\frac{3.8e^{-8.4\tau+7.3\tau^2}T_m}{K_m\tau_m}$, $\tau = \tau_m/(\tau_m + T_m)$	$5.2\tau_m e^{-2.5\tau-1.4\tau^2}$ or $0.46T_m e^{2.8\tau-2.1\tau^2}$	$0.89\tau_m e^{-0.37\tau-4.1\tau^2}$ or $0.077T_m e^{5.0\tau-4.8\tau^2}$	$b = 0.40e^{0.18\tau+2.8\tau^2}$; $M_s = 1.4$; $0.14 \le \frac{\tau_m}{T_m} \le 5.5$
	$\frac{8.4e^{-9.6\tau+9.8\tau^2}T_m}{K_m\tau_m}$	$3.2\tau_m e^{-1.5\tau-0.93\tau^2}$ or $0.28T_m e^{3.8\tau-1.6\tau^2}$	$0.86\tau_m e^{-1.9\tau-0.44\tau^2}$ or $0.076T_m e^{3.4\tau-1.1\tau^2}$	$b = 0.22e^{0.65\tau+0.051\tau^2}$; $M_s = 2.0$; $0.14 \le \frac{\tau_m}{T_m} \le 5.5$

Table 29: PID tuning rules - FOLPD model $\frac{K_m e^{-s\tau_m}}{1 + sT_m}$ - ideal controller with first order filter and set-point weighting $U(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{T_f s + 1}\left[R(s)\frac{1 + 0.4T_r s}{1 + sT_r} - Y(s)\right]$. *1 tuning rule*.

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Normey-Rico *et al*. [106a] *Model: Method not specified*	$\frac{0.375(\tau_m + 2T_m)}{K_m \tau_m}$	$T_m + 0.5\tau_m$	$\frac{T_m \tau_m}{2T_m + \tau_m}$	$T_f = 0.13\tau_m$ $T_r = 0.5\tau_m$

Table 30: PID tuning rules - FOLPD model $\frac{K_m e^{-s\tau_m}}{1+sT_m}$ - classical controller $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+T_d s}{1+\frac{T_d s}{N}}\right)$. **20.**

Rule	K_c	T_i	T_d	Comment
Process reaction				
Hang *et al*. [36] – *page 76* *Model: Method 1*	$\frac{0.83T_m}{K_m\tau_m}$	$1.5\tau_m$	$0.25\tau_m$	Foxboro EXACT controller 'pretune'; N=10
Witt and Waggoner [107] *Model: Method 1*	$[\frac{0.6T_m}{K_m\tau_m}, \frac{T_m}{K_m\tau_m}]$	τ_m	τ_m	Equivalent to Ziegler and Nichols [8]; $N=[10,20]$
Witt and Waggoner [107] *Model: Method 2*	$K_c^{(58)}$ [1]	$T_i^{(58)}$	$T_d^{(58)}$	Equivalent to Cohen and Coon [11]; N = [10,20]
St. Clair [15] – *page 21* *Model: Method 1*	$\frac{T_m}{K_m\tau_m}$	$5\tau_m$	$0.5\tau_m$	'aggressive' tuning;
	$\frac{0.5T_m}{K_m\tau_m}$	$5\tau_m$	$0.5\tau_m$	'conservative' tuning;
Shinskey [15a] *Model: Method 1*	$\frac{0.889T_m}{K_m\tau_m}$	$1.75\tau_m$	$0.70\tau_m$	$\frac{\tau_m}{T_m}=0.167$
Regulator tuning	**Minimum performance index**			
Minimum IAE - Kaya and Scheib [108] *Model: Method 3*	$\frac{0.98089}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.76167}$	$\frac{T_m}{0.91032}\left(\frac{\tau_m}{T_m}\right)^{1.05221}$	$0.59974T_m\left(\frac{\tau_m}{T_m}\right)^{0.89819}$	$0<\frac{\tau_m}{T_m}\le 1$; N=10
Minimum IAE – Witt and Waggoner [107] *Model: Method 1*	$K_c^{(59)}$ [2]	$T_i^{(59)}$	$T_d^{(59)}$	$0.1<\frac{\tau_m}{T_m}<0.258$; $N=[10,20]$

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[1] $$K_c^{(58)} = \frac{1.350\frac{T_m}{\tau_m}+0.25\pm\frac{T_m}{\tau_m}\sqrt{0.7425+0.0150\frac{\tau_m}{T_m}+0.0625\left(\frac{\tau_m}{T_m}\right)^2}}{2K_m}$$

$$T_i^{(58)} = \frac{T_m}{1.350\frac{T_m}{\tau_m}+0.25\mp\sqrt{0.7425+0.0150\frac{\tau_m}{T_m}+0.0625\left(\frac{\tau_m}{T_m}\right)^2}}$$

$$T_d^{(58)} = \frac{T_m}{1.350\frac{T_m}{\tau_m}+0.25\pm\sqrt{0.7425+0.0150\frac{\tau_m}{T_m}+0.0625\left(\frac{\tau_m}{T_m}\right)^2}}$$

[2] $$K_c^{(59)} = \frac{0.718}{K_m}\left(\frac{\tau_m}{T_m}\right)^{-0.921}\left[1\pm\sqrt{1-1.693\left(\frac{\tau_m}{T_m}\right)^{-1.886}}\right],$$

$$T_i^{(59)} = \frac{0.964T_m\left(\frac{\tau_m}{T_m}\right)^{1.137}}{1-\sqrt{1\mp 1.693\left(\frac{\tau_m}{T_m}\right)^{1.886}}}, \quad T_d^{(59)} = \frac{0.964T_m\left(\frac{\tau_m}{T_m}\right)^{1.137}}{1+\sqrt{1\pm 1.693\left(\frac{\tau_m}{T_m}\right)^{1.886}}}$$

Rule	K_c	T_i	T_d	Comment
Minimum ISE - Kaya and Scheib [108] *Model: Method 3*	$\frac{1.11907}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.89711}$	$\frac{T_m}{0.7987}\left(\frac{\tau_m}{T_m}\right)^{0.9548}$	$0.54766T_m\left(\frac{\tau_m}{T_m}\right)^{0.87798}$	$0<\frac{\tau_m}{T_m}\le 1$; N=10
Minimum ITAE - Kaya and Scheib [108] *Model: Method 3*	$\frac{0.77902}{K_m}\left(\frac{T_m}{\tau_m}\right)^{1.06401}$	$\frac{T_m}{1.14311}\left(\frac{\tau_m}{T_m}\right)^{0.70949}$	$0.57137T_m\left(\frac{\tau_m}{T_m}\right)^{1.03826}$	$0<\frac{\tau_m}{T_m}\le 1$; N=10
Minimum ITAE - Witt and Waggoner [107] *Model: Method 1*	$K_c^{(60)}$	$T_i^{(60)}$ [3]	$T_d^{(60)}$	$0.1<\frac{\tau_m}{T_m}<0.379$; $N=[10,20]$
Servo tuning	**Minimum performance index**			
Minimum IAE - Kaya and Scheib [108] *Model: Method 3*	$\frac{0.65}{K_m}\left(\frac{T_m}{\tau_m}\right)^{1.04432}$	$\frac{T_m}{0.9895+0.09539\frac{\tau_m}{T_m}}$	$0.50814T_m\left(\frac{\tau_m}{T_m}\right)^{1.08433}$	$0<\frac{\tau_m}{T_m}\le 1$; N=10
Minimum IAE - Witt and Waggoner [107] *Model: Method 1*	$K_c^{(61)}$	$T_i^{(61)}$	$T_d^{(61)}$	$0.1\le\frac{\tau_m}{T_m}\le 1$; $N=[10,20]$
Minimum ISE - Kaya and Scheib [108] *Model: Method 3*	$\frac{0.71959}{K_m}\left(\frac{T_m}{\tau_m}\right)^{1.03092}$	$\frac{T_m}{1.12666-0.18145\frac{\tau_m}{T_m}}$	$0.54568T_m\left(\frac{\tau_m}{T_m}\right)^{0.86411}$	$0<\frac{\tau_m}{T_m}\le 1$; N=10
Minimum ITAE - Kaya and Scheib [108] *Model: Method 3*	$\frac{1.12762}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.80368}$	$\frac{T_m}{0.99783+0.02860\frac{\tau_m}{T_m}}$	$0.42844T_m\left(\frac{\tau_m}{T_m}\right)^{1.0081}$	$0<\frac{\tau_m}{T_m}\le 1$; N=10

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[3] $K_c^{(60)}=\frac{0.679}{K_m}\left(\frac{\tau_m}{T_m}\right)^{0.947}\left[1\pm\sqrt{1-1.283\left(\frac{\tau_m}{T_m}\right)^{-1.733}}\right]$ $T_i^{(60)}=\frac{0.762T_m\left(\frac{\tau_m}{T_m}\right)^{0.995}}{1-\sqrt{1\mp 1.283\left(\frac{\tau_m}{T_m}\right)^{1.733}}}$,

$T_d^{(60)}=\frac{0.762T_m\left(\frac{\tau_m}{T_m}\right)^{0.995}}{1+\sqrt{1\pm 1.283\left(\frac{\tau_m}{T_m}\right)^{1.733}}}$.

$K_c^{(61)}=\frac{1.086}{K_m}\left(\frac{\tau_m}{T_m}\right)^{-0.869}\left[1\pm\sqrt{1-1.392\left(\frac{\tau_m}{T_m}\right)^{0.914}\left\{0.74-0.13\frac{\tau_m}{T_m}\right\}}\right]$,

$T_i^{(61)}=\frac{0.696T_m\left(\frac{\tau_m}{T_m}\right)^{0.914}}{1\mp\sqrt{1-1.392\left(\frac{\tau_m}{T_m}\right)^{0.869}\left\{0.74-0.13\frac{\tau_m}{T_m}\right\}}}$, $T_d^{(61)}=\frac{0.696T_m\left(\frac{\tau_m}{T_m}\right)^{0.914}}{1\pm\sqrt{1-1.392\left(\frac{\tau_m}{T_m}\right)^{0.869}\left\{0.74-0.13\frac{\tau_m}{T_m}\right\}}}$

<table>
<tr><th>Rule</th><th colspan="4">K_c</th><th colspan="4">T_i</th><th colspan="4">T_d</th><th>Comment</th></tr>
<tr><td>Minimum ITAE - Witt and Waggoner [107]
Model: Method 1</td><td colspan="4">$K_c^{(62)}$ [4]</td><td colspan="4">$T_i^{(62)}$</td><td colspan="4">$T_d^{(62)}$</td><td>$0.1 \le \frac{\tau_m}{T_m} \le 1$; $N = [10,20]$</td></tr>
<tr><td>**Direct synthesis**</td><td colspan="13"></td></tr>
<tr><td>Tsang and Rad [109]
Model: Method 13</td><td colspan="4">$\frac{0.809 T_m}{K_m \tau_m}$</td><td colspan="4">$T_m$</td><td colspan="4">$0.5\tau_m$</td><td>Overshoot = 16%; N=5</td></tr>
<tr><td rowspan="4">Tsang *et al*. [111]

Model: Method 6</td><td colspan="4">$\frac{a T_m}{K_m \tau_m}$</td><td colspan="4">T_m</td><td colspan="4">$0.25\tau_m$</td><td>N = 2.5</td></tr>
<tr><td>a</td><td>ξ</td><td>a</td><td>ξ</td><td>a</td><td>ξ</td><td>a</td><td>ξ</td><td>a</td><td>ξ</td><td>a</td><td>ξ</td><td></td></tr>
<tr><td>1.6818</td><td>0.0</td><td>1.1610</td><td>0.2</td><td>0.8594</td><td>0.4</td><td>0.6693</td><td>0.6</td><td>0.5429</td><td>0.8</td><td>0.4569</td><td>1.0</td><td></td></tr>
<tr><td>1.3829</td><td>0.1</td><td>0.9916</td><td>0.3</td><td>0.7542</td><td>0.5</td><td>0.6000</td><td>0.7</td><td>0.4957</td><td>0.9</td><td></td><td></td><td></td></tr>
<tr><td>**Robust**</td><td colspan="13"></td></tr>
<tr><td rowspan="2">Chien [50]

Model: Method 6</td><td colspan="4">$\frac{1}{K_m}\left(\frac{T_m}{\lambda + 0.5\tau_m}\right)$</td><td colspan="4">$T_m$</td><td colspan="4">$0.5\tau_m$</td><td>$\lambda = [\tau_m, T_m]$; N=10</td></tr>
<tr><td colspan="4">$\frac{1}{K_m}\left(\frac{0.5\tau_m}{\lambda + 0.5\tau_m}\right)$</td><td colspan="4">$0.5\tau_m$</td><td colspan="4">$T_m$</td><td>$\lambda = [\tau_m, T_m]$; N=10</td></tr>
<tr><td>**Ultimate cycle**</td><td colspan="13"></td></tr>
<tr><td>Minimum IAE regulator – Shinskey [59] – *page 167*.
Model: Method not specified</td><td colspan="4">$\frac{K_m \tau_m}{3\tau_m - 0.32 T_u}$</td><td colspan="4">$T_u\left(0.15\frac{T_u}{\tau_m} - 0.05\right)$</td><td colspan="4">$0.14 T_u$</td><td></td></tr>
<tr><td rowspan="4">Minimum IAE regulator - Shinskey [16] – *page 143*.
Model: Method 6</td><td colspan="4">$0.95 T_m / K_m \tau_m$</td><td colspan="4">$1.43\tau_m$</td><td colspan="4">$0.52\tau_m$</td><td>$\tau_m / T_m = 0.2$</td></tr>
<tr><td colspan="4">$0.95 T_m / K_m \tau_m$</td><td colspan="4">$1.17\tau_m$</td><td colspan="4">$0.48\tau_m$</td><td>$\tau_m / T_m = 0.5$</td></tr>
<tr><td colspan="4">$1.14 T_m / K_m \tau_m$</td><td colspan="4">$1.03\tau_m$</td><td colspan="4">$0.40\tau_m$</td><td>$\tau_m / T_m = 1$</td></tr>
<tr><td colspan="4">$1.39 T_m / K_m \tau_m$</td><td colspan="4">$0.77\tau_m$</td><td colspan="4">$0.35\tau_m$</td><td>$\tau_m / T_m = 2$</td></tr>
</table>

[4] $K_c^{(62)} = \frac{0.965}{K_m}\left(\frac{\tau_m}{T_m}\right)^{-0.85}\left[1 \pm \sqrt{1 - 1.232\left(\frac{\tau_m}{T_m}\right)^{0.929}\left\{0.796 - 0.1465\frac{\tau_m}{T_m}\right\}}\right]$,

$$T_i^{(62)} = \frac{0.616 T_m \left(\frac{\tau_m}{T_m}\right)^{0.929}}{1 \mp \sqrt{1 - 1.232\left(\frac{\tau_m}{T_m}\right)^{0.85}\left\{0.796 - 0.1465\frac{\tau_m}{T_m}\right\}}}, \; T_d^{(62)} = \frac{0.616 T_m \left(\frac{\tau_m}{T_m}\right)^{0.929}}{1 \pm \sqrt{1 - 1.232\left(\frac{\tau_m}{T_m}\right)^{0.85}\left\{0.796 - 0.1465\frac{\tau_m}{T_m}\right\}}}$$

Table 31: PID tuning rules - FOLPD model $\dfrac{K_m e^{-s\tau_m}}{1+sT_m}$ - non-interacting controller

$$U(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(E(s) - \frac{T_d s}{1+\frac{T_d s}{N}}Y(s)\right)$$. *2 tuning rules.*

Rule	K_c	T_i	T_d	Comment
Regulator tuning	**Minimum performance index**			
Minimum IAE – Huang *et al*. [18] *Model: Method 6*	$K_c^{(63)}$ [5]	$T_i^{(63)}$	$T_d^{(63)}$	$0.1 \le \frac{\tau_m}{T_{m1}} \le 1$; $N = 10$
Servo tuning	**Minimum performance index**			
Minimum IAE - Huang *et al*. [18] *Model: Method 6*	$K_c^{(64)}$ [6]	$T_i^{(64)}$	$T_d^{(64)}$	$0.1 \le \frac{\tau_m}{T_m} \le 1$; $N = 10$

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[5] $$K_c^{(63)} = \frac{1}{K_m}\left[0.1098 - 8.6290\frac{\tau_m}{T_m} + 1.1863\left(\frac{\tau_m}{T_m}\right)^{-0.8058} + 23.1098\left(\frac{\tau_m}{T_m}\right)^{0.6642} + 20.3519\left(\frac{\tau_m}{T_m}\right)^{2.1482} - 19.1463e^{\frac{\tau_m}{T_m}}\right]$$

$$T_i^{(63)} = T_m\left[-0.0145 + 2.0555\frac{\tau_m}{T_m} - 4.4805\left(\frac{\tau_m}{T_m}\right)^2 + 7.7916\left(\frac{\tau_m}{T_m}\right)^3 - 7.0263\left(\frac{\tau_m}{T_m}\right)^4 + 2.4311\left(\frac{\tau_m}{T_m}\right)^5\right]$$

$$T_d^{(63)} = \frac{T_m}{K_c^{(63)}}\left[-0.0206 + 0.9385\frac{\tau_m}{T_m} - 2.3820\left(\frac{\tau_m}{T_m}\right)^2 + 7.2774\left(\frac{\tau_m}{T_m}\right)^3 - 11.1018\left(\frac{\tau_m}{T_m}\right)^4 + 8.0849\left(\frac{\tau_m}{T_m}\right)^5 - 2.274\left(\frac{\tau_m}{T_m}\right)^6\right]$$

[6] $$K_c^{(64)} = \frac{1}{K_m}\left[7.0636 + 66.6512\frac{\tau_m}{T_m} + 26.1928\left(\frac{\tau_m}{T_m}\right)^{0.0865} + 7.3453\left(\frac{\tau_m}{T_m}\right)^{-0.4062} + 33.6578\left(\frac{\tau_m}{T_m}\right)^{2.6405} - 57.937e^{\frac{\tau_m}{T_m}}\right]$$

$$T_i^{(64)} = T_m\left[0.9923 + 0.2819\frac{\tau_m}{T_m} - 1.4510\left(\frac{\tau_m}{T_m}\right)^2 + 2.504\left(\frac{\tau_m}{T_m}\right)^3 - 1.8759\left(\frac{\tau_m}{T_m}\right)^4 + 0.5862\left(\frac{\tau_m}{T_m}\right)^5\right]$$

$$T_d^{(64)} = \frac{T_m}{K_c^{(64)}}\left[0.0075 + 0.3449\frac{\tau_m}{T_m} - 0.0793\left(\frac{\tau_m}{T_m}\right)^2 + 0.8089\left(\frac{\tau_m}{T_m}\right)^3 - 1.0884\left(\frac{\tau_m}{T_m}\right)^4 + 0.352\left(\frac{\tau_m}{T_m}\right)^5 + 0.0471\left(\frac{\tau_m}{T_m}\right)^6\right]$$

Table 32: PID tuning rules - FOLPD model $\frac{K_m e^{-s\tau_m}}{1+sT_m}$ - non-interacting controller

$U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - \frac{T_d s}{1+\frac{sT_d}{N}}Y(s)$. *5 tuning rules.*

Rule	K_c	T_i	T_d	Comment
Servo tuning	**Minimum performance index**			
Minimum ISE - Zhuang and Atherton [20] *Model: Method 6*	$\frac{1.260}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.887}$	$\frac{T_m}{0.701-0.147\frac{\tau_m}{T_m}}$	$0.375T_m\left(\frac{\tau_m}{T_m}\right)^{0.886}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$; N=10
	$\frac{1.295}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.619}$	$\frac{T_m}{0.661-0.110\frac{\tau_m}{T_m}}$	$0.378T_m\left(\frac{\tau_m}{T_m}\right)^{0.756}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$; N=10
Minimum ISTSE - Zhuang and Atherton [20] *Model: Method 6*	$\frac{1.053}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.930}$	$\frac{T_m}{0.736-0.126\frac{\tau_m}{T_m}}$	$0.349T_m\left(\frac{\tau_m}{T_m}\right)^{0.907}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$; N=10
	$\frac{1.120}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.625}$	$\frac{T_m}{0.720-0.114\frac{\tau_m}{T_m}}$	$0.350T_m\left(\frac{\tau_m}{T_m}\right)^{0.811}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$; N=10
Minimum ISTES - Zhuang and Atherton [20] *Model: Method 6*	$\frac{0.942}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.933}$	$\frac{T_m}{0.770-0.130\frac{\tau_m}{T_m}}$	$0.308T_m\left(\frac{\tau_m}{T_m}\right)^{0.897}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$; N=10
	$\frac{1.001}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.624}$	$\frac{T_m}{0.754-0.116\frac{\tau_m}{T_m}}$	$0.308T_m\left(\frac{\tau_m}{T_m}\right)^{0.813}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$; N=10
Ultimate cycle				
Servo – minimum ISTSE – Zhuang and Atherton [20] *Model: Method not relevant*	$\frac{4.437K_mK_u-1.587}{8.024K_mK_u-1.435}K_u$	$0.037(5.89K_mK_u+1)T_u$	$0.112T_u$	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$; N=10
Regulator - minimum IAE - Shinskey [16] – *page 148.* *Model: Method 6*	$0.5556K_u$	$0.39T_u$	$0.14T_u$	$\tau_m/T_m = 0.2$
	$0.4926K_u$	$0.34T_u$	$0.14T_u$	$\tau_m/T_m = 0.5$
	$0.5051K_u$	$0.33T_u$	$0.13T_u$	$\tau_m/T_m = 1$
	$0.4608K_u$	$0.28T_u$	$0.13T_u$	$\tau_m/T_m = 2$

Table 33: PID tuning rules - FOLPD model $\frac{K_m e^{-s\tau_m}}{1+sT_m}$ - non-interacting controller

$U(s) = \left(K_c + \frac{1}{T_i s}\right)E(s) - \frac{T_d s}{1+\frac{sT_d}{N}}Y(s)$. *6 tuning rules.*

Rule	K_c	T_i	T_d	Comment
Regulator tuning	**Minimum performance index**			
Minimum IAE - Kaya and Scheib [108] *Model: Method 3*	$\frac{1.31509}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.8826}$	$\frac{T_m}{1.2587}\left(\frac{\tau_m}{T_m}\right)^{1.3756}$	$0.5655T_m\left(\frac{\tau_m}{T_m}\right)^{0.4576}$	$0<\frac{\tau_m}{T_m}\le 1$; N=10
Minimum ISE - Kaya and Scheib [108] *Model: Method 3*	$\frac{1.3466}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.9308}$	$\frac{T_m}{1.6585}\left(\frac{\tau_m}{T_m}\right)^{1.25738}$	$0.79715T_m\left(\frac{\tau_m}{T_m}\right)^{0.41941}$	$0<\frac{\tau_m}{T_m}\le 1$; N=10
Minimum ITAE - Kaya and Scheib [108] *Model: Method 3*	$\frac{1.3176}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.7937}$	$\frac{T_m}{1.12499}\left(\frac{\tau_m}{T_m}\right)^{1.42603}$	$0.49547T_m\left(\frac{\tau_m}{T_m}\right)^{0.41932}$	$0<\frac{\tau_m}{T_m}\le 1$; N=10
Servo tuning	**Minimum performance index**			
Minimum IAE - Kaya and Scheib [108] *Model: Method 3*	$\frac{1.13031}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.81314}$	$\frac{T_m}{5.7527-5.7241\frac{\tau_m}{T_m}}$	$0.32175T_m\left(\frac{\tau_m}{T_m}\right)^{0.17707}$	$0<\frac{\tau_m}{T_m}\le 1$; N=10
Minimum ISE - Kaya and Scheib [108] *Model: Method 3*	$\frac{1.26239}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.8388}$	$\frac{T_m}{6.0356-6.0191\frac{\tau_m}{T_m}}$	$0.47617T_m\left(\frac{\tau_m}{T_m}\right)^{0.24572}$	$0<\frac{\tau_m}{T_m}\le 1$; N=10
Minimum ITAE - Kaya and Scheib [108] *Model: Method 3*	$\frac{0.98384}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.49851}$	$\frac{T_m}{2.71348-2.29778\frac{\tau_m}{T_m}}$	$0.21443T_m\left(\frac{\tau_m}{T_m}\right)^{0.16768}$	$0<\frac{\tau_m}{T_m}\le 1$; N=10

Table 32: PID tuning rules - FOLPD model $\frac{K_m e^{-s\tau_m}}{1+sT_m}$ - non-interacting controller with set-point weighting $U(s)=K_c\left(b+\frac{1}{T_i s}\right)E(s)-\frac{K_c T_d s}{1+\frac{sT_d}{N}}Y(s)+K_c(b-1)Y(s)$. *3 tuning rules*.

Rule	K_c	T_i	T_d	Comment
Ultimate cycle				
Hang and Astrom [111] N=10 *Model: Method 1*	$0.6K_u$	$0.5T_u$	$0.125T_u$	$\frac{\tau_m}{T_m}<0.3$. $b=2(x-0.1)+\frac{1.66\tau_m}{T_m}$, x = overshoot.
	$0.6K_u$	$0.5T_u$	$0.125T_u$	$0.3\le\frac{\tau_m}{T_m}<0.6$. $b=2x+\frac{\tau_m}{T_m}$
	$0.6K_u$	$0.5\left(1.5-.0.83\frac{\tau_m}{T_m}\right)T_u$	$0.125T_u$	$0.6\le\frac{\tau_m}{T_m}<0.8$. $b=1.6-\frac{\tau_m}{T_m}$
	$0.6K_u$	$0.5\left(1.5-.0.83\frac{\tau_m}{T_m}\right)T_u$	$0.125T_u$	$0.8\le\frac{\tau_m}{T_m}<1.0$; b=0.8
	$0.6K_u$	$0.335T_u$	$0.125T_u$	$1.0<\frac{\tau_m}{T_m}$; b=0.8
Hang *et al*. [65] *Model: Method 1*	$0.6K_u$ $b=\frac{15-K'}{15+K'}$, 10% overshoot - servo	$0.5T_u$ $b=\frac{36}{27+5K'}$, 20% overshoot - servo	$0.125T_u$ $K'=2\left(\frac{11[\tau_m/T_m]+13}{37[\tau_m/T_m]-4}\right)$	$0.16\le\frac{\tau_m}{T_m}<0.57$; N=10
	$0.6K_u$ $b=\frac{8}{17}\left(\frac{4}{9}K'+1\right)$, 20% overshoot, 10% undershoot; servo	$0.222K'T_u$ $K'=2\left(\frac{11[\tau_m/T_m]+13}{37[\tau_m/T_m]-4}\right)$	$0.125T_u$	$0.57\le\frac{\tau_m}{T_m}<0.96$; N=10
Hang and Cao [112] *Model: Method 11*	$0.6K_u$	$\left(0.53-0.22\frac{\tau_m}{T_m}\right)T_u$	$\left(0.53-0.22\frac{\tau_m}{T_m}\right)\frac{T_u}{4}$	$0.1\le\frac{\tau_m}{T_m}<0.5$; N=10

Table 33: PID tuning rules - FOLPD model $\frac{K_m e^{-s\tau_m}}{1+sT_m}$ - industrial controller

$$U(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(R(s) - \frac{1+T_d s}{1+\frac{T_d s}{N}}Y(s)\right)$$. *6 tuning rules.*

Rule	K_c	T_i	T_d	Comment
Regulator tuning	**Minimum performance index**			
Minimum IAE - Kaya and Scheib [108] *Model: Method 3*	$\frac{0.91}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.7938}$	$\frac{T_m}{1.01495}\left(\frac{\tau_m}{T_m}\right)^{1.00403}$	$0.5414T_m\left(\frac{\tau_m}{T_m}\right)^{0.7848}$	$0<\frac{\tau_m}{T_m}\le 1$; N=10
Minimum ISE - Kaya and Scheib [108] *Model: Method 3*	$\frac{1.1147}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.8992}$	$\frac{T_m}{0.9324}\left(\frac{\tau_m}{T_m}\right)^{0.8753}$	$0.56508T_m\left(\frac{\tau_m}{T_m}\right)^{0.91107}$	$0<\frac{\tau_m}{T_m}\le 1$; N=10
Minimum ITAE - Kaya and Scheib [108] *Model: Method 3*	$\frac{0.7058}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.8872}$	$\frac{T_m}{1.03326}\left(\frac{\tau_m}{T_m}\right)^{0.99138}$	$0.60006T_m\left(\frac{\tau_m}{T_m}\right)^{0.971}$	$0<\frac{\tau_m}{T_m}\le 1$; N=10
Servo tuning	**Minimum performance index**			
Minimum IAE - Kaya and Scheib [108] *Model: Method 3*	$\frac{0.81699}{K_m}\left(\frac{T_m}{\tau_m}\right)^{1.004}$	$\frac{T_m}{1.09112-0.22387\frac{\tau_m}{T_m}}$	$0.44278T_m\left(\frac{\tau_m}{T_m}\right)^{0.97186}$	$0<\frac{\tau_m}{T_m}\le 1$; N=10
Minimum ISE - Kaya and Scheib [108] *Model: Method 3*	$\frac{1.1427}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.9365}$	$\frac{T_m}{0.99223-0.35269\frac{\tau_m}{T_m}}$	$0.35308T_m\left(\frac{\tau_m}{T_m}\right)^{0.78088}$	$0<\frac{\tau_m}{T_m}\le 1$; N=10
Minimum ITAE - Kaya and Scheib [108] *Model: Method 3*	$\frac{0.8326}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.7607}$	$\frac{T_m}{1.00268+0.00854\frac{\tau_m}{T_m}}$	$0.44243T_m\left(\frac{\tau_m}{T_m}\right)^{1.11499}$	$0<\frac{\tau_m}{T_m}\le 1$; N=10

Table 34: PID tuning rules - FOLPD model $\frac{K_m e^{-s\tau_m}}{1+sT_m}$ - series controller $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)(1+sT_d)$. *3 tuning rules*.

<table>
<tr><th>Rule</th><th colspan="4">K_c</th><th colspan="4">T_i</th><th colspan="4">T_d</th><th>Comment</th></tr>
<tr><td>Autotuning **</td><td colspan="13"></td></tr>
<tr><td>Astrom and Hagglund [3] – page 246
Model: Not specified</td><td colspan="4">$\frac{5T_m}{6K_m\tau_m}$</td><td colspan="4">$1.5\tau_m$</td><td colspan="4">$0.25\tau_m$</td><td>Foxboro EXACT controller</td></tr>
<tr><td>Ultimate cycle</td><td colspan="13"></td></tr>
<tr><td>Pessen [63]
Model: Method 6</td><td colspan="4">$0.35K_u$</td><td colspan="4">$\frac{0.25}{T_u}$</td><td colspan="4">$\frac{0.25}{T_u}$</td><td>$0.1 \le \frac{\tau_m}{T_m} \le 1$</td></tr>
<tr><td>Direct synthesis</td><td colspan="13"></td></tr>
<tr><td rowspan="4">Tsang et al. [110]

Model: Method 13</td><td colspan="4">$\frac{aT_m}{K_m\tau_m}$</td><td colspan="4">T_m</td><td colspan="4">$0.25\tau_m$</td><td></td></tr>
<tr><td>a</td><td>ξ</td><td>a</td><td>ξ</td><td>a</td><td>ξ</td><td>a</td><td>ξ</td><td>a</td><td>ξ</td><td>a</td><td>ξ</td><td></td></tr>
<tr><td>1.8194</td><td>0.0</td><td>1.2690</td><td>0.2</td><td>0.9492</td><td>0.4</td><td>0.7482</td><td>0.6</td><td>0.6170</td><td>0.8</td><td>0.5413</td><td>1.0</td><td></td></tr>
<tr><td>1.5039</td><td>0.1</td><td>1.0894</td><td>0.3</td><td>0.8378</td><td>0.5</td><td>0.6756</td><td>0.7</td><td>0.5709</td><td>0.9</td><td></td><td></td><td></td></tr>
</table>

Table 35: PID tuning rules – FOLPD model $\frac{K_m e^{-s\tau_m}}{1+sT_m}$ - series controller with filtered derivative

$$G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(1+\frac{sT_d}{1+\frac{sT_d}{N}}\right)$$. *1 tuning rule*.

Rule	K_c	T_i	T_d	Comment
Robust				
Chien [50]	$\frac{T_m}{K_m(\lambda+0.5\tau_m)}$	T_m	$0.5\tau_m$	$\lambda=[\tau_m, T_m]$; N=10
Model: Method 6	$\frac{0.5\tau_m}{K_m(\lambda+0.5\tau_m)}$	$0.5\tau_m$	T_m	$\lambda=[\tau_m, T_m]$; N=10

Table 36: PID tuning rules – FOLPD model $\frac{K_m e^{-s\tau_m}}{1+sT_m}$ - controller with filtered derivative $G_c(s)=K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+s\frac{T_d}{N}}\right)$. *3 tuning rules.*

Rule	K_c	T_i	T_d	Comment
Robust				
Chien [50] *Model: Method 6*	$\frac{T_m+0.5\tau_m}{K_m(\lambda+0.5\tau_m)}$	$T_m+0.5\tau_m$	$\frac{T_m\tau_m}{2T_m+\tau_m}$	$\lambda=[\tau_m, T_m]$; N=10
Gong *et al*. [113] *Model: Method 6*	$\frac{T_m+0.3866\tau_m}{K_m(\lambda+1.0009\tau_m)}$	$T_m+0.3866\tau_m$	$\frac{0.3866T_m\tau_m}{T_m+0.3866\tau_m}$	$N=[3,10]$
	$\lambda=\frac{(0.1388+0.1247N)T_m+0.0482N\tau_m}{0.3866(N-1)T_m+0.1495N\tau_m}\tau_m$			
Direct synthesis				
Davydov *et al*. [31] *Model: Method 12*	$\frac{1}{K_m\left(1.552\frac{\tau_m}{T_m}+0.078\right)}$	$\left(0.186\frac{\tau_m}{T_m}+0.532\right)T_m$	$0.25\left(0.186\frac{\tau_m}{T_m}+0.532\right)T_m$	Closed loop response damping factor = 0.9; $0.2\le\tau_m/T_m\le 1$; $N=K_m$
	$\frac{1}{K_m\left(1.209\frac{\tau_m}{T_m}+0.103\right)}$	$\left(0.382\frac{\tau_m}{T_m}+0.338\right)T_m$	$0.4\left(0.382\frac{\tau_m}{T_m}+0.338\right)T_m$	Closed loop response damping factor = 0.9; $0.2\le\tau_m/T_m\le 1$; $N=K_m$

Table 37: PID tuning rules – FOLPD model $\frac{K_m e^{-s\tau_m}}{1+sT_m}$ - alternative non-interacting controller 1 - $U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - K_c T_d sY(s)$. *6 tuning rules.*

Rule	K_c	T_i	T_d	Comment
Ultimate cycle				
Regulator - minimum IAE - Shinskey [59] – *page 167. Model: method not specified*	$\frac{K_u}{3.73-0.69\frac{T_u}{\tau_m}}$	$0.125\frac{T_u^2}{\tau_m}$	$0.12T_u$	$\frac{T_u}{\tau_m} < 2.7$
	$\frac{K_u}{2.62-0.35\frac{T_u}{\tau_m}}$	$0.125\frac{T_u^2}{\tau_m}$	$0.12T_u$	$\frac{T_u}{\tau_m} \ge 2.7$
Minimum IAE - Shinskey [17] – *page 117. Model: Method 6*	$1.32T_m/K_m\tau_m$	$1.80\tau_m$	$0.44\tau_m$	$\tau_m/T_m = 0.1$
Minimum IAE - Shinskey [16] – *page 143. Model: Method 6*	$1.32T_m/K_m\tau_m$	$1.77\tau_m$	$0.41\tau_m$	$\tau_m/T_m = 0.2$
	$1.35T_m/K_m\tau_m$	$1.43\tau_m$	$0.41\tau_m$	$\tau_m/T_m = 0.5$
	$1.49T_m/K_m\tau_m$	$1.17\tau_m$	$0.37\tau_m$	$\tau_m/T_m = 1$
	$1.82T_m/K_m\tau_m$	$0.92\tau_m$	$0.32\tau_m$	$\tau_m/T_m = 2$
Regulator - minimum IAE - Shinskey [16] – *page 148. Model: Method 6*	$0.7692K_u$	$0.48T_u$	$0.11T_u$	$\tau_m/T_m = 0.2$
	$0.6993K_u$	$0.42T_u$	$0.12T_u$	$\tau_m/T_m = 0.5$
	$0.6623K_u$	$0.38T_u$	$0.12T_u$	$\tau_m/T_m = 1$
	$0.6024K_u$	$0.34T_u$	$0.12T_u$	$\tau_m/T_m = 2$
Regulator – minimum IAE - Shinskey [17] – *page 121. Model: method not specified*	$0.7576K_u$	$0.48T_u$	$0.11T_u$	$\tau_m/T_m = 0.2$
Process reaction – VanDoren [114] *Model: Method 1*	$\frac{1.5T_m}{K_m\tau_m}$	$2.5\tau_m$	$0.4\tau_m$	

Table 38: PID tuning rules – FOLPD model $\frac{K_m e^{-s\tau_m}}{1+sT_m}$ - Alternative filtered derivative controller -

$$G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+0.5\tau_m s+0.0833\tau_m{}^2 s^2}{\left[1+0.1\tau_m s\right]^2}\right)$$. *1 tuning rule*.

Rule	K_c				T_i				T_d				Comment
Direct synthesis													
Tsang *et al.* [110] *Model: Method 13*	$\frac{aT_m}{K_m\tau_m}$				T_m				$0.25T_m$				
	a	ξ	a	ξ	a	ξ	a	ξ	a	ξ	a	ξ	
	1.8512	0.0	1.3293	0.2	1.0280	0.4	0.8411	0.6	0.6953	0.8	0.5527	1.0	
	1.5520	0.1	1.1595	0.3	0.9246	0.5	0.7680	0.7	0.6219	0.9			

Table 39: PID tuning rules – FOLPD model $\frac{K_m e^{-s\tau_m}}{1+sT_m}$ - I-PD controller $U(s)=\frac{K_c}{T_i s}E(s)-K_c(1+T_d s)Y(s)$. *2 tuning rules.*

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Chien *et al*. [74] *Model: Method 6*	$K_c^{(64a)}$ [1]	$T_i^{(64a)}$	$T_d^{(64a)}$	Underdamped system response - $\xi = 0.707$. $\tau_m \le 0.2T_m$
Minimum ISE – Argelaguet *et al*. [114a]. *Model: Method not defined*	$\frac{2T_m+\tau_m}{2K_m\tau_m}$	$T_m + 0.5\tau_m$	$\frac{T_m\tau_m}{2T_m+\tau_m}$	First order Pade approximation for τ_m

[1] $K_c^{(64a)} = \frac{1.414T_{CL}T_m + \tau_m T_m + 0.25\tau_m^2 - T_{CL}^2}{K_m\left(T_{CL}^2 + 0.707T_{CL}\tau_m + 0.25\tau_m^2\right)}$, $T_i^{(64a)} = \frac{1.414T_{CL}T_m + \tau_m T_m + 0.25\tau_m^2 - T_{CL}^2}{T_m + 0.5\tau_m}$

$T_d^{(64a)} = \frac{0.707T_m T_{CL}\tau_m + 0.25T_m\tau_m^2 - 0.5\tau_m T_{CL}^2}{T_m\tau_m + 0.25\tau_m^2 + 1.414T_{CL}T_m - T_{CL}^2}$

Table 40: PID tuning rules - FOLPD model - $G_m(s) = \frac{K_m e^{-s\tau_m}}{1+sT_m}$ – Two degree of freedom controller:

$$U(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N}s}\right)E(s) - K_c\left(\alpha + \frac{\beta T_d s}{1 + \frac{T_d}{N}s}\right)R(s)$$. *1 tuning rule*.

Rule	K_c	T_i	T_d	Comment
Servo/regulator tuning	**Minimum performance index**			
Taguchi and Araki [61a] *Model: ideal process*	$K_c^{(64b)}$ [2]	$T_i^{(64b)}$	$T_d^{(64b)}$	$\frac{\tau_m}{T_m} \le 1.0$; Overshoot (servo step) ≤ 20% ; settling time ≤ settling time of tuning rules of Chien *et al*. [10]

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[2] $K_c^{(64b)} = \frac{1}{K_c}\left(0.1415 + \frac{1.224}{\frac{\tau_m}{T_m} - 0.001582}\right)$, $T_i^{(64b)} = T_m\left(0.01353 + 2.200\frac{\tau_m}{T_m} - 1.452\left[\frac{\tau_m}{T_m}\right]^2 + 0.4824\left[\frac{\tau_m}{T_m}\right]^3\right)$

$T_d^{(64b)} = T_m\left(0.0002783 + 0.4119\frac{\tau_m}{T_m} - 0.04943\left[\frac{\tau_m}{T_m}\right]^2\right)$, $\alpha = 0.6656 - 0.2786\frac{\tau_m}{T_m} + 0.03966\left[\frac{\tau_m}{T_m}\right]^2$,

$\beta = 0.6816 - 0.2054\frac{\tau_m}{T_m} + 0.03936\left[\frac{\tau_m}{T_m}\right]^2$

Table 41: PID tuning rules - non-model specific – ideal controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$. *25 tuning rules.*

Rule	K_c	T_i	T_d	Comment
Ultimate cycle				
Ziegler and Nichols [8]	$[0.6K_u, K_u]$	$0.5T_u$	$0.125T_u$	Quarter decay ratio
Blickley [115]	$0.5K_u$	T_u	$[0.125T_u, 0.167T_u]$	Quarter decay ratio
Parr [64] – *pages 190-191*	$0.5K_u$	T_u	$0.2T_u$	Overshoot to servo response $\approx$ 20%
	$0.5K_u$	$0.34T_u$	$0.08T_u$	Quarter decay ratio
De Paor [116]	$0.866K_u$	$0.5T_u$	$0.125T_u$	phase margin = 30^0
Corripio [117] – *page 27*	$0.75K_u$	$0.63T_u$	$0.1T_u$	Quarter decay ratio
Mantz andTacconi [118]	$0.906K_u$	$0.5T_u$	$0.125T_u$	phase margin = 25^0
Astrom and Hagglund [3] – *page 142*	$0.4698K_u$	$0.4546T_u$	$0.1136T_u$	Gain margin = 2, phase margin = 20^0
	$0.1988K_u$	$1.2308T_u$	$0.3077T_u$	Gain margin = 2.44, phase margin = 61^0
	$0.2015K_u$	$0.7878T_u$	$0.1970T_u$	Gain margin = 3.45, phase margin = 46^0
Astrom and Hagglund [93]	$0.35K_u$	$0.77T_u$	$0.19T_u$	Gain margin ≥ 2, phase margin $\geq 45^0$
Atkinson and Davey [119]	$0.25K_u$	$0.75T_u$	$0.25T_u$	20% overshoot - servo response
** Perry and Chilton [120]	$0.33K_u$	$0.5T_u$	$0.33T_u$	'Some' overshoot
	$0.2K_u$	$0.5T_u$	$0.33T_u$	'No' overshoot
Yu [122] – *page 11*	$0.33K_u$	$0.5T_u$	$0.125T_u$	'Some' overshoot
	$0.2K_u$	$0.5T_u$	$0.125T_u$	'No' overshoot
Luo *et al*. [121]	$0.48K_u$	$0.5T_u$	$0.125T_u$	
McMillan [14] – *page 90*	$0.5K_u$	$0.5T_u$	$0.125T_u$	
McAvoy and Johnson [83]	$0.54K_u$	T_u	$0.2T_u$	
Karaboga and Kalinli [123]	$[0.32K_u, 0.6K_u]$	$\left[\frac{0.4267K_u}{T_u}, \frac{1.5K_u}{T_u}\right]$	$[0.08K_uT_u, 0.15K_uT_u]$	
Hang and Astrom [124]	$K_u \sin\phi_m$	$\frac{T_u(1-\cos\phi_m)}{\pi\sin\phi_m}$	$\frac{T_u(1-\cos\phi_m)}{4\pi\sin\phi_m}$	ϕ_m = phase margin
Astrom *et al*. [30]	$K_u \cos\phi_m$	$\frac{T_u}{4\pi}\left(\tan\phi_m + \sqrt{1+\tan^2\phi_m}\right)$	$\frac{T_u}{16\pi}\left(\tan\phi_m + \sqrt{1+\tan^2\phi_m}\right)$	'small' time delay
St. Clair [15] - *page 17*	$0.5K_u$	$1.2T_u$	$0.125T_u$	'aggressive' tuning
	$0.25K_u$	$1.2T_u$	$0.125T_u$	'conservative' tuning

Rule	K_c	T_i	T_d	Comment
Pole placement - Shin *et al*. [125]	$K_c^{(65)}$ [1]	$T_i^{(65)}$	$\alpha T_i^{(65)}$	Typical $\alpha : 0.1$ Typical $\xi : [0.3, 0.7]$
Other rules				
Harriott [126] – *pages 179-180*	$K_{25\%}$	$0.167 T_{25\%}$	$0.667 T_{25\%}$	Quarter decay ratio
Parr [64] – *pages 191, 193*	$K_{25\%}$	$0.67 T_{25\%}$	$0.17 T_{25\%}$	Quarter decay ratio
	$\dfrac{0.5}{\lvert G_p(j\omega_u)\rvert}$	T_u	$0.25 T_u$	
McMillan [14] - *Page 43*	$0.83 K_{25\%}$	$0.5 T_{25\%}$	$0.1 T_{25\%}$	'Fast' tuning
	$0.67 K_{25\%}$	$0.5 T_{25\%}$	$0.1 T_{25\%}$	'Slow' tuning
Calcev and Gorez [69]	$\dfrac{1}{2\sqrt{2}\lvert G_p(j\omega_u)\rvert}$	$\dfrac{1}{\omega_u}$	$\dfrac{T_i}{4}$	$\phi_m = 45^0$, 'small' τ_m $\phi_m = 15^0$, 'large' τ_m
Zhang *et al*. [127]	$K_c^{(66)}$ [2]	$T_i^{(66)}$	$T_d^{(66)}$	

[1] $$K_c^{(65)} = \frac{\rho}{a_1\left(\alpha\omega_u T_i^{(65)} - \dfrac{1}{\omega_u T_i^{(65)}}\right) - a_2\left(\alpha\omega_1 T_i^{(65)} - \dfrac{1}{\omega_1 T_i^{(65)}}\right) - b_2 - \rho a_2},$$

$$T_i^{(65)} = \frac{(a_2 - a_1) + \sqrt{(a_1 - a_2)^2 + 4(\alpha\rho a_1\omega_u + \alpha b_2\omega_1)\left(\dfrac{\rho a_1}{\omega_u} + \dfrac{b_2}{\omega_1}\right)}}{2(\alpha\rho a_1\omega_u + \alpha b_2\omega_1)}$$

$$a_2 = \frac{1}{K_1}\cos\left(\angle G_p(j\omega_1)\right),\ \rho = \frac{(\omega_u - \omega_1)}{\omega_u}\frac{\sqrt{1-\xi^2}}{\xi},\ b_2 = \frac{1}{K_1}\sin\left(\angle G_p(j\omega_1)\right),\ a_1 = -\frac{1}{K_u}$$

K_1, ω_1 = modified ultimate gain and corresponding angular frequency

[2] $$K_c^{(66)} = \frac{1}{\sqrt{1+\tan^2(\phi_m - \phi_p)}\sqrt{\dfrac{\pi^2(A^2 - \varepsilon^2)}{16d^2} + \sin^2\phi_m}}$$, d, ε = relay amp. and deadband, A = limit cycle amp.

<u>Crossing point of the Nyquist curve and relay with hysteresis is outside the unit circle</u>:

$$T_i^{(66)} = \frac{\left(\dfrac{\omega_u}{\omega_c}\right)^2 - 1}{\omega_u\left(\beta - \dfrac{\omega_u}{\omega_c}\tan(\phi_m - \phi_p)\right)},\ 1.0 + \frac{\omega_u}{\omega_c}\tan(\phi_m - \phi_p) \le \beta \le 1.2 + \frac{\omega_u}{\omega_c}\tan(\phi_m - \phi_p)$$

$$T_d^{(66)} = \frac{\beta\omega_u - \omega_c\tan(\phi_m - \phi_p)}{\omega_u^2 - \omega_c^2}$$, ω_c = frequency when the open loop gain equals unity.

<u>Crossing point of the Nyquist curve and relay with hysteresis is within the unit circle</u>:

$$T_i^{(66)} = \frac{\left(\dfrac{\omega_u}{\omega_c}\right)^2 - 1}{\omega_u\left(\beta + \dfrac{\omega_u}{\omega_c}\tan(\phi_p - \phi_m)\right)},\ \frac{\omega_u - \omega_r}{\omega_r} - 0.4 \le \beta \le \frac{\omega_u - \omega_r}{\omega_r} + 0.4$$

$$T_d^{(66)} = \frac{\beta\omega_u - \omega_c\tan(\phi_m - \phi_p)}{\omega_u^2 - \omega_c^2}$$, ω_r = oscillation frequency when a pure relay is switched into closed loop.

Rule	K_c	T_i	T_d	Comment
Garcia and Castelo [127a]	$K_c^{(66a)}$ [3]	$T_i^{(66a)}$	$T_d^{(66a)}$	

[3] $K_c^{(66a)} = \frac{\cos\left(180^0 + \phi_m - \angle G_p(j\omega_1)\right)}{\left|G_p(j\omega_1)\right|}$, $T_i^{(66a)} = \frac{2[\sin\left(180^0 + \phi_m - \angle G_p(j\omega_1)\right) + 1]}{\omega_1 \cos\left(180^0 + \phi_m - \angle G_p(j\omega_1)\right)}$,

$T_d^{(66a)} = \frac{\sin\left(180^0 + \phi_m - \angle G_p(j\omega_1)\right) + 1}{2\omega_1 \cos\left(180^0 + \phi_m - \angle G_p(j\omega_1)\right)}$, ω_1 = oscillation frequency when a sine function is placed in series with the process in closed loop; $\omega_1 < \omega_u$.

Table 42: PID tuning rules - non-model specific – controller with filtered derivative

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d s}{N}}\right).\ \textit{8 tuning rules.}$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Vrancic [72]	$\frac{T_i}{2(A_1 - T_i)}$	$\frac{A_3}{A_2 - T_d^{(67)} A_1 - \frac{T_d^{(67)2}}{N}}$	$T_d^{(67)}$ [4]	$N < 10$
	$\frac{A_3}{2(A_1 A_2 - A_3 K_m - T_d A_1^{\,2})}$	$\frac{A_3}{A_2 - T_d A_1}$	$\frac{A_3 A_4 - A_2 A_5}{A_3^{\,2} - A_1 A_5}$	$N \ge 10$
	$\frac{0.5 T_i}{A_1 - K_m T_i}$	$\frac{A_2 - \sqrt{A_2^{\,2} - 4\chi A_1 A_3}}{2\chi A_1}$	χT_i	$N = 10$ $\chi = [0.2, 0.25]$
Vrancic [73]	$K_c^{(67a)}$	$\frac{A_3}{A_2 - T_d^{(67a)} A_1 - \frac{T_d^{(67a)2}}{N} K_m}$	$T_d^{(67a)}$	$8 \le N \le 20$

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[4] $$T_d^{(67)} = \frac{-A_3^{\,2} - A_5 A_1 + \sqrt{\left(A_3^{\,2} - A_5 A_1\right)^2 - \frac{4}{N}(A_3 A_2 - A_5)(A_5 A_2 - A_4 A_3)}}{\frac{2}{N}(A_3 A_2 - A_5)}$$

$$y_1(t) = \int_0^t \left(K_m - \frac{y(\tau)}{\Delta u}\right) d\tau\ ,\ y_2(t) = \int_0^t \left(A_1 - y_1(\tau)\right) d\tau\ ,\ y_3(t) = \int_0^t \left(A_2 - y_2(\tau)\right) d\tau\ ,\ y_4(t) = \int_0^t \left[A_3 - y_3(\tau)\right] d\tau\ ,$$

$$y_5(t) = \int_0^t \left[A_4 - y_4(\tau)\right] d\tau,\ A_1 = y_1(\infty),\ A_2 = y_2(\infty),\ A_3 = y_3(\infty),\ A_4 = y_4(\infty),\ A_5 = y_5(\infty)$$

Alternatively, if the process model is $G_m(s) = K_m \frac{1 + b_1 s + b_2 s^2 + b_3 s^3 + b_4 s^4 + b_5 s^5}{1 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5} e^{-s\tau_m}$, then

$$A_1 = K_m(a_1 - b_1 + \tau_m)\ ,\ A_2 = K_m\left(b_2 - a_2 + A_1 a_1 - b_1 \tau_m + 0.5\tau_m^{\,2}\right)\ ,$$

$$A_3 = K_m\left(a_3 - b_3 + A_2 a_1 - A_1 a_2 + b_2 \tau_m - 0.5 b_1 \tau_m^{\,2} + 0.167 \tau_m^{\,3}\right),$$

$$A_4 = K_m\left(b_4 - a_4 + A_3 a_1 - A_2 a_2 + A_1 a_3 - b_3 \tau_m + 0.5 b_2 \tau_m^{\,2} + 0.167 b_1 \tau_m^{\,3} + 0.042 \tau_m^{\,4}\right),$$

$$A_5 = K_m\left(a_5 - b_5 + A_4 a_1 - A_3 a_2 + A_2 a_3 - A_1 a_4 + b_4 \tau_m - 0.5 b_3 \tau_m^{\,2} + 0.167 b_2 \tau_m^{\,3} - 0.042 b_1 \tau_m^{\,4} + 0.008 \tau_m^{\,5}\right)$$

$$K_c^{(67a)} = \frac{A_3}{2\left(A_1 A_2 - A_0 A_3 - T_d^{(67a)} A_1^{\,2} - \frac{[T_d^{(67a)}]^2}{N} A_0 A_1\right)}$$

$T_d^{(67a)}$ is obtained from a solution of the following equation:

$$\frac{A_0 A_3}{N^3}\left[T_d^{(67a)}\right]^4 + \frac{A_1 A_3}{N^2}\left[T_d^{(67a)}\right]^3 - \frac{A_0 A_5 - A_2 A_3}{N}\left[T_d^{(67a)}\right]^2 + \left(A_3^{\,2} - A_1 A_5\right)\left[T_d^{(67a)}\right] + \left(A_2 A_5 - A_3 A_4\right) = 0$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Lennartson and Kristiansson [157] *Model: Method 1*	$K_c^{(111)}$ [5]	$T_i^{(111)}$	$0.4T_i^{(111)}$	$K_uK_m \le 0.6$
Kristiansson and Lennartson [157] *Model: Method 1*	$K_c^{(111a)}$	$T_i^{(111a)}$	$0.4T_i^{(111a)}$	$K_uK_m > 10$ $\omega_u / K_uK_m \le 0.4$
	$K_c^{(111b)}$	$T_i^{(111b)}$	$0.4T_i^{(111b)}$	$K_uK_m > 10$ $\omega_u / K_uK_m \ge 0.4$
	$K_c^{(111c)}$ [6]	$T_i^{(111c)}$	$0.4T_i^{(111c)}$	$K_uK_m > 10$ $\omega_u / K_uK_m \le 0.4$
	$K_c^{(111d)}$	$T_i^{(111d)}$	$0.4T_i^{(111d)}$	$K_uK_m > 10$ $\omega_u / K_uK_m \ge 0.4$
	$K_c^{(111e)}$	$T_i^{(111e)}$	$0.4T_i^{(111e)}$	$K_uK_m > 1.67$ $\omega_u / K_uK_m > 0.45$ $N = 2.5$

[5] $K_c^{(111)} = K_uK_m \dfrac{12K_u^2K_m^2 - 35K_uK_m + 30}{K_uK_m + 2.5\left[12K_u^2K_m^2 - 35K_uK_m + 30\right]}$, $T_i^{(111)} = \dfrac{K_c^{(111)}}{-0.053\omega_u^3 + 0.47\omega_u^2 - 0.14\omega_u + 0.11}$,

$$N = \frac{2.5}{K_mK_u}\left[12 - \frac{35}{K_mK_u} + \frac{30}{K_m^2K_u^2}\right]$$

$$K_c^{(111a)} = K_uK_m \frac{12K_u^2K_m^2 - 35K_uK_m + 30}{K_uK_m + 3\left[12K_u^2K_m^2 - 35K_uK_m + 30\right]},\ T_i^{(111a)} = \frac{K_c^{(111a)}}{-0.525\omega_u^3 + 0.473\omega_u^2 - 0.143\omega_u + 0.113},$$

$$N = \frac{3}{K_mK_u}\left[12 - \frac{35}{K_mK_u} + \frac{30}{K_m^2K_u^2}\right]$$

$$K_c^{(111b)} = K_uK_m \frac{12K_u^2K_m^2 - 35K_uK_m + 30}{K_uK_m + 3\left[12K_u^2K_m^2 - 35K_uK_m + 30\right]},\ T_i^{(111b)} = \frac{K_c^{(111b)}}{-0.185\omega_u^3 + 1.052\omega_u^2 - 0.854\omega_u + 0.309},$$

$$N = \frac{3}{K_mK_u}\left[12 - \frac{35}{K_mK_u} + \frac{30}{K_m^2K_u^2}\right]$$

[6] $K_c^{(111c)} = K_uK_m \dfrac{12K_u^2K_m^2 - 35K_uK_m + 30}{K_uK_m + 2.5\left[12K_u^2K_m^2 - 35K_uK_m + 30\right]}$, $T_i^{(111c)} = \dfrac{K_c^{(111c)}}{-0.525\omega_u^3 + 0.473\omega_u^2 - 0.143\omega_u + 0.113}$,

$$N = \frac{2.5}{K_mK_u}\left[12 - \frac{35}{K_mK_u} + \frac{30}{K_m^2K_u^2}\right]$$

$$K_c^{(111d)} = K_uK_m \frac{12K_u^2K_m^2 - 35K_uK_m + 30}{K_uK_m + 2.5\left[12K_u^2K_m^2 - 35K_uK_m + 30\right]},\ T_i^{(111d)} = \frac{K_c^{(111d)}}{-0.185\omega_u^3 + 1.052\omega_u^2 - 0.854\omega_u + 0.309},$$

$$N = \frac{2.5}{K_mK_u}\left[12 - \frac{35}{K_mK_u} + \frac{30}{K_m^2K_u^2}\right]$$

$$K_c^{(111e)} = \frac{7.71}{K_m^2K_u^2} - \frac{9.14}{K_mK_u} + 3.14,\ T_i^{(111e)} = \frac{K_c^{(111e)}}{-0.63\omega_u^3 + 0.39\omega_u^2 + 0.15\omega_u + 0.0082}$$

Rule	K_c	T_i	T_d	Comment
Kristiansson and Lennartson [158] *Model: Method 1*	$K_c^{(111f)}$ [7]	$T_i^{(111f)}$	$0.4T_i^{(111f)}$	
Kristiansson and Lennartson [158a]	$K_c^{(111g)}$	$T_i^{(111g)}$	$T_d^{(111g)}$	$T_f^{(111g)}$ given below; $0.1 \le K_u K_m \le 0.5$
Model: Method not specified	$K_c^{(111h)}$ [8]	$T_i^{(111h)}$	$T_d^{(111h)}$	$T_f^{(111h)}$ given below; $K_u K_m > 0.5$

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[7] $K_c^{(111f)} = \dfrac{12K_u^3K_m^2 - 35K_u^2K_m + 30K_u}{K_m^3K_u^3 + 2.5\left[12K_u^2K_m^2 - 35K_uK_m + 30\right]}$, $T_i^{(111f)} = \dfrac{K_c^{(111f)}K_m^3K_u^2}{\omega_u\left[0.95K_m^2K_u^2 - 2K_mK_u + 1.4\right]}$,

$$N = \frac{2.5}{K_mK_u}\left[12 - \frac{35}{K_mK_u} + \frac{30}{K_m^2K_u^2}\right]$$

$$K_c^{(111g)} = \frac{(1.1K_u^2K_m^2 - 2.3K_uK_m + 1.6)^2}{K_m^2K_u(-20 + 13K_mK_u)(1 + 0.37K_mK_u)^2}\left[\frac{1.6(-20 + 13K_mK_u)(1 + 0.37K_mK_u)}{1.1K_m^2K_u^2 - 2.3K_mK_u + 1.6} - 1\right]$$

$$T_i^{(111g)} = \frac{K_mK_u(1.1K_u^2K_m^2 - 2.3K_uK_m + 1.6)}{\omega_u(-20 + 13K_mK_u)(1 + 0.37K_mK_u)^2}\left[\frac{1.6(-20 + 13K_mK_u)(1 + 0.37K_mK_u)}{1.1K_m^2K_u^2 - 2.3K_mK_u + 1.6} - 1\right]$$

$$T_d^{(111g)} = \frac{K_mK_u(1.1K_u^2K_m^2 - 2.3K_uK_m + 1.6)}{\omega_u(-20 + 13K_mK_u)(1 + 0.37K_mK_u)^2}\left[\frac{\dfrac{(-20 + 13K_mK_u)^2(1 + 0.37K_mK_u)^2}{(1.1K_m^2K_u^2 - 2.3K_mK_u + 1.6)^2}}{\dfrac{1.6(-20 + 13K_mK_u)(1 + 0.37K_mK_u)}{1.1K_m^2K_u^2 - 2.3K_mK_u + 1.6} - 1} - 1\right]$$

$$T_f^{(111g)} = \frac{K_mK_u(1.1K_u^2K_m^2 - 2.3K_uK_m + 1.6)}{\omega_u(-20 + 13K_mK_u)(1 + 0.37K_mK_u)^2}$$

[8] $K_c^{(111h)} = \dfrac{(1.1K_u^2K_m^2 - 2.3K_uK_m + 1.6)^2}{3\omega_uK_m^2K_u^2(1 + 0.37K_mK_u)^2}\left[\dfrac{4.8K_mK_u(1 + 0.37K_mK_u)}{1.1K_m^2K_u^2 - 2.3K_mK_u + 1.6} - 1\right]$

$$T_i^{(111h)} = \frac{(1.1K_u^2K_m^2 - 2.3K_uK_m + 1.6)}{3\omega_u(1 + 0.37K_mK_u)^2}\left[\frac{4.8K_mK_u(1 + 0.37K_mK_u)}{1.1K_m^2K_u^2 - 2.3K_mK_u + 1.6} - 1\right]$$

$$T_d^{(111h)} = \frac{(1.1K_u^2K_m^2 - 2.3K_uK_m + 1.6)}{3\omega_u(1 + 0.37K_mK_u)^2}\left[\frac{\dfrac{3K_m^2K_u^2(1 + 0.37K_mK_u)^2}{(1.1K_m^2K_u^2 - 2.3K_mK_u + 1.6)^2}}{\dfrac{4.8K_mK_u(1 + 0.37K_mK_u)}{1.1K_m^2K_u^2 - 2.3K_mK_u + 1.6} - 1} - 1\right]$$

$$T_f^{(111h)} = \frac{1.1K_u^2K_m^2 - 2.3K_uK_m + 1.6}{3\omega_u(1 + 0.37K_mK_u)^2}$$

Rule	K_c	T_i	T_d	Comment
Kristiansson and Lennartson [158a] (continued) *Model: Method not specified*	$K_c^{(111i)}$ [9]	$T_i^{(111i)}$	$T_d^{(111i)}$	$T_f^{(111i)}$ given below; $K_u K_m < 0.1$
Other				
Leva [67]	$\frac{\omega T_i}{\lvert G_p(j\omega)\rvert \sqrt{\left(1-\omega^2 T_i T_d\right)^2 + \omega^2 T_i^2}}$	$\frac{\tan\left(\phi_m - \phi_\omega - 0.5\pi\right)}{\omega\sqrt{1+\frac{2\pi}{\beta}\tan\left(\phi_m - \phi_\omega - 0.5\pi\right)}}$	$\frac{2\pi}{\beta\omega}$	$\beta = 10$ $\phi_\omega > \phi_m - \pi$
	$\frac{\omega T_i}{\lvert G_p(j\omega)\rvert \sqrt{\left(1-\omega^2 T_i T_d\right)^2 + \omega^2 T_i^2}}$	$\alpha T_d^{(68)}$ [10]	$T_d^{(68)}$	$6 \le \alpha \le 10$ $\phi_\omega < \phi_m - \pi$
Astrom [68]	$K_u \cos\phi_m$	$\frac{2\left[\tan\phi_m + \sqrt{1+\tan^2\phi_m}\right]}{\omega_u}$	$\frac{T_i}{4}$	Parameters determined at $\phi_m = 30^0, 45^0, 60^0$

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[9] $$K_c^{(111i)} = \frac{(-6+3.7K_{135^0}K_m)^2}{13K_m(1.8+0.3K_m K_{135^0})^2}\left[\frac{20.8(1.8+0.3K_m K_{135^0})}{(-6+3.7K_m K_{135^0})} - 1\right]$$

$$T_i^{(111i)} = \frac{(-6+3.7K_{135^0}K_m)K_{135^0}K_m}{13\omega_{135^0}(1.8+0.3K_m K_{135^0})^2}\left[\frac{20.8(1.8+0.3K_m K_{135^0})}{(-6+3.7K_m K_{135^0})} - 1\right]$$

$$T_d^{(111i)} = \frac{(-6+3.7K_m K_{135^0})K_m K_{135^0}}{13\omega_{135^0}(1.8+0.3K_m K_{135^0})^2}\left[\frac{\frac{169(1.8+0.3K_m K_{135^0})^2}{(-6+3.7K_m K_{135^0})^2} - 1}{\frac{20.8(1.8+0.3K_m K_{135^0})}{(-6+3.7K_m K_{135^0})} - 1}\right], T_f^{(111i)} = \frac{(-6+3.7K_m K_{135^0})K_m K_{135^0}}{13\omega_{135^0}(1.8+0.3K_m K_{135^0})^2}$$

[10] $$T_d^{(68)} = \frac{-\alpha\omega + \sqrt{\alpha^2\omega^2 + 4\alpha\omega^2\tan^2\left(\phi_m - \phi_\omega - 0.5\pi\right)}}{2\alpha\omega^2\tan\left(\phi_m - \phi_\omega - 0.5\pi\right)}$$

Table 43: PID tuning rules - non-model specific – ideal controller with set-point weighting

$$U(s) = K_c\left(F_p R(s) - Y(s)\right) + \frac{1}{T_i s}\left(F_i R(s) - Y(s)\right) + T_d s\left(F_d R(s) - Y(s)\right)$$. *1 tuning rule*.

Rule	K_c	T_i	T_d	Comment
Ultimate cycle				
Mantz and Tacconi [118]	$0.6K_u$ $F_p = 0.17$	$0.5T_u$ $F_i = 1$	$0.125T_u$ $F_d = 0.654$	Quarter decay ratio

Table 44: PID tuning rules - non-model specific – ideal controller with proportional weighting $G_c(s) = K_c\left(b + \frac{1}{T_i s} + T_d s\right)$. *1 tuning rule*.

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Astrom and Hagglund [3] – *page 217*	$\left(0.33e^{-0.31\kappa-\kappa^2}\right)K_u$, $\kappa = 1/K_m K_u$	$\left(0.76e^{-1.6\kappa-0.36\kappa^2}\right)T_u$	$\left(0.17e^{-0.46\kappa-2.1\kappa^2}\right)T_u$	$b = 0.58e^{-1.3\kappa+3.5\kappa^2}$ $0 < K_m K_u < \infty$ maximum $M_s = 1.4$
	$\left(0.72e^{-1.6\kappa+1.2\kappa^2}\right)K_u$	$\left(0.59e^{-1.3\kappa+0.38\kappa^2}\right)T_u$	$\left(0.15e^{-1.4\kappa+0.56\kappa^2}\right)T_u$	$b = 0.25e^{0.56\kappa-0.12\kappa^2}$ $0 < K_m K_u < \infty$ maximum $M_s = 2.0$

Table 45: PID tuning rules - non-model specific – non-interacting controller $U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - \frac{K_c T_d s}{1 + \frac{sT_d}{N}}Y(s)$. *1 tuning rule*.

Rule	K_c	T_i	T_d	Comment
Ultimate cycle				
Fu *et al*. [128]	$0.5K_u$	$0.34T_u$	$0.08T_u$	

Table 46: PID tuning rules - non-model specific – series controller $U(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(1+sT_d\right)$. *3 tuning rules*.

Rule	K_c	T_i	T_d	Comment
Ultimate cycle				
Pessen [131]	$0.25K_u$	$0.33T_u$	$\frac{0.5}{T_u}$	'optimum' servo response
	$0.2K_u$	$0.25T_u$	$\frac{0.5}{T_u}$	'optimum' regulator response - step changes
Pessen [129]	$0.2K_u$	$0.5T_u$	$0.33T_u$	No overshoot; close to optimum regulator
	$0.33K_u$	$0.33T_u$	$0.5T_u$	'Some' overshoot
Grabbe *et al*. [130]	$0.25K_u$	$0.33T_u$	$0.5T_u$	

Table 47: PID tuning rules - non-model specific – series controller with filtered derivative

$$U(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(1+\frac{sT_d}{1+\frac{sT_d}{N}}\right)$$. *1 tuning rule*.

Rule	K_c	T_i	T_d	Comment
Ultimate cycle				
Hang *et al*. [36] - *page 58*	$0.35K_u$	$1.13T_u$	$0.20T_u$	

Table 48: PID tuning rules - non-model specific – classical controller $U(s) = K_c\left(1 + \frac{1}{T_i s}\right)\frac{1 + sT_d}{1 + s\frac{T_d}{N}}$. *1 tuning rule*.

Rule	K_c	T_i	T_d	Comment
Ultimate cycle				
Corripio [117] – *page 27*	$0.6K_u$	$0.5T_u$	$0.125T_u$	$10 \le N \le 20$

Table 49: PID tuning rules - non-model specific – non-interacting controller
$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - K_c T_d s Y(s)$. *1 tuning rule*.

Rule	K_c	T_i	T_d	Comment
Ultimate cycle				
VanDoren [114]	$0.75K_u$	$0.625T_u$	$0.1T_u$	

Table 50: PID tuning rules - IPD model $\frac{K_m e^{-s\tau_m}}{s}$ - ideal controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$. *5 tuning rules.*

Rule	K_c		T_i		T_d		Comment
Direct synthesis							
Wang and Cluett [76] – *deduced from graph* *Model: Method 2*	$\frac{\alpha}{K_m\tau_m}$		$\beta\tau_m$		$\chi\tau_m$		
	Closed loop time constant	Damping Factor, ξ	Gain margin A_m	Phase margin ϕ_m [degrees]	α	β	χ
	τ_m	0.707	2.0	32	0.9056	2.6096	0.3209
	$2\tau_m$	0.707	3.1	40	0.5501	4.0116	0.2205
	$3\tau_m$	0.707	4.4	46	0.3950	5.4136	0.1681
	$4\tau_m$	0.707	5.5	49	0.3081	6.8156	0.1357
	$5\tau_m$	0.707	6.7	52	0.2526	8.2176	0.1139
	$6\tau_m$	0.707	7.8	54	0.2140	9.6196	0.0980
	$7\tau_m$	0.707	8.9	55	0.1856	11.0216	0.0861
	$8\tau_m$	0.707	10.0	56	0.1639	12.4236	0.0767
	$9\tau_m$	0.707	11.2	57	0.1467	13.8256	0.0692
	$10\tau_m$	0.707	12.2	58	0.1328	15.2276	0.0630
	$11\tau_m$	0.707	13.4	59	0.1213	16.6296	0.0579
	$12\tau_m$	0.707	14.5	59	0.1117	18.0316	0.0535
	$13\tau_m$	0.707	15.6	59	0.1034	19.4336	0.0497
	$14\tau_m$	0.707	16.7	60	0.0963	20.8356	0.0464
	$15\tau_m$	0.707	17.8	60	0.0901	22.2376	0.0436
	$16\tau_m$	0.707	19.0	60	0.0847	23.6396	0.0410
	τ_m	1.0	2.0	37	0.8859	3.2120	0.3541
	$2\tau_m$	1.0	2.9	46	0.6109	5.2005	0.2612
	$3\tau_m$	1.0	3.8	52	0.4662	7.1890	0.2069
	$4\tau_m$	1.0	4.6	56	0.3770	9.1775	0.1713
	$5\tau_m$	1.0	5.5	58	0.3164	11.1660	0.1462
	$6\tau_m$	1.0	6.4	61	0.2726	13.1545	0.1275
	$7\tau_m$	1.0	7.1	62	0.2394	15.1430	0.1311
	$8\tau_m$	1.0	8.0	64	0.2135	17.1315	0.1015
	$9\tau_m$	1.0	8.7	65	0.1926	19.1200	0.0921
	$10\tau_m$	1.0	9.5	66	0.1754	21.1085	0.0843
	$11\tau_m$	1.0	10.4	67	0.1611	23.0970	0.0777
	$12\tau_m$	1.0	11.1	67	0.1489	25.0855	0.0721
	$13\tau_m$	1.0	12.0	68	0.1384	27.0740	0.0672
	$14\tau_m$	1.0	12.8	68	0.1293	29.0625	0.0630
	$15\tau_m$	1.0	13.4	69	0.1213	31.0510	0.0592
	$16\tau_m$	1.0	14.4	69	0.1143	33.0395	0.0559

Rule	K_c	T_i	T_d	Comment
Cluett and Wang [44] *Model: Method 2*	$\frac{0.9588}{K_m \tau_m}$	$3.0425\tau_m$	$0.3912\tau_m$	Closed loop time constant = τ_m
	$\frac{0.6232}{K_m \tau_m}$	$5.2586\tau_m$	$0.2632\tau_m$	Closed loop time constant = $2\tau_m$
	$\frac{0.4668}{K_m \tau_m}$	$7.2291\tau_m$	$0.2058\tau_m$	Closed loop time constant = $3\tau_m$
	$\frac{0.3752}{K_m \tau_m}$	$9.1925\tau_m$	$0.1702\tau_m$	Closed loop time constant = $4\tau_m$
	$\frac{0.3144}{K_m \tau_m}$	$11.1637\tau_m$	$0.1453\tau_m$	Closed loop time constant = $5\tau_m$
	$\frac{0.2709}{K_m \tau_m}$	$13.1416\tau_m$	$0.1269\tau_m$	Closed loop time constant = $6\tau_m$
Rotach [77] *Model: Method 4*	$\frac{1.21}{K_m \tau_m}$	$1.60\tau_m$	$0.48\tau_m$	Damping factor for oscillations to a disturbance input = 0.75.
Process reaction				
Ford [132] *Model: Method 2*	$\frac{1.48}{K_m \tau_m}$	$2\tau_m$	$0.37\tau_m$	Decay ratio: 2.7:1
Astrom and Hagglund [3] – *page 139* *Model: Method not relevant*	$\frac{0.94}{K_m \tau_m}$	$2\tau_m$	$0.5\tau_m$	Ultimate cycle Ziegler-Nichols equivalent

Table 51: PID tuning rules - IPD model $\frac{K_m e^{-s\tau_m}}{s}$ - Ideal controller with first order filter, set-point weighting and output feedback $U(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{1}{T_f s + 1}\left[R(s)\frac{1 + 0.4T_r s}{1 + sT_r} - Y(s)\right] - K_0 Y(s)$. *1 tuning rule*.

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Normey-Rico *et al*. [106a] *Model: Method not specified*	$\frac{0.563}{K_m \tau_m}$	$1.5\tau_m$	$0.667\,\tau_m$	$T_f = 0.13\tau_m$ $K_0 = \frac{1}{2K_m\tau_m}$ $T_r = 0.75\tau_m$

Table 52: PID tuning rules - IPD model $\frac{K_m e^{-s\tau_m}}{s}$ - ideal controller with filtered derivative

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{sT_d}{N}}\right)$$. *1 tuning rule.*

Rule	K_c	T_i	T_d	Comment
Robust				
Chien [50] *Model: Method 2*	$\frac{2}{K_m(\lambda + 0.5\tau_m)}$	$2\lambda + \tau_m$	$\frac{\tau_m(\lambda + 0.25\tau_m)}{2\lambda + \tau_m}$	$\lambda = \frac{1}{K_m}$; N=10

Table 53: PID tuning rules - IPD model $\frac{K_m e^{-s\tau_m}}{s}$ - series controller with filtered derivative

$$G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(1+\frac{T_d s}{1+\frac{T_d s}{N}}\right)$$. *1 tuning rule*.

Rule	K_c	T_i	T_d	Comment
Robust				
Chien [50]	$\frac{1}{K_m}\left(\frac{2\lambda+0.5\tau_m}{[\lambda+0.5\tau_m]^2}\right)$	$2\lambda+0.5\tau_m$	$0.5\tau_m$	$\lambda=\left[\frac{1}{K_m},\tau_m\right]$; N=10
Model: Method 2	$\frac{1}{K_m}\left(\frac{0.5\tau_m}{[\lambda+0.5\tau_m]^2}\right)$	$05\tau_m$	$2\lambda+0.5\tau_m$	$\lambda=\left[\frac{1}{K_m},\tau_m\right]$; N=10

Table 54: PID tuning rules - IPD model $\frac{K_m e^{-s\tau_m}}{s}$ - classical controller $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+T_d s}{1+\frac{T_d s}{N}}\right)$.

5 tuning rules.

Rule	K_c	T_i	T_d	Comment
Ultimate cycle				
Luyben [133] *Model: Method 2*	$0.46K_u$	$2.2T_u$	$0.16T_u$	maximum closed loop log modulus of +2dB ; N=10
Belanger and Luyben [134] *Model: Method 2*	$3.11K_u$	$2.2T_u$	$3.64T_u$	N=10
Robust				
Chien [50] *Model: Method 2*	$\frac{1}{K_m}\left(\frac{0.5\tau_m}{[\lambda+0.5\tau_m]^2}\right)$	$0.5\tau_m$	$2\lambda+0.5\tau_m$	$\lambda=\left[\frac{1}{K_m},\tau_m\right]$; N=10
	$\frac{1}{K_m}\left(\frac{2\lambda+0.5\tau_m}{[\lambda+0.5\tau_m]^2}\right)$	$2\lambda+0.5\tau_m$	$0.5\tau_m$	$\lambda=\left[\frac{1}{K_m},\tau_m\right]$; N=10
Regulator tuning	**Minimum performance index**			
Minimum IAE - Shinskey [17] – *page 121. Model: Method not specified*	$0.56K_u$	$0.39T_u$	$0.15T_u$	
Minimum IAE - Shinskey [17] – *page 117. Model: Method 1*	$\frac{0.93}{K_m\tau_m}$	$1.57\tau_m$	$0.56\tau_m$	
Minimum IAE – Shinskey [59] – *page 74 Model: Method not specified*	$\frac{0.9259}{K_m\tau_m}$	$1.60\tau_m$	$0.58\tau_m$	N=10
	$\frac{0.9259}{K_m\tau_m}$	$1.48\tau_m$	$0.63\tau_m$	N=20

Table 55: PID tuning rules - IPD model $\frac{K_m e^{-s\tau_m}}{s}$ - Alternative non-interacting controller 1 - $U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - K_c T_d sY(s)$. *2 tuning rules*.

Rule	K_c	T_i	T_d	Comment
Regulator tuning	**Minimum performance index**			
Minimum IAE - Shinskey [59] – *page 74. Model: Method not specified*	$\frac{1.2821}{K_m\tau_m}$	$1.9\tau_m$	$0.46\tau_m$	
Minimum IAE – Shinskey [17] – *page 121. Model: Method not specified*	$0.77K_u$	$0.48T_u$	$0.15\tau_m$	
Minimum IAE – Shinskey [17] – *page 117. Model: Method 1*	$\frac{1.28}{K_m\tau_m}$	$1.90\tau_m$	$0.48\tau_m$	

Table 56: PID tuning rules - IPD model $\frac{K_m e^{-s\tau_m}}{s}$ - I-PD controller $U(s) = \frac{K_c}{T_i s} E(s) - K_c(1 + T_d s)Y(s)$.

1 tuning rule.

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Chien *et al*. [74] *Model: Method 1*	$K_c^{(68a)}$ [1]	$1.414T_{CL} + \tau_m$	$\frac{0.25\tau_m^2 + 0.707T_{CL}\tau_m}{1.414T_{CL} + \tau_m}$	Underdamped system response - $\xi = 0.707$. $\tau_m \le 0.2T_m$

[1] $K_c^{(68a)}\ \frac{1.414T_{CL} + \tau_m}{K_m\left(T_{CL}^2 + 0.707T_{CL}\tau_m + 0.25\tau_m^2\right)}$

Table 57: PID tuning rules - IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$ - controller $U(s) = K_c(1 + \frac{1}{T_i s})E(s) + K_c(b-1)R(s) - K_c T_d s Y(s)$. *1 tuning rule*.

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Hansen [91a] *Model: Method not specified*	$0.938 K_m \tau_m$	$2.7\tau_m$	$0.313\tau_m$	b = 0.167

Table 58: PID tuning rules - IPD model $\frac{K_m e^{-s\tau_m}}{s}$ - Two degree of freedom controller:

$$U(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N}s}\right)E(s) - K_c\left(\alpha + \frac{\beta T_d s}{1 + \frac{T_d}{N}s}\right)R(s)$$. *2 tuning rules*.

Rule	K_c	T_i	T_d	Comment
Servo/regulator tuning	**Minimum performance index**			
Taguchi and Araki [61a] *Model: ideal process*	$\frac{1}{K_m}\left(\frac{1.253}{\tau_m}\right)$ $\alpha = 0.6642$	$2.388\tau_m$ $\beta = 0.6797$	$0.4137\tau_m$	$\frac{\tau_m}{T_m} \le 1.0$; Overshoot (servo step) $\le 20\%$; settling time $\le$ settling time of tuning rules of Chien *et al*. [10]
Minimum ITAE - Pecharroman and Pagola [134a] ϕ_c = phase corresponding to the crossover frequency; $K_m = 1$ *Model: Method 6*	$1.672K_u$	$0.366T_u$	$0.136T_u$	$\alpha = 0.601$, $\beta = 1$, $N = 10$, $\phi_c = -164^0$
	$1.236K_u$	$0.427T_u$	$0.149T_u$	$\alpha = 0.607$, $\beta = 1$, $N = 10$, $\phi_c = -160^0$
	$0.994K_u$	$0.486T_u$	$0.155T_u$	$\alpha = 0.610$, $\beta = 1$, $N = 10$, $\phi_c = -155^0$
	$0.842K_u$	$0.538T_u$	$0.154T_u$	$\alpha = 0.616$, $\beta = 1$, $N = 10$, $\phi_c = -150^0$
	$0.752K_u$	$0.567T_u$	$0.157T_u$	$\alpha = 0.605$, $\beta = 1$, $N = 10$, $\phi_c = -145^0$
	$0.679K_u$	$0.610T_u$	$0.149T_u$	$\alpha = 0.610$, $\beta = 1$, $N = 10$, $\phi_c = -140^0$
	$0.635K_u$	$0.637T_u$	$0.142T_u$	$\alpha = 0.612$, $\beta = 1$, $N = 10$, $\phi_c = -135^0$
	$0.590K_u$	$0.669T_u$	$0.133T_u$	$\alpha = 0.610$, $\beta = 1$, $N = 10$, $\phi_c = -130^0$
	$0.551K_u$	$0.690T_u$	$0.114T_u$	$\alpha = 0.616$, $\beta = 1$, $N = 10$, $\phi_c = -125^0$
	$0.520K_u$	$0.776T_u$	$0.087T_u$	$\alpha = 0.609$, $\beta = 1$, $N = 10$, $\phi_c = -120^0$
	$0.509K_u$	$0.810T_u$	$0.068T_u$	$\alpha = 0.611$, $\beta = 1$, $N = 10$, $\phi_c = -118^0$

Table 59: PID tuning rules - FOLIPD model $\frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$ - ideal controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$.

1 tuning rule.

Rule	K_c	T_i	T_d	Comment
Ultimate cycle				
McMillan [58] *Model: Method not relevant*	$K_c^{(69)}$ [1]	$T_i^{(69)}$	$T_d^{(69)}$	Tuning rules developed from K_u, T_u

[1] $K_c^{(69)} = \frac{1.111}{K_m}\frac{T_m}{\tau_m^2}\left\{\frac{1}{1+\left(\frac{T_m}{\tau_m}\right)^{0.65}}\right\}^2$, $T_i^{(69)} = 2\tau_m\left\{1+\left(\frac{T_m}{\tau_m}\right)^{0.65}\right\}$, $T_d^{(69)} = 0.5\tau_m\left\{1+\left(\frac{T_m}{\tau_m}\right)^{0.65}\right\}$

Table 60: PID tuning rules - FOLIPD model $\frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$ - ideal controller with filter

$G_c(s) = K_c\left(1+\frac{1}{T_i s}+T_d s\right)\frac{1}{1+T_f s}$. *3 tuning rules.*

Rule	K_c	T_i	T_d	Comment
Robust				
Tan *et al.* [81] *Model: Method 2*	$[\frac{0.463\lambda+0.277}{K_m\tau_m{}^2}]$ $([0.238\lambda+0.123]T_m+\tau_m)$	$T_m+\frac{\tau_m}{0.238\lambda+0.123}$	$\frac{T_m\tau_m}{(0.238\lambda+0.123)T_m+\tau_m}$	$T_f=\frac{\tau_m}{5.750\lambda+0.590}$ $\lambda=0.5$ - performance $\lambda=0.1$ - robustness $\lambda=0.25$ - acceptable
Zhang *et al.* [135] *Model: Method 2*	$\frac{3\lambda+T_m+\tau_m}{K_m\left(3\lambda^2+3\lambda\tau_m+\tau_m{}^2\right)}$	$3\lambda+T_m+\tau_m$	$\frac{(3\lambda+\tau_m)T_m}{3\lambda+\tau_m+T_m}$	$T_f=\frac{\lambda^3}{3\lambda^2+3\lambda\tau_m+\tau_m{}^2}$ $1.5\tau_m\le\lambda\le4.5\tau_m$
	$\lambda=1.5\tau_m$ ….Overshoot = 58%, Settling time = $6\tau_m$ $\lambda=2.5\tau_m$ ….Overshoot = 35%, Settling time = $11\tau_m$ $\lambda=3.5\tau_m$ ….Overshoot = 26%, Settling time = $16\tau_m$ $\lambda=4.5\tau_m$ ….Overshoot = 22%, Settling time = $20\tau_m$			Obtained from graph
Tan *et al.* [136] *Model: Method 2*	$\frac{0.0337T_m}{K_m\tau_m{}^2}\left(1+\frac{\tau_m}{0.1225T_m}\right)$	$T_m+8.1633\tau_m$	$\frac{T_m\tau_m}{0.1225T_m+\tau_m}$	$T_f=0.5549\tau_m$
	$\frac{0.0754T_m}{K_m\tau_m{}^2}\left(1+\frac{\tau_m}{0.1863T_m}\right)$	$T_m+5.3677\tau_m$	$\frac{T_m\tau_m}{0.1863T_m+\tau_m}$	$T_f=0.4482\tau_m$
	$\frac{0.1344T_m}{K_m\tau_m{}^2}\left(1+\frac{\tau_m}{0.2523T_m}\right)$	$T_m+3.9635\tau_m$	$\frac{T_m\tau_m}{0.2523T_m+\tau_m}$	$T_f=0.2863\tau_m$

Table 61: PID tuning rules - FOLIPD model $\frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$ - ideal controller with set-point weighting

$$G_c(s) = K_c\left(b + \frac{1}{T_i s} + T_d s\right)$$. *1 tuning rule.*

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Astrom and Hagglund [3] - *pages 212-213* *Model: Method 1 or Method 2.*	$\frac{5.6e^{-8.8\tau+6.8\tau^2}}{K_m(T_m+\tau_m)}$, $\tau = \tau_m/(\tau_m+T_m)$	$1.1\tau_m e^{6.7\tau-4.4\tau^2}$	$1.7\tau_m e^{-6.4\tau+2.0\tau^2}$	Maximum $M_s = 1.4$ $b = 0.12e^{6.9\tau-6.6\tau^2}$
	$\frac{8.6e^{-7.1\tau+5.4\tau^2}}{K_m(T_m+\tau_m)}$	$1.0\tau_m e^{3.3\tau-2.33\tau^2}$	$0.38\tau_m e^{0.056\tau-0.60\tau^2}$	Maximum $M_s = 2.0$ $b = 0.56e^{-2.2\tau+1.2\tau^2}$

Table 62: PID tuning rules - FOLIPD model $\frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$ - classical controller $G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+T_d s}{1+\frac{T_d s}{N}}\right)$.

5 tuning rules.

Rule	K_c	T_i	T_d	Comment
Robust				
Chien [50] *Model: Method 2*	$\frac{1}{K_m}\left(\frac{T_m}{[\lambda+\tau_m]^2}\right)$	T_m	$2\lambda+\tau_m$	$\lambda=[\tau_m, T_m]$; N=10
	$\frac{1}{K_m}\left(\frac{2\lambda+\tau_m}{[\lambda+\tau_m]^2}\right)$	$2\lambda+\tau_m$	T_m	$\lambda=[\tau_m, T_m]$; N=10
Regulator tuning	**Minimum performance index**			
Minimum IAE – Shinskey [59] – *page 75.* *Model: Method not specified*	$\frac{0.78}{K_m(\tau_m+T_m)}$	$1.38(\tau_m+T_m)$	$0.66(\tau_m+T_m)$	$\tau_m = T_m$; N=10
Minimum IAE - Shinskey [59] – *pages 158-159* *Model: Open loop method not specified*	$\frac{100}{108K_m\tau_m\left(1.22-0.03\frac{T_m}{\tau_m}\right)}$	$1.57\tau_m\left(1+1.2\left[1-e^{-\frac{T_m}{\tau_m}}\right]\right)$	$0.56\tau_m+0.75T_m$	$\frac{T_m}{\tau_m}>0.5$
	$\frac{100}{108K_m\tau_m\left(1+0.4\frac{T_m}{\tau_m}\right)}$	$1.57\tau_m\left(1+1.2\left[1-e^{-\frac{T_m}{\tau_m}}\right]\right)$	$0.56\tau_m+0.75T_m$	$\frac{T_m}{\tau_m}\le 0.5$
Minimum ITAE - Poulin and Pomerleau [82], [92] – *deduced from graph* *Model: Method 2*	$\frac{b}{K_m(\tau_m+T_m)}\sqrt{\frac{T_m^2}{a(\tau_m+T_m)^2}+1}$	$a(\tau_m+T_m)$	T_m	$0\le\frac{\tau_m}{(T_d/N)}\le 2$; $0.1T_m\le\frac{T_d}{N}\le 0.33T_m$

	$\tau_m/(T_d/N)$	a	b	$\tau_m/(T_d/N)$	a	b
Output step load disturbance	0.2	5.0728	0.5231	1.2	4.7565	0.5250
	0.4	4.9688	0.5237	1.4	4.7293	0.5252
	0.6	4.8983	0.5241	1.6	4.7107	0.5254
	0.8	4.8218	0.5245	1.8	4.6837	0.5256
	1.0	4.7839	0.5249	2.0	4.6669	0.5257
Input step load disturbance	0.2	3.9465	0.5320	1.2	4.0397	0.5312
	0.4	3.9981	0.5315	1.4	4.0278	0.5312
	0.6	4.0397	0.5311	1.6	4.0278	0.5312
	0.8	4.0397	0.5311	1.8	4.0218	0.5313
	1.0	4.0397	0.5311	2.0	4.0099	0.5314

Table 63: PID tuning rules - FOLIPD model $\frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$ - series controller with derivative filtering

$$G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(1+\frac{T_d s}{1+\frac{T_d s}{N}}\right)$$. *1 tuning rule*.

Rule	K_c	T_i	T_d	Comment
Robust				
Chien [50] *Model: Method 2*	$\frac{1}{K_m}\left(\frac{2\lambda+\tau_m}{[\lambda+\tau_m]^2}\right)$	$2\lambda+\tau_m$	T_m	$\lambda=[\tau_m, T_m]$; N=10 (Chien and Fruehauf [137])
	$\frac{1}{K_m}\left(\frac{T_m}{[\lambda+\tau_m]^2}\right)$	T_m	$2\lambda+\tau_m$	$\lambda=[\tau_m, T_m]$; N=10

Table 64: PID tuning rules - FOLIPD model $\frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$ - alternative non-interacting controller 1

$U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - K_c T_d sY(s)$. *2 tuning rules.*

Rule	K_c	T_i	T_d	Comment
Regulator tuning	**Minimum performance index**			
Minimum IAE - Shinskey [59] – *page 75. Model: Method not specified*	$\frac{1.18}{K_m(\tau_m + T_m)}$	$1.38(\tau_m + T_m)$	$0.55(\tau_m + T_m)$	
Minimum IAE - Shinskey [59] – *page 159. Model: Open loop method not defined*	$\frac{1.28}{K_m\tau_m(1+0.24\frac{T_m}{\tau_m} - 0.14\left[\frac{T_m}{\tau_m}\right]^2)}$	$1.9\tau_m\left(1+0.75[1-e^{-\frac{T_m}{\tau_m}}]\right)$	$0.48\tau_m + 0.7T_m$	

Table 65: PID tuning rules - FOLIPD model $\dfrac{K_m e^{-s\tau_m}}{s(1+sT_m)}$ - ideal controller with filtered derivative $G_c(s) = K_c\left(1 + \dfrac{1}{T_i s} + \dfrac{T_d s}{1 + s\dfrac{T_d}{N}}\right)$. *1 tuning rule*.

Rule	K_c	T_i	T_d	Comment
Robust				
Chien [50] *Model: Method 2*	$\dfrac{2\lambda + \tau_m + T_m}{K_m(\lambda + \tau_m)^2}$	$2\lambda + T_m + \tau_m$	$\dfrac{T_m(2\lambda + \tau_m)}{2\lambda + T_m + \tau_m}$	$\lambda = T_m$; $N = 10$

Table 66: PID tuning rules - FOLIPD model $\frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$ - ideal controller with set-point weighting

$U(s) = K_c\left(F_p R(s) - Y(s)\right) + \frac{K_c}{T_i s}\left[F_i R(s) - Y(s)\right] + K_c T_d s\left[F_d R(s) - Y(s)\right]$. *1 tuning rule.*

Rule	K_c	T_i	T_d	Comment
Ultimate cycle				
Oubrahim and Leonard [138] *Model: Method not relevant*	$0.6K_u$ $F_p = 0.1$	$0.5T_u$ $F_i = 1$	$0.125T_u$ $F_d = 0.01$	$0.05 < \frac{\tau_m}{T_m} < 0.8$; 20% overshoot

Table 67: PID tuning rules - FOLIPD model $\frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$ - Alternative classical controller

$$G_c(s) = K_c\left(\frac{1+T_i s}{1+\frac{T_d s}{N}}\right)\left(\frac{1+T_d s}{1+\frac{T_d s}{N}}\right)$$. *1 tuning rule*.

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Tsang and Rad [109] *Model: Method 3*	$\frac{0.809}{K_m \tau_m}$	T_m	$0.5\tau_m$	Maximum overshoot = 16%; N = 8.33

Table 68: PID tuning rules – FOLIPD model $\frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$ - Two degree of freedom controller:

$$U(s)=K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+\frac{T_d}{N}s}\right)E(s)-K_c\left(\alpha+\frac{\beta T_d s}{1+\frac{T_d}{N}s}\right)R(s)$$. *2 tuning rules*.

Rule	K_c	T_i	T_d	Comment
Servo/regulator tuning	**Minimum performance index**			
Taguchi and Araki [61a] *Model: ideal process*	$K_c^{(69a)}$ [2]	$T_i^{(69a)}$	$T_d^{(69a)}$	$\frac{\tau_m}{T_m}\le 1.0$; Overshoot (servo step) $\le 20\%$; settling time $\le$ settling time of tuning rules of Chien *et al*. [10]
Minimum ITAE - Pecharroman and Pagola [134a] $\phi_c =$ phase corresponding to the crossover frequency; $K_m = 1$; $T_m = 1$; $0.05 < \tau_m < 0.8$. *Model: Method 4*	$1.672 K_u$	$0.366 T_u$	$0.136 T_u$	$\alpha = 0.601$, $\beta = 1$, $N = 10$, $\phi_c = -164^0$
	$1.236 K_u$	$0.427 T_u$	$0.149 T_u$	$\alpha = 0.607$, $\beta = 1$, $N = 10$, $\phi_c = -160^0$
	$0.994 K_u$	$0.486 T_u$	$0.155 T_u$	$\alpha = 0.610$, $\beta = 1$, $N = 10$, $\phi_c = -155^0$
	$0.842 K_u$	$0.538 T_u$	$0.154 T_u$	$\alpha = 0.616$, $\beta = 1$, $N = 10$, $\phi_c = -150^0$
	$0.752 K_u$	$0.567 T_u$	$0.157 T_u$	$\alpha = 0.605$, $\beta = 1$, $N = 10$, $\phi_c = -145^0$
	$0.679 K_u$	$0.610 T_u$	$0.149 T_u$	$\alpha = 0.610$, $\beta = 1$, $N = 10$, $\phi_c = -140^0$
	$0.635 K_u$	$0.637 T_u$	$0.142 T_u$	$\alpha = 0.612$, $\beta = 1$, $N = 10$, $\phi_c = -135^0$
	$0.590 K_u$	$0.669 T_u$	$0.133 T_u$	$\alpha = 0.610$, $\beta = 1$, $N = 10$, $\phi_c = -130^0$
	$0.551 K_u$	$0.690 T_u$	$0.114 T_u$	$\alpha = 0.616$, $\beta = 1$, $N = 10$, $\phi_c = -125^0$

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[2] $K_c^{(69a)} = \frac{1}{K_c}\left(0.7608+\frac{0.5184}{[\frac{\tau_m}{T_m}-0.01308]^2}\right)$, $T_i^{(69a)} = T_m\left(0.03330+3.997\frac{\tau_m}{T_m}-0.5517\left[\frac{\tau_m}{T_m}\right]^2\right)$

$T_d^{(69a)} = T_m\left(0.03432+2.058\frac{\tau_m}{T_m}-1.774\left[\frac{\tau_m}{T_m}\right]^2+0.6878\left[\frac{\tau_m}{T_m}\right]^3\right)$,

$\alpha = 0.6647$, $\beta = 0.8653-0.1277\frac{\tau_m}{T_m}+0.03330\left[\frac{\tau_m}{T_m}\right]^2$

Rule	K_c	T_i	T_d	Comment
Minimum ITAE - Pecharroman and Pagola [134a] - continued	$0.520 K_u$	$0.776 T_u$	$0.087 T_u$	$\alpha = 0.609$, $\beta = 1$, $N = 10$, $\phi_c = -120^0$
	$0.509 K_u$	$0.810 T_u$	$0.068 T_u$	$\alpha = 0.611$, $\beta = 1$, $N = 10$, $\phi_c = -118^0$

Table 69: PID tuning rules - SOSPD model $\frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ or $\frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$ - ideal controller

$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$. *27 tuning rules.*

Rule	K_c	T_i	T_d	Comment
Servo tuning	**Minimum performance index**			
Minimum ITAE – Sung *et al.* [139] *Model: Method 2*	$K_c^{(70)}$	$T_i^{(70)}$	$T_d^{(70)}$	$0.05 < \frac{\tau_m}{T_{m1}} \le 2$
Regulator tuning	**Minimum performance index**			
Minimum ITAE – Sung *et al.* [139] *Model: Method 2*	$K_c^{(71)}$	$T_i^{(71)}$	$T_d^{(71)}$	$0.05 < \frac{\tau_m}{T_{m1}} \le 2$

[1]

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[1] $K_c^{(70)} = \frac{1}{K_m}\left[-0.04 + \left[0.333 + 0.949\left(\frac{\tau_m}{T_{m1}}\right)^{-0.983}\right]\xi_m\right]$, $\xi_m \le 0.9$ or

$$K_c^{(70)} = \frac{1}{K_m}\left[-0.544 + 0.308\frac{\tau_m}{T_{m1}} + 1.408\left(\frac{\tau_m}{T_{m1}}\right)^{-0.832}\xi_m\right],\ \xi_m > 0.9.$$

$$T_i^{(70)} = T_{m1}\left[2.055 + 0.072\frac{\tau_m}{T_{m1}}\right]\xi_m,\ \frac{\tau_m}{T_{m1}} \le 1 \text{ or } T_i^{(70)} = T_{m1}\left[1.768 + 0.329\frac{\tau_m}{T_{m1}}\right]\xi_m,\ \frac{\tau_m}{T_{m1}} > 1$$

$$T_d^{(70)} = \frac{T_{m1}}{\left[1 - e^{-\frac{\left(\frac{\tau_m}{T_{m1}}\right)^{1.060}\xi_m}{0.870}}\right]\left[0.55 + 1.683\left(\frac{T_{m1}}{\tau_m}\right)^{1.090}\right]}$$

$$K_c^{(71)} = \frac{1}{K_m}\left[-0.67 + 0.297\left(\frac{\tau_m}{T_{m1}}\right)^{-2.001} + 2.189\left(\frac{\tau_m}{T_{m1}}\right)^{-0.766}\xi_m\right],\ \frac{\tau_m}{T_{m1}} < 0.9 \text{ or}$$

$$K_c^{(71)} = \frac{1}{K_m}\left[-0.365 + 0.260\left(\frac{\tau_m}{T_{m1}} - 1.4\right)^2 + 2.189\left(\frac{\tau_m}{T_{m1}}\right)^{-0.766}\xi_m\right],\ \frac{\tau_m}{T_{m1}} \ge 0.9$$

$$T_i^{(71)} = T_{m1}\left[2.212\left(\frac{\tau_m}{T_{m1}}\right)^{0.520} - 0.3\right],\ \frac{\tau_m}{T_{m1}} < 0.4 \text{ or}$$

$$T_i^{(71)} = T_{m1}\left\{-0.975 + 0.910\left(\frac{\tau_m}{T_{m1}} - 1.845\right)^2 + \left[1 - e^{-\frac{\xi_m}{0.15 + 0.33\frac{\tau_m}{T_{m1}}}}\right]\left[5.25 - 0.88\left(\frac{\tau_m}{T_{m1}} - 2.8\right)^2\right]\right\},\ \frac{\tau_m}{T_{m1}} \ge 0.4$$

$$T_d^{(71)} = \frac{T_{m1}}{\left[1 - e^{-\frac{\xi_m}{-0.15 + 0.939\left(\frac{\tau_m}{T_{m1}}\right)^{-1.121}}}\right]\left[1.45 + 0.969\left(\frac{T_{m1}}{\tau_m}\right)^{1.171}\right] - 1.9 + 1.576\left(\frac{\tau_m}{T_{m1}}\right)^{-0.530}}$$

Rule	K_c		T_i		T_d	Comment
	ξ_m	τ_m / T_{m1}	K_c	T_i	T_d	
Minimum ITAE – Lopez *et al*. [84] - *taken from plots* *Model: Method 12*	0.5	0.1	$\frac{25}{K_m}$	$0.5T_{m1}$	$0.25T_{m1}$	Representative results
	0.5	1.0	$\frac{0.7}{K_m}$	$1.3T_{m1}$	$1.2T_{m1}$	
	0.5	10.0	$\frac{0.35}{K_m}$	$5T_{m1}$	$1.0T_{m1}$	
	1.0	0.1	$\frac{25}{K_m}$	$0.5T_{m1}$	$0.2T_{m1}$	
	1.0	1.0	$\frac{1.8}{K_m}$	$1.7T_{m1}$	$0.7T_{m1}$	
	4.0	1.0	$\frac{9.0}{K_m}$	$2T_{m1}$	$0.45T_{m1}$	
Ultimate cycle						
Regulator – nearly minimum IAE, ISE, ITAE – Hwang [60] *Model: Method 3*	$K_c^{(72)}$ [2]		$T_i^{(72)}$		$\frac{1.45(1+K_uK_m)}{K_m^2\omega_u}\left(1-\frac{1.16}{\varepsilon_0}\right)\left(1-0.612\omega_u\tau_m+0.103\omega_u^2\tau_m^2\right)$	Decay ratio = 0.15 - $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$ and $0.6 \le \xi_m \le 4.2$; $\varepsilon < 2.4$
	$K_c^{(73)}$		$T_i^{(73)}$		$\frac{1.45(1+K_uK_m)}{K_m^2\omega_u}\left(1-\frac{1.16}{\varepsilon_0}\right)\left(1-0.612\omega_u\tau_m+0.103\omega_u^2\tau_m^2\right)$	Decay ratio = 0.15 - $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$ and $0.6 \le \xi_m \le 4.2$; $2.4 \le \varepsilon < 3$

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[2] $$K_H = \frac{9}{2K_m\tau_m^2}\left[\frac{\tau_m^2}{18} - T_{m1}^2 - \frac{\xi_m\tau_m T_m}{9} + \frac{4K_mT_dK_c\tau_m}{9\omega_u}\right] +$$
$$\frac{9}{2K_m\tau_m^2}\left[\sqrt{T_{m1}^4 + \frac{\tau_m^4}{324} + \frac{49\tau_m^2\xi_m^2T_m^2}{81} - \frac{\tau_m^2T_{m1}^2}{9} + \frac{7T_m\xi_m\tau_m^3}{81} + \frac{10\tau_{m1}T_{m1}^3\xi_m}{9} - K_cT_dK_m\left[\frac{10\tau_m^3}{81} + \frac{4T_{m1}\tau_m(\xi_m\tau_m + T_{m1})}{9}\right] - \frac{8K_c^2K_m^2T_d^2\tau_m^2}{81\omega_u^2}}\right]$$
$$\omega_H = \sqrt{\frac{1+K_HK_m}{T_{m1}^2 + \frac{2\xi_m\tau_mT_m}{3} + \frac{K_HK_m\tau_m^2}{6} - \frac{2K_cK_mT_d\tau_m}{3\omega_u}}}\ ,\ \varepsilon_0 = \frac{6T_{m1}^2 + 4T_{m1}\xi_m\tau_m + K_uK_m\tau_m^2}{2\tau_mT_{m1}^2\omega_u}$$
$$\varepsilon = \frac{6T_{m1}^2 + 4T_{m1}\xi_m\tau_m + K_HK_m\tau_m^2 - 4\tau_mK_mK_cT_d}{(K_mK_cT_d\tau_m^2 + 2\tau_mT_{m1}^2)\omega_H}$$
$$K_c^{(72)} = \left(K_H - \frac{0.674\left[1-0.447\omega_H\tau_m + 0.0607\omega_H^2\tau_m^2\right]}{K_m/(1+K_HK_m)}\right),\ T_i^{(72)} = \frac{K_c^{(72)}(1+K_HK_m)}{\omega_HK_m0.0607(1+1.05\omega_H\tau_m - 0.233\omega_H^2\tau_m^2)}$$
$$K_c^{(73)} = \left(K_H - \frac{0.778\left[1-0.467\omega_H\tau_m + 0.0609\omega_H^2\tau_m^2\right]}{K_m/(1+K_HK_m)}\right),\ T_i^{(73)} = \frac{K_c^{(73)}(1+K_HK_m)}{\omega_HK_m0.0309(1+2.84\omega_H\tau_m - 0.532\omega_H^2\tau_m^2)}$$

Rule	K_c	T_i	T_d	Comment
Regulator – nearly minimum IAE, ISE, ITAE – Hwang [60] – continued Model: Method 3	$K_c^{(74)}$ [3]	$T_i^{(74)}$	$\frac{1.45(1+K_uK_m)}{K_m^2\omega_u}\left(1-\frac{1.16}{\varepsilon_0}\right)\left(1-0.612\omega_u\tau_m+0.103\omega_u^2\tau_m^2\right)$	Decay ratio = 0.15 - $0.2\le\frac{\tau_m}{T_{m1}}\le 2.0$ and $0.6\le\xi_m\le 4.2$; $3\le\varepsilon<20$
	$K_c^{(75)}$	$T_i^{(75)}$	$\frac{1.45(1+K_uK_m)}{K_m^2\omega_u}\left(1-\frac{1.16}{\varepsilon_0}\right)\left(1-0.612\omega_u\tau_m+0.103\omega_u^2\tau_m^2\right)$	Decay ratio = 0.15 - $0.2\le\frac{\tau_m}{T_{m1}}\le 2.0$ and $0.6\le\xi_m\le 4.2$; $\varepsilon\ge 20$
	$K_c^{(76)}$	$T_i^{(76)}$	$\frac{1.45(1+K_uK_m)}{K_m^2\omega_u}\left(1-\frac{1.16}{\varepsilon_0}\right)\left(1-0.612\omega_u\tau_m+0.103\omega_u^2\tau_m^2\right)$	Decay ratio = 0.2 - $0.2\le\frac{\tau_m}{T_{m1}}\le 2.0$ and $0.6\le\xi_m\le 4.2$; $\varepsilon<2.4$
	$K_c^{(77)}$	$T_i^{(77)}$	$\frac{1.45(1+K_uK_m)}{K_m^2\omega_u}\left(1-\frac{1.16}{\varepsilon_0}\right)\left(1-0.612\omega_u\tau_m+0.103\omega_u^2\tau_m^2\right)$	Decay ratio = 0.2 - $0.2\le\frac{\tau_m}{T_{m1}}\le 2.0$ and $0.6\le\xi_m\le 4.2$; $2.4\le\varepsilon<3$
	$K_c^{(78)}$	$T_i^{(78)}$	$\frac{1.45(1+K_uK_m)}{K_m^2\omega_u}\left(1-\frac{1.16}{\varepsilon_0}\right)\left(1-0.612\omega_u\tau_m+0.103\omega_u^2\tau_m^2\right)$	Decay ratio = 0.2 - $0.2\le\frac{\tau_m}{T_{m1}}\le 2.0$ and $0.6\le\xi_m\le 4.2$; $3\le\varepsilon<20$
	$K_c^{(79)}$	$T_i^{(79)}$	$\frac{1.45(1+K_uK_m)}{K_m^2\omega_u}\left(1-\frac{1.16}{\varepsilon_0}\right)\left(1-0.612\omega_u\tau_m+0.103\omega_u^2\tau_m^2\right)$	Decay ratio = 0.2 - $0.2\le\frac{\tau_m}{T_{m1}}\le 2.0$ and $0.6\le\xi_m\le 4.2$; $\varepsilon\ge 20$

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[3] $$K_c^{(74)}=\left(K_H-\frac{1.31(0.519)^{\omega_H\tau_m}\left[1-1.03/\varepsilon+0.514/\varepsilon^2\right]}{K_m/(1+K_HK_m)}\right),\ T_i^{(74)}=\frac{K_c^{(74)}(1+K_HK_m)}{\omega_HK_m0.0603(1+0.929\ln[\omega_H\tau_m])(1+2.01/\varepsilon-1.2/\varepsilon^2)}$$

$$K_c^{(75)}=\left(K_H-\frac{1.14\left[1-0.482\omega_H\tau_m+0.068\omega_H^2\tau_m^2\right]}{K_m/(1+K_HK_m)}\right),\ T_i^{(75)}=\frac{K_c^{(75)}(1+K_HK_m)}{\omega_HK_m0.0694(-1+2.1\omega_H\tau_m-0.367\omega_H^2\tau_m^2)}$$

$$K_c^{(76)}=\left(K_H-\frac{0.622\left[1-0.435\omega_H\tau_m+0.052\omega_H^2\tau_m^2\right]}{K_m/(1+K_HK_m)}\right),\ T_i^{(76)}=\frac{K_c^{(76)}(1+K_HK_m)}{\omega_HK_m0.0697(1+0.752\omega_H\tau_m-0.145\omega_H^2\tau_m^2)}$$

$$K_c^{(77)}=\left(K_H-\frac{0.724\left[1-0.469\omega_H\tau_m+0.0609\omega_H^2\tau_m^2\right]}{K_m/(1+K_HK_m)}\right),\ T_i^{(77)}=\frac{K_c^{(77)}(1+K_HK_m)}{\omega_HK_m0.0405(1+1.93\omega_H\tau_m-0.363\omega_H^2\tau_m^2)}$$

$$K_c^{(78)}=\left(K_H-\frac{1.26(0.506)^{\omega_H\tau_m}\left[1-1.07/\varepsilon+0.616/\varepsilon^2\right]}{K_m/(1+K_HK_m)}\right),\ T_i^{(78)}=\frac{K_c^{(78)}(1+K_HK_m)}{\omega_HK_m0.0661(1+0.824\ln[\omega_H\tau_m])(1+1.71/\varepsilon-1.17/\varepsilon^2)}$$

$$K_c^{(79)}=\left(K_H-\frac{1.09\left[1-0.497\omega_H\tau_m+0.0724\omega_H^2\tau_m^2\right]}{K_m/(1+K_HK_m)}\right),\ T_i^{(79)}=\frac{K_c^{(79)}(1+K_HK_m)}{\omega_HK_m0.054(-1+2.54\omega_H\tau_m-0.457\omega_H^2\tau_m^2)}$$

Rule	K_c	T_i	T_d	Comment
Regulator – nearly minimum IAE, ISE, ITAE – Hwang [60] – continued Model: Method 3	$K_c^{(80)}$ [4]	$T_i^{(80)}$	$\frac{1.45(1+K_uK_m)}{K_m^2\omega_u}\left(1-\frac{1.16}{\varepsilon_0}\right)\left(1-0.612\omega_u\tau_m+0.103\omega_u^2\tau_m^2\right)$	Decay ratio = 0.25 - $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$ and $0.6 \le \xi_m \le 4.2$; $\varepsilon < 2.4$
	$K_c^{(81)}$	$T_i^{(81)}$	$\frac{1.45(1+K_uK_m)}{K_m^2\omega_u}\left(1-\frac{1.16}{\varepsilon_0}\right)\left(1-0.612\omega_u\tau_m+0.103\omega_u^2\tau_m^2\right)$	Decay ratio = 0.25 - $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$ and $0.6 \le \xi_m \le 4.2$; $2.4 \le \varepsilon < 3$
	$K_c^{(82)}$	$T_i^{(82)}$	$\frac{1.45(1+K_uK_m)}{K_m^2\omega_u}\left(1-\frac{1.16}{\varepsilon_0}\right)\left(1-0.612\omega_u\tau_m+0.103\omega_u^2\tau_m^2\right)$	Decay ratio = 0.25 - $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$ and $0.6 \le \xi_m \le 4.2$; $3 \le \varepsilon < 20$
	$K_c^{(83)}$	$T_i^{(83)}$	$\frac{1.45(1+K_uK_m)}{K_m^2\omega_u}\left(1-\frac{1.16}{\varepsilon_0}\right)\left(1-0.612\omega_u\tau_m+0.103\omega_u^2\tau_m^2\right)$	Decay ratio = 0.25 - $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$ and $0.6 \le \xi_m \le 4.2$; $\varepsilon \ge 20$
Servo– nearly minimum IAE, ISE, ITAE – Hwang [60]. In all, decay ratio = 0.1 with $0.2 \le \frac{\tau_m}{T_{m1}} \le 2.0$ and $0.6 \le \xi_m \le 4.2$ *Model: Method 3*	$K_c^{(84)}$	$T_i^{(84)}$	$\frac{0.471K_u}{K_m\omega_u}$	$\xi \le 0.613 + 0.613\frac{\tau_m}{T_{m1}} + 0.117\left(\frac{\tau_m}{T_{m1}}\right)^2$
	$K_c^{(85)}$	$T_i^{(85)}$	$\frac{0.471K_u}{K_m\omega_u}$	$\xi \le 0.613 + 0.613\frac{\tau_m}{T_{m1}} + 0.117\left(\frac{\tau_m}{T_{m1}}\right)^2$

4 $K_c^{(80)} = \left(K_H - \frac{0.584\left[1-0.439\omega_H\tau_m+0.0514\omega_H^2\tau_m^2\right]}{K_m/(1+K_HK_m)}\right)$, $T_i^{(80)} = \frac{K_c^{(80)}(1+K_HK_m)}{\omega_HK_m0.0714\left(1+0.685\omega_H\tau_m-0.131\omega_H^2\tau_m^2\right)}$

$$K_c^{(81)} = \left(K_H - \frac{0.675\left[1-0.472\omega_H\tau_m+0.061\omega_H^2\tau_m^2\right]}{K_m/(1+K_HK_m)}\right),\ T_i^{(81)} = \frac{K_c^{(81)}(1+K_HK_m)}{\omega_HK_m0.0484\left(1+1.43\omega_H\tau_m-0.273\omega_H^2\tau_m^2\right)}$$

$$K_c^{(82)} = \left(K_H - \frac{1.2(0.495)^{\omega_H\tau_m}\left[1-1.1/\varepsilon+0.698/\varepsilon^2\right]}{K_m/(1+K_HK_m)}\right),\ T_i^{(82)} = \frac{K_c^{(82)}(1+K_HK_m)}{\omega_HK_m0.0702\left(1+0.734\ln[\omega_H\tau_m]\right)\left(1+1.48/\varepsilon-1.1/\varepsilon^2\right)}$$

$$K_c^{(83)} = \left(K_H - \frac{1.03\left[1-0.51\omega_H\tau_m+0.0759\omega_H^2\tau_m^2\right]}{K_m/(1+K_HK_m)}\right),\ T_i^{(83)} = \frac{K_c^{(83)}(1+K_HK_m)}{\omega_HK_m0.0386\left(-1+3.26\omega_H\tau_m-0.6\omega_H^2\tau_m^2\right)}$$

$$K_c^{(84)} = \left(K_H - \frac{0.822\left[1-0.549\omega_H\tau_m+0.112\omega_H^2\tau_m^2\right]}{K_m/(1+K_HK_m)}\right),\ T_i^{(84)} = \frac{K_c^{(84)}(1+K_HK_m)}{\omega_HK_m0.0142\left(1+6.96\omega_H\tau_m-1.77\omega_H^2\tau_m^2\right)}$$

$$K_c^{(85)} = \left(K_H - \frac{0.786\left[1-0.441\omega_H\tau_m+0.0569\omega_H^2\tau_m^2\right]}{K_m/(1+K_HK_m)}\right),\ T_i^{(85)} = \frac{K_c^{(85)}(1+K_HK_m)}{\omega_HK_m0.0172\left(1+4.62\omega_H\tau_m-0.823\omega_H^2\tau_m^2\right)}$$

Rule	K_c	T_i	T_d	Comment
Servo– nearly minimum IAE, ISE, ITAE – Hwang [60] (continued)	$K_c^{(86)}$ [5]	$T_i^{(86)}$	$\frac{0.471K_u}{K_m\omega_u}$	$\xi \le 0.613 + 0.613\frac{\tau_m}{T_{m1}} + 0.117\left(\frac{\tau_m}{T_{m1}}\right)^2$
	$K_c^{(87)}$	$T_i^{(87)}$	$\frac{0.471K_u}{K_m\omega_u}$	$\xi \le 0.613 + 0.613\frac{\tau_m}{T_{m1}} + 0.117\left(\frac{\tau_m}{T_{m1}}\right)^2$
Servo – nearly minimum IAE, ISE, ITAE – Hwang [60] [6] *Model: Method 3*	$K_c^{(88)}$	$T_i^{(88)}$	$\frac{0.471K_u}{K_m\omega_u}$	$\xi > 0.649 + 0.58\frac{\tau_m}{T_{m1}} - 0.005\left(\frac{\tau_m}{T_{m1}}\right)^2$
	$K_c^{(89)}$	$T_i^{(89)}$	$\frac{0.471K_u}{K_m\omega_u}$	$\xi > 0.649 + 0.58\frac{\tau_m}{T_{m1}} - 0.005\left(\frac{\tau_m}{T_{m1}}\right)^2$
	$K_c^{(90)}$	$T_i^{(90)}$	$\frac{0.471K_u}{K_m\omega_u}$	$\xi > 0.649 + 0.58\frac{\tau_m}{T_{m1}} - 0.005\left(\frac{\tau_m}{T_{m1}}\right)^2$
	$K_c^{(91)}$	$T_i^{(91)}$	$\frac{0.471K_u}{K_m\omega_u}$	$\xi > 0.649 + 0.58\frac{\tau_m}{T_{m1}} - 0.005\left(\frac{\tau_m}{T_{m1}}\right)^2$
	$K_c^{(92)}$	$T_i^{(92)}$	$\frac{0.471K_u}{K_m\omega_u}$	$\xi \le 0.649 + 0.58\frac{\tau_m}{T_{m1}} - 0.005\left(\frac{\tau_m}{T_{m1}}\right)^2$, $\xi > 0.613 + 0.613\frac{\tau_m}{T_{m1}} + 0.117\left(\frac{\tau_m}{T_{m1}}\right)^2$
	$K_c^{(93)}$	$T_i^{(93)}$	$\frac{0.471K_u}{K_m\omega_u}$	$\xi \le 0.649 + 0.58\frac{\tau_m}{T_{m1}} - 0.005\left(\frac{\tau_m}{T_{m1}}\right)^2$, $\xi > 0.613 + 0.613\frac{\tau_m}{T_{m1}} + 0.117\left(\frac{\tau_m}{T_{m1}}\right)^2$

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[5] $$K_c^{(86)} = \left(K_H - \frac{1.28(0.542)^{\omega_H\tau_m}\left[1 - 0.986/\varepsilon + 0.558/\varepsilon^2\right]}{K_m/(1+K_HK_m)}\right),\ T_i^{(86)} = \frac{K_c^{(86)}(1+K_HK_m)}{\omega_HK_m0.0476(1+0.996\ln[\omega_H\tau_m])(1+2.13/\varepsilon - 1.13/\varepsilon^2)}$$

$$K_c^{(87)} = \left(K_H - \frac{1.14\left[1 - 0.466\omega_H\tau_m + 0.0647\omega_H{}^2\tau_m{}^2\right]}{K_m/(1+K_HK_m)}\right),\ T_i^{(87)} = \frac{K_c^{(87)}(1+K_HK_m)}{\omega_HK_m0.0609(-1+1.97\omega_H\tau_m - 0.323\omega_H{}^2\tau_m{}^2)}$$

[6] $$K_c^{(88)} = \left(K_H - \frac{0.794\left[1 - 0.541\omega_H\tau_m + 0.126\omega_H{}^2\tau_m{}^2\right]}{K_m/(1+K_HK_m)}\right),\ T_i^{(88)} = \frac{K_c^{(88)}(1+K_HK_m)}{\omega_HK_m0.0078(1+8.38\omega_H\tau_m - 1.97\omega_H{}^2\tau_m{}^2)}$$

$$K_c^{(89)} = \left(K_H - \frac{0.738\left[1 - 0.415\omega_H\tau_m + 0.0575\omega_H{}^2\tau_m{}^2\right]}{K_m/(1+K_HK_m)}\right),\ T_i^{(89)} = \frac{K_c^{(89)}(1+K_HK_m)}{\omega_HK_m0.0124(1+4.05\omega_H\tau_m - 0.63\omega_H{}^2\tau_m{}^2)}$$

$$K_c^{(90)} = \left(K_H - \frac{1.15(0.564)^{\omega_H\tau_m}\left[1 - 0.959/\varepsilon + 0.773/\varepsilon^2\right]}{K_m/(1+K_HK_m)}\right),\ T_i^{(90)} = \frac{K_c^{(90)}(1+K_HK_m)}{\omega_HK_m0.0335(1+0.947\ln[\omega_H\tau_m])(1+1.9/\varepsilon - 1.07/\varepsilon^2)}$$

$$K_c^{(91)} = \left(K_H - \frac{1.07\left[1 - 0.466\omega_H\tau_m + 0.0667\omega_H{}^2\tau_m{}^2\right]}{K_m/(1+K_HK_m)}\right),\ T_i^{(91)} = \frac{K_c^{(91)}(1+K_HK_m)}{\omega_HK_m0.0328(-1+2.21\omega_H\tau_m - 0.338\omega_H{}^2\tau_m{}^2)}$$

$$K_c^{(92)} = \left(K_H - \frac{0.789\left[1 - 0.527\omega_H\tau_m + 0.11\omega_H{}^2\tau_m{}^2\right]}{K_m/(1+K_HK_m)}\right),\ T_i^{(92)} = \frac{K_c^{(92)}(1+K_HK_m)}{\omega_HK_m0.009(1+9.7\omega_H\tau_m - 2.4\omega_H{}^2\tau_m{}^2)}$$

$$K_c^{(93)} = \left(K_H - \frac{0.76\left[1 - 0.426\omega_H\tau_m + 0.0551\omega_H{}^2\tau_m{}^2\right]}{K_m/(1+K_HK_m)}\right),\ T_i^{(93)} = \frac{K_c^{(93)}(1+K_HK_m)}{\omega_HK_m0.0153(1+4.37\omega_H\tau_m - 0.743\omega_H{}^2\tau_m{}^2)}$$

Rule	K_c	T_i	T_d	Comment
Servo – nearly minimum IAE, ISE, ITAE – Hwang [60] (continued)	$K_c^{(94)}$ [7]	$T_i^{(94)}$	$\frac{0.471K_u}{K_m\omega_u}$	$\xi \le 0.649 + 0.58\frac{\tau_m}{T_{m1}} - 0.005\left(\frac{\tau_m}{T_{m1}}\right)^2$, $\xi > 0.613 + 0.613\frac{\tau_m}{T_{m1}} + 0.117\left(\frac{\tau_m}{T_{m1}}\right)^2$
	$K_c^{(95)}$	$T_i^{(95)}$	$\frac{0.471K_u}{K_m\omega_u}$	$\xi \le 0.649 + 0.58\frac{\tau_m}{T_{m1}} - 0.005\left(\frac{\tau_m}{T_{m1}}\right)^2$, $\xi > 0.613 + 0.613\frac{\tau_m}{T_{m1}} + 0.117\left(\frac{\tau_m}{T_{m1}}\right)^2$
Minimum IAE regulator – ultimate cycle - Shinskey [16] – *page 151*. *Model: Method 10.*	$0.6173K_u$	$0.38T_u$	$0.15T_u$	$T_{m2}/T_{m2} + \tau_m = 0.25$
	$0.6766K_u$	$0.33T_u$	$0.19T_u$	$T_{m2}/T_{m2} + \tau_m = 0.5$
	$0.7874K_u$	$0.26T_u$	$0.21T_u$	$T_{m2}/T_{m2} + \tau_m = 0.75$
Direct synthesis				
Gain and phase margin – Hang *et al*. [35] *Model: Method 4*	$\frac{\pi T_{m1}}{A_m K_m \tau_m}$,	$2T_{m1}$,	$0.5T_{m1}$	Sample A_m, ϕ_m provided. Model has a repeated pole (T_{m1})
	$A_m = 1.5, \phi_m = 30^o$ $A_m = 2.0, \phi_m = 45^o$	$A_m = 3.0, \phi_m = 60^o$ $A_m = 4.0, \phi_m = 67.5^o$	$A_m = 5.0, \phi_m = 72^o$	
Gain and phase margin – Ho *et al*. [140] *Model: Method 1*	$\frac{\omega_p T_{m1}}{A_m K_m}$	$\frac{1}{2\omega_p - \frac{4\omega_p^2\tau_m}{\pi} + \frac{1}{T_{m1}}}$	T_{m2}	$T_{m1} > T_{m2}$. Sample A_m, ϕ_m provided $\omega_p = \frac{A_m\phi_m + 0.5\pi A_m(A_m - 1)}{(A_m^2 - 1)\tau_m}$
	$A_m = 2.0, \phi_m = 45^o$	$A_m = 3.0, \phi_m = 60^o$	$A_m = 4.0, \phi_m = 70^o$	
Gain and phase margin – Ho *et al*. [141] *Model: Method 1*	$K_c^{(96)}$	$T_i^{(96)}$	$T_d^{(96)}$	$2\xi_m \ge \frac{\tau_m}{T_m}$, $\frac{\tau_m}{T_m} \le 1$ Sample A_m, ϕ_m provided *
	$A_m = 2.0, \phi_m = 45^o$	$A_m = 3.0, \phi_m = 60^o$	$A_m = 4.0, \phi_m = 70^o$	
Gain and phase margin – Ho *et al*. [142] *Model: Method 1*	$K_c^{(97)}$	$T_i^{(97)}$	$T_d^{(97)}$	$\omega_n\tau_m < 2\xi_m$; Sample A_m, ϕ_m provided
	$A_m = 2.0, \phi_m = 45^o$	$A_m = 3.0, \phi_m = 60^o$	$A_m = 4.0, \phi_m = 60^o$	

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[7] $K_c^{(94)} = \left(K_H - \frac{1.22(0.55)^{\omega_H\tau_m}\left[1 - 0.978/\varepsilon + 0.659/\varepsilon^2\right]}{K_m/(1 + K_H K_m)}\right)$, $T_i^{(94)} = \frac{K_c^{(94)}(1 + K_H K_m)}{\omega_H K_m 0.0421(1 + 0.969\ln[\omega_H\tau_m])(1 + 2.02/\varepsilon - 1.11/\varepsilon^2)}$

$K_c^{(95)} = \left(K_H - \frac{1.11\left[1 - 0.467\omega_H\tau_m + 0.0657\omega_H^2\tau_m^2\right]}{K_m/(1 + K_H K_m)}\right)$, $T_i^{(95)} = \frac{K_c^{(95)}(1 + K_H K_m)}{\omega_H K_m 0.0477(-1 + 2.07\omega_H\tau_m - 0.333\omega_H^2\tau_m^2)}$

$K_c^{(96)} = \frac{2}{\pi A_m K_m}\left(\pi\xi_m + \pi - 2\frac{\tau_m}{T_{m1}}\right)$, $T_i^{(96)} = \frac{2}{\pi}T_{m1}(\pi\xi_m + \pi - 2\tau_m)$, $T_d^{(96)} = \frac{\pi T_{m1}^2}{2(\pi\xi_m T_{m1} + \pi T_{m1} - 2\tau_m)}$

$K_c^{(97)} = \frac{2\omega_p T_{m1}^2}{\pi A_m}\left(\frac{\pi\xi_m}{\omega_p T_{m1}} + \pi - 2\omega_p\tau_m\right)$, $T_i^{(97)} = \frac{2}{\pi}T_{m1}^2\left(\frac{\pi\xi_m}{T_{m1}\omega_p} + \pi - 2\omega_p\tau_m\right)$,

$T_d^{(97)} = \frac{\pi T_{m1}}{2\left(\frac{\pi\xi_m}{\omega_p} + \pi T_{m1} - 2T_{m1}\omega_p\tau_m\right)}$

$$* \phi_m < \frac{\left(\pi + \sqrt{\pi^2 + \frac{8\pi\tau_m\xi_m}{T_{m1}}}\right)(A_m^2 - 1) - 2\pi A_m(A_m - 1)}{4A_m}$$

Rule	K_c	T_i	T_d	Comment
Wang *et al*. [143] *Model: Method 8*	$\frac{\xi_m T_{m1}}{K_m \tau_m}$	$2\xi_m T_{m1}$	$\frac{0.5T_{m1}}{\xi_m}$	$\xi_m > 0.7071$ or $0.05 < \frac{0.7071\tau_m}{T_{m1}\sqrt{2\xi_m^2 - 1}} < 0.15$ $\frac{0.7071\tau_m}{T_{m1}\sqrt{2\xi_m^2 - 1}} > 1, \xi_m \geq 1$ or $0.05 < \frac{\xi_m \tau_m}{T_{m1}} < 0.15$, $\frac{\xi_m \tau_m}{T_{m1}} > 1, \xi_m < 1$
	Minimum of $\frac{2T_{m1}^2}{K_m} e^{-\frac{T_m \tau_m}{\xi_m}}, \frac{0.7358\xi_m T_m}{K_m \tau_m}$	$2\xi_m T_{m1}$	$\frac{0.5T_{m1}}{\xi_m}$	$\xi_m \leq 0.7071$ and $0.15 \leq \frac{\xi_m \tau_m}{T_{m1}} \leq 1$
Gain and phase margin – Leva *et al*. [34] *Model: Method 5*	$K_c^{(98)}$ [8]	$T_i^{(98)}$	$0.25T_i^{(98)}$	$\xi_m \leq 1$ **or** $\xi_m > 1$ with $0.5\pi - \phi_m >$ $\frac{3\tau_m}{T_{m1}}\left[\xi_m + \sqrt{\xi_m^2 - 1}\right]$
	$K_c^{(99)}$	$T_i^{(99)}$	$T_d^{(99)}$	$\xi_m > 1$ with $\frac{3\tau_m}{T_{m1}}\left[\xi_m - \sqrt{\xi_m^2 - 1}\right]$ $< 0.5\pi - \phi_m \leq$ $\frac{3\tau_m}{T_{m1}}\left[\xi_m + \sqrt{\xi_m^2 - 1}\right]$, $\phi_m = 70^0$ at least

[8] $K_c^{(98)} = \frac{\omega_{cn} T_i}{K_m}\sqrt{\frac{\left(1 + \omega_{cn}^2 T_{m1}^2\right)^2 + 4\xi_m^2 \omega_{cn}^2 T_{m1}^2}{\left(1 - T_i T_d \omega_{cn}^2\right)^2 + T_i^2 \omega_{cn}^2}}$, $\omega_{cn} = \frac{1}{\tau_m}\left[4.07 - \phi_m + \tan^{-1}\left\{\frac{2\xi_m \tau_m T_{m1}(0.5\pi - \phi_m)}{(0.5\pi - \phi_m)^2 T_{m1}^2 - \tau_m^2}\right\}\right]$

$T_i^{(98)} = \frac{2}{\omega_{cn}}\tan\left[0.5\left\{\omega_{cn}\tau_m + \phi_m - 0.5\pi - \tan^{-1}\left(\frac{2\xi_m \omega_{cn} T_{m1}}{\omega_{cn}^2 T_{m1}^2 - 1}\right)\right\}\right]$, $K_c^{(99)} = \frac{\omega_{cn} T_{m1}^2}{K_m T_d^{(99)}}\sqrt{\frac{\omega_{cn}^2 + \frac{1}{T_{m1}^2}\left(2\xi_m\sqrt{\xi_m^2 - 1} - 1\right)}{\omega_{cn}^2 + z^2}}$,

$z = \frac{\omega_{cn}}{\tan\left[\phi_m - 0.5\pi + \omega_{cn}\tau_m + \tan^{-1}\left\{\frac{\omega_{cn} T_{m1}}{\left(\xi_m + \sqrt{\xi_m^2 - 1}\right)}\right\}\right]}$, $T_i^{(99)} = \frac{T_{m1} z + \left(\xi_m + \sqrt{\xi_m^2 - 1}\right)}{z\left(\xi_m + \sqrt{\xi_m^2 - 1}\right)}$, $T_d^{(99)} = \frac{T_{m1}}{T_{m1} z + \left(\xi_m + \sqrt{\xi_m^2 - 1}\right)}$

Rule	K_c	T_i	T_d	Comment
Gain and phase margin – Wang and Shao [144] *Model: Method 8* *Authors quote tuning rule for* $A_m = 3$, $\phi_m = 60^0$; *other tuning rules obtained using authors method*	$1.047\frac{\xi_m T_{m1}}{K_m \tau_m}$	$2\xi_m T_{m1}$	$0.5\frac{T_{m1}}{\xi_m}$	$A_m = 3$, $\phi_m = 60^0$
	$2.094\frac{\xi_m T_{m1}}{K_m \tau_m}$	$2\xi_m T_{m1}$	$0.5\frac{T_{m1}}{\xi_m}$	$A_m = 1.5$, $\phi_m = 30^0$
	$1.571\frac{\xi_m T_{m1}}{K_m \tau_m}$	$2\xi_m T_{m1}$	$0.5\frac{T_{m1}}{\xi_m}$	$A_m = 2$, $\phi_m = 45^0$
	$0.785\frac{\xi_m T_{m1}}{K_m \tau_m}$	$2\xi_m T_{m1}$	$0.5\frac{T_{m1}}{\xi_m}$	$A_m = 4$, $\phi_m = 67.5^0$
	$0.628\frac{\xi_m T_{m1}}{K_m \tau_m}$	$2\xi_m T_{m1}$	$0.5\frac{T_{m1}}{\xi_m}$	$A_m = 5$, $\phi_m = 72^0$
Pemberton [145] *Model: Method 1*	$\frac{2(T_{m1} + T_{m2})}{3K_m \tau_m}$	$T_{m1} + T_{m2}$	$\frac{T_{m1}T_{m2}}{T_{m1} + T_{m2}}$	
Pemberton [24] *Model: Method 1*	$\frac{(T_{m1} + T_{m2})}{K_m \tau_m}$	$T_{m1} + T_{m2}$	$\frac{T_{m1}T_{m2}}{T_{m1} + T_{m2}}$	$0.1 \le \frac{\tau_m}{T_{m1}} \le 1.0$; $0.2 \le \frac{\tau_m}{T_{m2}} \le 1.0$
Pemberton [145] *Model: Method 1*	$\frac{2(T_{m1} + T_{m2})}{3K_m \tau_m}$	$T_{m1} + T_{m2}$	$\frac{T_{m1} + T_{m2}}{4}$	$0.1 \le \frac{\tau_m}{T_{m1}} \le 1.0$ $0.2 \le \frac{\tau_m}{T_{m2}} \le 1.0$
Suyama [100] *Model: Method 1*	$\frac{T_{m1} + T_{m2}}{2K_m \tau_m}$	$T_{m1} + T_{m2}$	$\frac{T_{m1}T_{m2}}{T_{m1} + T_{m2}}$	
Smith *et al*. [146] *Model: Method not stated*	$\frac{T_{m1} + T_{m2}}{K_m(\lambda + \tau_m)}$	$T_{m1} + T_{m2}$	$\frac{T_{m1}T_{m2}}{T_{m1} + T_{m2}}$	
Chiu *et al*. [29] *Model: Method 6*	$\frac{\lambda T_{m1} + T_{m2}}{K_m(1 + \lambda\tau_m)}$	$T_{m1} + T_{m2}$	$\frac{T_{m1}T_{m2}}{T_{m1} + T_{m2}}$	λ variable; suggested values are 0.2, 0.4, 0.6 and 1.0.
Wang and Clemens [147] *Model: Method 9*	$\frac{\lambda T_{m1}}{K_m(1 + \lambda\tau_m)}$	T_{m1}	T_{m2}	$T_{m1} > T_{m2}$. λ = pole of specified closed loop overdamped response.
	$\frac{2\lambda\xi_m T_{m1}}{K_m(1 + \lambda\tau_m)}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	Underdamped response
Gorez and Klan [147a] *Model: Ideal*	$\frac{2\xi_m T_{m1}}{K_m(2\xi_m T_{m1} + \tau_m)}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	Non-dominant time delay
Miluse *et al*. [27a] *Model: Method not specified* Overdamped process; $T_{m1} > T_{m2}$	$\frac{0.368(T_{m1} + T_{m2})}{K_m \tau_m}$	$T_{m1} + T_{m2}$	$\frac{T_{m1}T_{m2}}{T_{m1} + T_{m2}}$	Closed loop overshoot = 0%
	$\frac{0.514(T_{m1} + T_{m2})}{K_m \tau_m}$	$T_{m1} + T_{m2}$	$\frac{T_{m1}T_{m2}}{T_{m1} + T_{m2}}$	Closed loop overshoot = 5%
	$\frac{0.581(T_{m1} + T_{m2})}{K_m \tau_m}$	$T_{m1} + T_{m2}$	$\frac{T_{m1}T_{m2}}{T_{m1} + T_{m2}}$	Closed loop overshoot = 10%
	$\frac{0.641(T_{m1} + T_{m2})}{K_m \tau_m}$	$T_{m1} + T_{m2}$	$\frac{T_{m1}T_{m2}}{T_{m1} + T_{m2}}$	Closed loop overshoot = 15%

Rule	K_c	T_i	T_d	Comment
Miluse *et al*. [27a] (continued)	$\frac{0.696(T_{m1}+T_{m2})}{K_m\tau_m}$	$T_{m1}+T_{m2}$	$\frac{T_{m1}T_{m2}}{T_{m1}+T_{m2}}$	Closed loop overshoot = 20%
	$\frac{0.748(T_{m1}+T_{m2})}{K_m\tau_m}$	$T_{m1}+T_{m2}$	$\frac{T_{m1}T_{m2}}{T_{m1}+T_{m2}}$	Closed loop overshoot = 25%
	$\frac{0.801(T_{m1}+T_{m2})}{K_m\tau_m}$	$T_{m1}+T_{m2}$	$\frac{T_{m1}T_{m2}}{T_{m1}+T_{m2}}$	Closed loop overshoot = 30%
	$\frac{0.853(T_{m1}+T_{m2})}{K_m\tau_m}$	$T_{m1}+T_{m2}$	$\frac{T_{m1}T_{m2}}{T_{m1}+T_{m2}}$	Closed loop overshoot = 35%
	$\frac{0.906(T_{m1}+T_{m2})}{K_m\tau_m}$	$T_{m1}+T_{m2}$	$\frac{T_{m1}T_{m2}}{T_{m1}+T_{m2}}$	Closed loop overshoot = 40%
	$\frac{0.957(T_{m1}+T_{m2})}{K_m\tau_m}$	$T_{m1}+T_{m2}$	$\frac{T_{m1}T_{m2}}{T_{m1}+T_{m2}}$	Closed loop overshoot = 45%
	$\frac{1.008(T_{m1}+T_{m2})}{K_m\tau_m}$	$T_{m1}+T_{m2}$	$\frac{T_{m1}T_{m2}}{T_{m1}+T_{m2}}$	Closed loop overshoot = 50%
Miluse *et al*. [27a] *Model: Method not specified* Underdamped process; $0.5<\xi_m\le 1$	$\frac{0.736\xi_m T_{m1}}{K_m\tau_m}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	Closed loop overshoot = 0%
	$\frac{1.028\xi_m T_{m1}}{K_m\tau_m}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	Closed loop overshoot = 5%
	$\frac{1.162\xi_m T_{m1}}{K_m\tau_m}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	Closed loop overshoot = 10%
	$\frac{1.282\xi_m T_{m1}}{K_m\tau_m}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	Closed loop overshoot = 15%
	$\frac{1.392\xi_m T_{m1}}{K_m\tau_m}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	Closed loop overshoot = 20%
	$\frac{1.496\xi_m T_{m1}}{K_m\tau_m}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	Closed loop overshoot = 25%
	$\frac{1.602\xi_m T_{m1}}{K_m\tau_m}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	Closed loop overshoot = 30%
	$\frac{1.706\xi_m T_{m1}}{K_m\tau_m}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	Closed loop overshoot = 35%
	$\frac{1.812\xi_m T_{m1}}{K_m\tau_m}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	Closed loop overshoot = 40%
	$\frac{1.914\xi_m T_{m1}}{K_m\tau_m}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	Closed loop overshoot = 45%
	$\frac{2.016\xi_m T_{m1}}{K_m\tau_m}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	Closed loop overshoot = 50%
Miluse *et al*. [27b] *Model: Method 14*	$\frac{0.736 T_{m1}}{K_m\tau_m}$	$2T_{m1}$	$0.5T_{m1}$	$T_{m1}=T_{m2}$
Seki *et al*. [147b] *Model: Method not specified*	$\lvert G_c(j\omega_0)\rvert \frac{\omega_c^{\,2}}{\omega_0^{\,2}}\cos\theta_c$	$\frac{\tan\theta_c+\sqrt{2+\tan^2\theta_c}}{\omega_c}$	$0.5T_i$	'Re-tuning' rule. ω_0 = freq. when controlled system goes unstable (crossover freq.); ω_c = new crossover freq. ; θ_c = phase lag

Rule	K_c	T_i	T_d	Comment
Autotuning – Landau and Voda [148] *Model: Method not relevant*	$\frac{4+\beta}{4}\frac{\beta}{2\sqrt{2}\left\lvert G_p(j\omega_{135^0})\right\rvert}$	$\frac{4+\beta}{\beta}\frac{1}{\omega_{135^0}}$	$\frac{4}{4+\beta}\frac{1}{\omega_{135^0}}$	$1 \le \beta \le 2$; $\tau_m < 0.25T_{m1}$
	$\frac{3}{\left\lvert G_p(j\omega_u)\right\rvert}$	$\frac{3.2}{\omega_u}$	$\frac{0.8}{\omega_u}$	$\omega_u\tau_m \le 0.16$
	$\frac{1.9}{\left\lvert G_p(j\omega_u)\right\rvert}$	$\frac{4}{\omega_u}$	$\frac{1}{\omega_u}$	$0.16 < \omega_u\tau_m \le 0.2$
Robust				
Brambilla *et al.* [48] – *values deduced from graph* *Model: Method 1*	$\frac{T_{m1}+T_{m2}+0.5\tau_m}{K_m\tau_m(2\lambda+1)}$	$T_{m1}+T_{m2}+0.5\tau_m$	$\frac{T_{m1}T_{m2}+0.5(T_{m1}+T_{m2})\tau_m}{T_{m1}+T_{m2}+0.5\tau_m}$	λ varies graphically with $\tau_m/(T_{m1}+T_{m2})$ $0.1 \le \tau_m/(T_{m1}+T_{m2}) \le 10$
	$\tau_m/(T_{m1}+T_{m2})$, λ	$\tau_m/(T_{m1}+T_{m2})$, λ	$\tau_m/(T_{m1}+T_{m2})$, λ	
	0.1, 0.50	1.0, 0.29	10.0, 0.14	
	0.2, 0.47	2.0, 0.22		
	0.5, 0.39	5.0, 0.16		
Chen *et al.* [53] *Model: Method 1*	$\frac{1.00\xi_m T_m}{\tau_m K_m}$	$2\xi_m\tau_m$	$\frac{\tau_m}{2\xi_m}$	$A_m = 3.14$, $\phi_m = 61.4^0$, $M_s = 1.00$
	$\frac{1.22\xi_m T_m}{\tau_m K_m}$	$2\xi_m\tau_m$	$\frac{\tau_m}{2\xi_m}$	$A_m = 2.58$, $\phi_m = 55.0^0$, $M_s = 1.10$
	$\frac{1.34\xi_m T_m}{\tau_m K_m}$	$2\xi_m\tau_m$	$\frac{\tau_m}{2\xi_m}$	$A_m = 2.34$, $\phi_m = 51.6^0$, $M_s = 1.20$
	$\frac{1.40\xi_m T_m}{\tau_m K_m}$	$2\xi_m\tau_m$	$\frac{\tau_m}{2\xi_m}$	$A_m = 2.24$, $\phi_m = 50.0^0$, $M_s = 1.26$
	$\frac{1.44\xi_m T_m}{\tau_m K_m}$	$2\xi_m\tau_m$	$\frac{\tau_m}{2\xi_m}$	$A_m = 2.18$, $\phi_m = 48.7^0$, $M_s = 1.30$
	$\frac{1.52\xi_m T_m}{\tau_m K_m}$	$2\xi_m\tau_m$	$\frac{\tau_m}{2\xi_m}$	$A_m = 2.07$, $\phi_m = 46.5^0$, $M_s = 1.40$
	$\frac{1.60\xi_m T_m}{\tau_m K_m}$	$2\xi_m\tau_m$	$\frac{\tau_m}{2\xi_m}$	$A_m = 1.96$, $\phi_m = 44.1^0$, $M_s = 1.50$
Lee *et al.* [55] *Model: Method 1*	$\frac{T_i}{K_m(2\lambda+\tau_m)}$	$2\xi_m T_{m1} - \frac{2\lambda^2-\tau_m{}^2}{2(2\lambda+\tau_m)}$	$T_i - 2\xi_m T_{m1} + \frac{T_{m1}{}^2}{T_i} - \frac{\tau_m{}^3}{6T_i(2\lambda+\tau_m)}$	Desired closed loop response = $\frac{e^{-\tau_m s}}{(\lambda s+1)^2}$

Rule	K_c	T_i	T_d	Comment
Lee *et al*. [55] (continued)	$\frac{T_i}{K_m(2\lambda + \tau_m)}$	$T_{m1} + T_{m2} - \frac{2\lambda^2 - \tau_m^{\;2}}{2(2\lambda + \tau_m)}$	$T_i - T_{m1} - T_{m2} + \frac{T_{m1}T_{m2}}{T_i} - \frac{\tau_m^{\;3}}{6T_i(2\lambda + \tau_m)}$	Desired closed loop response = $\frac{e^{-\tau_m s}}{(\lambda s + 1)^2}$
	$\frac{T_i}{K_m(\lambda + \tau_m)}$	$2\xi_m T_{m1} + \frac{\tau_m^{\;2}}{2(\lambda + \tau_m)}$	$T_i - 2\xi_m T_{m1}\left(\frac{T_{m1}^{\;2} - \frac{\tau_m^{\;3}}{6(\lambda + \tau_m)}}{T_i}\right)$	Desired closed loop response = $\frac{e^{-\tau_m s}}{(\lambda s + 1)}$
	$\frac{T_i}{K_m(\lambda + \tau_m)}$	$T_{m1} + T_{m2} + \frac{\tau_m^{\;2}}{2(\lambda + \tau_m)}$	$T_i - (T_{m1} + T_{m2})\left(\frac{T_{m1}T_{m2} - \frac{\tau_m^{\;3}}{6(\lambda + \tau_m)}}{T_i}\right)$	Desired closed loop response = $\frac{e^{-\tau_m s}}{(\lambda s + 1)}$

Table 70: PID tuning rules - SOSPD model $\frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ or $\frac{K_m e^{-s\tau_m}}{T_{m1}{}^2 s^2 + 2\xi_m T_{m1}s + 1}$ - filtered controller

$G_c(s) = K_c\left(1+\frac{1}{T_i s}+T_d s\right)\frac{1}{T_f s+1}$. *1 tuning rule.*

Rule	K_c	T_i	T_d	Comment
Robust				
Hang *et al*. [35] *Model: Method 4*	$\frac{2T_{m1}+\tau_m}{2(\lambda+\tau_m)K_m}$	$T_{m1}+0.5\tau_m$	$\frac{T_{m1}\tau_m}{2T_{m1}+\tau_m}$	Model has a repeated pole (T_{m1}). $\lambda > 0.25\tau_m$. $T_f = \frac{\lambda\tau_m}{2(\lambda+\tau_m)}$

Table 71: PID tuning rules - SOSPD model $\frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ or $\frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1}s + 1}$ - filtered controller

$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)\frac{b_1 s + 1}{a_1 s + 1}$. *1 tuning rule.*

Rule	K_c	T_i	T_d	Comment
Robust				
Jahanmiri and Fallahi [149] *Model: Method 6*	$\frac{2\xi_m T_{m1}}{K_m(\tau_m + \lambda)}$ $\lambda = 0.25\tau_m + 0.1\xi_m T_{m1}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	$b_1 = 0.5\tau_m$ $a_1 = \frac{\lambda\tau_m}{2(\tau_m + \lambda)}$

Table 72: PID tuning rules - SOSPD model $\frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ or $\frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$ - classical controller

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)\left(\frac{1 + T_d s}{1 + \frac{T_d s}{N}}\right)$$. *7 tuning rules.*

Rule	K_c	T_i	T_d	Comment
Regulator tuning	**Minimum performance index**			
Minimum IAE - Shinskey [59] – *page 159* *Model: Open loop method not specified*	[1] $K_c^{(100)}$	$\tau_m\left(1.5 - e^{-\frac{T_{m1}}{1.5\tau_m}}\right)\left(1 + 0.9\left[1 - e^{-\frac{T_{m2}}{\tau_m}}\right]\right)$	$0.56\tau_m\left(1 - e^{-\frac{1.2T_{m1}}{\tau_m}}\right) + 0.6T_{m2}$	$\frac{T_{m2}}{\tau_m} \le 3$
	$\frac{2.5T_{m1}}{K_m\tau_m}$	$\tau_m + 0.2T_{m2}$	$\tau_m + 0.2T_{m2}$	$\frac{T_{m2}}{\tau_m} > 3$
** Minimum IAE - Shinskey [59]	$\frac{0.800T_{m1}}{K_m\tau_m}$	$1.5(T_{m2} + \tau_m)$	$0.60(T_{m2} + \tau_m)$	$\frac{T_{m2}}{T_{m2} + \tau_m} = 0.25$
	$\frac{0.770T_{m1}}{K_m\tau_m}$	$1.2(T_{m2} + \tau_m)$	$0.70(T_{m2} + \tau_m)$	$\frac{T_{m2}}{T_{m2} + \tau_m} = 0.5$
	$\frac{0.833T_{m1}}{K_m\tau_m}$	$0.75(T_{m2} + \tau_m)$	$0.60(T_{m2} + \tau_m)$	$\frac{T_{m2}}{T_{m2} + \tau_m} = 0.75$
** Minimum IAE - Shinskey [17]	$0.59K_u$	$0.36T_u$	$0.26T_u$	$\frac{\tau_m}{T_{m1}} = 0.2$, $\frac{T_{m2}}{T_{m1}} = 0.2$
** Minimum IAE Shinskey [17]	$\frac{0.85T_{m1}}{K_m\tau_m}$	$1.98\tau_m$	$0.86\tau_m$	$\frac{\tau_m}{T_{m1}} = 0.2$, $\frac{T_{m2}}{T_{m1}} = 0.1$
	$\frac{0.87T_{m1}}{K_m\tau_m}$	$2.30\tau_m$	$1.65\tau_m$	$\frac{T_{m2}}{T_{m1}} = 0.2$
	$\frac{1.00T_{m1}}{K_m\tau_m}$	$2.50\tau_m$	$2.00\tau_m$	$\frac{T_{m2}}{T_{m1}} = 0.5$
	$\frac{1.25T_{m1}}{K_m\tau_m}$	$2.75\tau_m$	$2.75\tau_m$	$\frac{T_{m2}}{T_{m1}} = 1.0$

[1] $$K_c^{(100)} = \frac{100}{\left(48 + 57\left[1 - e^{-\frac{1.2T_{m1}}{\tau_m}}\right]\right)\frac{K_m\tau_m}{T_{m1}}\left(1 + 0.34\frac{T_{m2}}{\tau_m} - 0.2\left[\frac{T_{m2}}{\tau_m}\right]^2\right)}$$

Rule	K_c				T_i				T_d				Comment			
Minimum ISE – McAvoy and Johnson [83] – *deduced from graph*	$\frac{\alpha}{K_m}$				$\beta\tau_m$				$\chi\tau_m$							
Model: Method 1	ξ_m	$\frac{\tau_m}{T_{m1}}$	α	β	χ	ξ_m	$\frac{\tau_m}{T_{m1}}$	α	β	χ	ξ_m	$\frac{\tau_m}{T_{m1}}$	α	β	χ	
N = 20	1	0.5	0.7	0.97	0.75	4	0.5	3.0	1.16	0.85	7	0.5	5.4	1.19	0.85	
	1	4.0	7.6	3.33	2.03	4	4.0	22.7	1.89	1.28	7	4.0	40.0	1.64	1.14	
	1	10.0	34.3	5.00	2.7											
N = 10	1	0.5	0.9	1.10	0.64	4	0.5	3.2	1.33	0.78	7	0.5	5.9	1.39	1.04	
	1	4.0	8.0	4.00	1.83	4	4.0	23.9	2.17	1.17	7	4.0	42.9	1.89	1.37	
	1	10.0	33.5	6.25	2.43											

Rule	K_c			T_i			T_d			Comment
Direct synthesis										
Astrom *et al*. [30] – *Method 13*	$\frac{3}{K_m\left(1+\frac{3\tau_m}{T_{m1}+T_{m2}}\right)}$			$T_{m1}+T_{m2}$			$\frac{T_{m1}T_{m2}}{T_{m1}+T_{m2}}$			N = 8 - Honeywell UDC6000 controller
Smith *et al*. [26] – *deduced from graph*	$\frac{\alpha T_{m1}}{K_m\tau_m}$			T_{m1}			T_{m2}			N = 10
$[\rho=\tau_m/(T_{m1}+T_{m2})]$	ξ_m	ρ	α	ξ_m	ρ	α	ξ_m	ρ	α	
Model: Method not stated	6	≥ 0.02	0.51	3	≥ 0.04	0.50	2	≥ 0.06	0.45	
	1.75	0.27	0.46	1.75	0.13	0.42	1.75	0.07	0.39	
	1.75	0.07	0.36	1.5	0.33	0.44	1.5	0.17	0.38	
	1.5	0.11	0.33	1.5	0.08	0.28	1.0	0.50	0.40	
	1.0	0.25	0.46	1.0	0.17	0.48	1.0	0.13	0.49	

Table 73: PID tuning rules - SOSPD model $\dfrac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ or $\dfrac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$ - alternative classical controller $G_c(s) = K_c\left(1+\dfrac{1}{T_i s}\right)\left(\dfrac{1+NT_d s}{1+T_d s}\right)$. *1 tuning rule*.

Rule	K_c	T_i	T_d	Comment
******				
Hougen [85]	$\dfrac{0.8T_{m1}^{0.7}T_{m2}^{0.3}}{\tau_m}$	$0.5T_{m1} + T_{m2}$	$\sqrt[3]{\tau_m T_{m1} T_{m2}}$	N=10
Model: Method 1	$\dfrac{0.84T_{m1}^{0.8}T_{m2}^{0.2}}{\tau_m}$	$0.53T_{m1} + 1.3T_{m2}$	$0.08(\tau_m T_{m1} T_{m2})^{0.28}$	N=30

Table 74: PID tuning rules - SOSPD model $\dfrac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ or $\dfrac{K_m e^{-s\tau_m}}{T_{m1}{}^2 s^2 + 2\xi_m T_{m1}s + 1}$ - alternative non-interacting controller 1: $U(s) = K_c\left(1 + \dfrac{1}{T_i s}\right)E(s) - K_c T_d sY(s)$. *3 tuning rules.*

Rule	K_c	T_i	T_d	Comment
Regulator tuning	**Minimum performance index**			
Minimum IAE - Shinskey [59] – *page 159* *Model: open loop method not specified*	[1] $K_c^{(101)}$	$\tau_m\left(0.5 + 1.4\left[1 - e^{-\frac{T_{m1}}{1.5\tau_m}}\right]\right)\left(1 + 0.48\left[1 - e^{-\frac{T_{m2}}{\tau_m}}\right]\right)$	$0.42\tau_m\left(1 - e^{-\frac{1.2T_{m1}}{\tau_m}}\right) + 0.6T_{m2}$	$\dfrac{T_{m2}}{\tau_m} \le 3$
	$\dfrac{3.33T_{m1}}{K_m\tau_m}$	$\tau_m + 0.2T_{m2}$	$\tau_m + 0.2T_{m2}$	$\dfrac{T_{m2}}{\tau_m} > 3$
Minimum IAE - Shinskey [17] – *page 119. Model: method not specified.*	$\dfrac{1.18T_{m1}}{K_m\tau_m}$	$2.20\tau_m$	$0.72\tau_m$	$\dfrac{\tau_m}{T_{m1}} = 0.2$, $\dfrac{T_{m2}}{T_{m1}} = 0.1$
	$\dfrac{1.25T_{m1}}{K_m\tau_m}$	$2.20\tau_m$	$1.10\tau_m$	$\dfrac{T_{m2}}{T_{m1}} = 0.2$
	$\dfrac{1.67T_{m1}}{K_m\tau_m}$	$2.40\tau_m$	$1.65\tau_m$	$\dfrac{T_{m2}}{T_{m1}} = 0.5$
	$\dfrac{2.5T_{m1}}{K_m\tau_m}$	$2.15\tau_m$	$2.15\tau_m$	$\dfrac{T_{m2}}{T_{m1}} = 1.0$
Minimum IAE - Shinskey [17] – *page 121. Model: method not specified.*	$0.85K_u$	$0.35T_u$	$0.17T_u$	$\dfrac{\tau_m}{T_{m1}} = 0.2$, $\dfrac{T_{m2}}{T_{m1}} = 0.2$

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[1] $K_c^{(101)} = \dfrac{100}{\left(38 + 40\left[1 - e^{-\frac{1.5T_{m1}}{\tau_m}}\right]\right)\dfrac{K_m\tau_m}{T_{m1}}\left(1 + 0.34\dfrac{T_{m2}}{\tau_m} - 0.2\left[\dfrac{T_{m2}}{\tau_m}\right]^2\right)}$

Table 75: PID tuning rules - SOSPD model $\dfrac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ or $\dfrac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$ - series controller $G_c(s) = K_c\left(1+\dfrac{1}{T_i s}\right)(1+T_d s)$. *1 tuning rule.*

Rule	K_c	T_i	T_d	Comment
Regulator tuning	**Minimum performance index**			
Least mean square error - Haalman [23] *Model: Method 1*	$\dfrac{2T_{m1}}{3\tau_m K_m}$	T_{m1}	T_{m2}	$T_{m1} > T_{m2}$. Maximum sensitivity = 1.9, Gain margin = 2.36, Phase margin = 50^0

<u>Table 76</u>: PID tuning rules - SOSPD model $\dfrac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ or $\dfrac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$ - non-interacting controller $U(s) = K_c\left(1+\dfrac{1}{T_i s}\right)\left(E(s) - \dfrac{T_d s}{1+\dfrac{T_d s}{N}} Y(s)\right)$. *2 tuning rules*.

Rule	K_c	T_i	T_d	Comment
Servo – Min. IAE - Huang *et al*. [18] *Model: Method 1*	$K_c^{(102)}$ [2]	$T_i^{(102)}$	$T_d^{(102)}$	$0 < \dfrac{T_{m2}}{T_{m1}} \le 1$; $0.1 \le \dfrac{\tau_m}{T_{m1}} \le 1$; N =10

[2] $$K_c^{(102)} = \frac{1}{K_m}\left[7.0636 + 66.6512\frac{\tau_m}{T_{m1}} - 137.8937\frac{T_{m2}}{T_{m1}} - 122.7832\frac{\tau_m T_{m2}}{T_{m1}^2} + 26.1928\left(\frac{\tau_m}{T_{m1}}\right)^{0.0865}\right]$$
$$+\frac{1}{K_m}\left[33.6578\left(\frac{\tau_m}{T_{m1}}\right)^{2.6405} + 3.0098\left(\frac{T_{m2}}{T_{m1}}\right)^{1.0309} - 10.9347\left(\frac{T_{m2}}{T_{m1}}\right)^{2.345} + 141.511\left(\frac{T_{m2}}{T_{m1}}\right)^{1.0570} + 29.4068\frac{T_{m2}}{\tau_m}\right]$$
$$+\frac{1}{K_m}\left[34.3156\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^{-0.9450} - 70.1035\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^{-0.9282} + 152.6392\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^{0.8866}\right]$$
$$+\frac{1}{K_m}\left[-47.9791\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^{0.8148} - 57.9370e^{\frac{\tau_m}{T_{m1}}} + 10.4002e^{\frac{T_{m2}}{T_{m1}}} + 6.7646e^{\frac{\tau_m T_{m2}}{T_{m1}^2}} + 7.3453\left(\frac{\tau_m}{T_m}\right)^{-0.4062}\right]$$

$$T_i^{(102)} = T_{m1}\left[0.9923 + 0.2819\frac{\tau_m}{T_{m1}} - 0.2679\frac{T_{m2}}{T_{m1}} - 1.4510\left(\frac{\tau_m}{T_{m1}}\right)^2 - 0.6712\frac{\tau_m T_{m2}}{T_{m1}^2} + 0.6424\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 2.504\left(\frac{\tau_m}{T_{m1}}\right)^3\right]$$
$$+T_{m1}\left[2.5324\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^2 + 2.3641\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 2.0500\left(\frac{T_{m2}}{T_{m1}}\right)^3 - 1.8759\left(\frac{\tau_m}{T_{m1}}\right)^4 + 0.8519\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^4\right]$$
$$+T_{m1}\left[-1.3496\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^3 - 3.4972\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^2 - 2.4216\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^3 - 3.1142\left(\frac{T_{m2}}{T_{m1}}\right)^4\right]$$
$$+T_{m1}\left[0.5862\left(\frac{\tau_m}{T_{m1}}\right)^5 + 0.0797\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^4 + 0.985\left(\frac{T_{m2}}{T_{m1}}\right)^2\left(\frac{\tau_m}{T_{m1}}\right)^3 + 1.2892\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^3 + 1.2108\left(\frac{T_{m2}}{T_{m1}}\right)^5\right]$$

$$T_d^{(102)} = T_{m1}\left[0.0075 + 0.3449\frac{\tau_m}{T_{m1}} + 0.3924\frac{T_{m2}}{T_{m1}} - 0.0793\left(\frac{\tau_m}{T_{m1}}\right)^2 + 2.7495\frac{\tau_m T_{m2}}{T_{m1}^2} + 0.6485\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 0.8089\left(\frac{\tau_m}{T_{m1}}\right)^3\right]$$
$$+T_{m1}\left[-9.7483\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^2 + 3.4679\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^2 - 5.8194\left(\frac{T_{m2}}{T_{m1}}\right)^3 - 1.0884\left(\frac{\tau_m}{T_{m1}}\right)^4\right]$$
$$+T_{m1}\left[12.0049\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^3 - 1.4056\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^2 - 3.7055\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^3 + 10.0045\left(\frac{T_{m2}}{T_{m1}}\right)^4\right]$$
$$+T_{m1}\left[0.3520\left(\frac{\tau_m}{T_{m1}}\right)^5 - 6.3603\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^4 - 3.2980\left(\frac{T_{m2}}{T_{m1}}\right)^2\left(\frac{\tau_m}{T_{m1}}\right)^3 + 7.0404\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^3\right]$$
$$+T_{m1}\left[1.4294\left(\frac{\tau_m}{T_{m1}}\right)\left(\frac{T_{m2}}{T_{m1}}\right)^4 - 6.9064\left(\frac{T_{m2}}{T_{m1}}\right)^5 + 0.0471\left(\frac{\tau_m}{T_{m1}}\right)^6 + 1.1839\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^5 + 1.7087\left(\frac{T_{m2}}{T_{m1}}\right)^6\right]$$

Rule	K_c	T_i	T_d	Comment
Servo-Min. IAE - Huang *et al*. [18]	$K_c^{(103)\,3}$ (N=10)	$T_i^{(103)}$	$T_d^{(103)}$	$0.4 \le \xi_m \le 1$; $0.05 \le \frac{\tau_m}{T_{m1}} \le 1$

$$+T_{m1}\left[1.7444\left(\frac{\tau_m}{T_{m1}}\right)^4\left(\frac{T_{m2}}{T_{m1}}\right)^2-1.2817\left(\frac{\tau_m}{T_{m1}}\right)^3\left(\frac{T_{m2}}{T_{m1}}\right)^3-2.1281\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^4+1.5121\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^5\right]$$

$$^3\,K_c^{(103)}=\frac{1}{K_m}\left[-8.1727-32.9042\frac{\tau_m}{T_{m1}}+31.9179\xi_m+38.3405\xi_m\left(\frac{\tau_m}{T_{m1}}\right)+0.2079\left(\frac{\tau_m}{T_{m1}}\right)^{-1.9009}\right]$$

$$+\frac{1}{K_m}\left[29.3215\left(\frac{\tau_m}{T_{m1}}\right)^{0.1571}+35.9456\left(\frac{\tau_m}{T_{m1}}\right)^{1.2234}-21.4045\xi_m{}^{0.1311}+5.1159\xi_m{}^{1.9814}-21.9381\xi_m{}^{1.737}\right]$$

$$+\frac{1}{K_m}\left[-17.7448\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^{-0.1303}+26.8655\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^{1.2008}-52.9156\frac{\tau_m}{T_{m1}}\xi_m{}^{1.1207}\right]$$

$$+\frac{1}{K_m}\left[-22.4297\frac{\tau_m}{T_{m1}}\xi_m{}^{0.3626}-3.3331e^{\frac{\tau_m}{T_{m1}}}+8.5175e^{\xi_m}-1.5312e^{\frac{\tau_m\xi_m}{T_{m1}}}+0.8906\frac{\xi_m T_{m1}}{\tau_m}\right]$$

$$T_i^{(103)}=T_{m1}\left[1.1731+6.3082\frac{\tau_m}{T_{m1}}-0.6937\xi_m+8.5271\left(\frac{\tau_m}{T_{m1}}\right)^2-24.7291\xi_m\frac{\tau_m}{T_{m1}}-6.7123\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^2+7.9559\xi_m{}^2\right]$$

$$+T_{m1}\left[-32.3937\left(\frac{\tau_m}{T_{m1}}\right)^3-27.1372\left(\frac{\tau_m}{T_{m1}}\right)^4+166.9272\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^3+36.3954\xi_m{}^2\left(\frac{\tau_m}{T_{m1}}\right)\right]$$

$$+T_{m1}\left[-94.8879\xi_m{}^2\left(\frac{\tau_m}{T_{m1}}\right)^2-22.6065\xi_m{}^3-1.6084\frac{\tau_m}{T_{m1}}\xi_m{}^3+29.9159\xi_m{}^4\right]$$

$$+T_{m1}\left[49.6314\left(\frac{\tau_m}{T_{m1}}\right)^5-84.3776\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^4-93.8912\xi_m{}^2\left(\frac{\tau_m}{T_{m1}}\right)^3+110.1706\left(\frac{\tau_m}{T_{m1}}\right)^2\xi_m{}^3\right]$$

$$+T_{m1}\left[-25.1896\left(\frac{\tau_m}{T_{m1}}\right)\xi_m{}^4-19.7569\xi_m{}^5-12.4348\left(\frac{\tau_m}{T_{m1}}\right)^6-11.7589\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^5+5.5268\xi_m{}^6\right]$$

$$+T_{m1}\left[68.3097\left(\frac{\tau_m}{T_{m1}}\right)^4\xi_m{}^2-17.8663\left(\frac{\tau_m}{T_{m1}}\right)^3\xi_m{}^3-22.5926\left(\frac{\tau_m}{T_{m1}}\right)^2\xi_m{}^4+9.5061\frac{\tau_m}{T_{m1}}\xi_m{}^5\right]$$

$$T_d^{(104)}=T_{m1}\left[0.0904+0.8637\frac{\tau_m}{T_{m1}}-0.1301\xi_m+4.9601\left(\frac{\tau_m}{T_{m1}}\right)^2+14.3899\xi_m\frac{\tau_m}{T_{m1}}+0.7170\xi_m{}^2-12.5311\left(\frac{\tau_m}{T_{m1}}\right)^3\right]$$

$$+T_{m1}\left[-42.5012\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^2-21.4907\xi_m{}^2\left(\frac{\tau_m}{T_{m1}}\right)-6.9555\xi_m{}^3-12.3016\left(\frac{\tau_m}{T_{m1}}\right)^4\right]$$

$$+T_{m1}\left[102.9447\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^3+7.5855\xi_m{}^2\left(\frac{\tau_m}{T_{m1}}\right)^2+19.1257\frac{\tau_m}{T_{m1}}\xi_m{}^3+17.0952\xi_m{}^4\right]$$

$$+T_{m1}\left[10.8688\left(\frac{\tau_m}{T_{m1}}\right)^5-17.2130\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^4-110.0342\xi_m{}^2\left(\frac{\tau_m}{T_{m1}}\right)^3+50.6455\left(\frac{\tau_m}{T_{m1}}\right)^2\xi_m{}^3\right]$$

$$+T_{m1}\left[-16.7073\left(\frac{\tau_m}{T_{m1}}\right)\xi_m{}^4-16.2013\xi_m{}^5-0.0979\left(\frac{\tau_m}{T_{m1}}\right)^6-10.9260\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^5+5.4409\xi_m{}^6\right]$$

Rule	K_c	T_i	T_d	Comment
Regulator – Min. IAE - Huang *et al*. [18] *Model: Method 1*	$K_c^{(104)}$ [4]	$T_i^{(104)}$	$T_d^{(104)}$	$0 < \dfrac{T_{m2}}{T_{m1}} \le 1$; $0.1 \le \dfrac{\tau_m}{T_{m1}} \le 1$; N =10

$$+T_{m1}\left[29.4445\left(\frac{\tau_m}{T_{m1}}\right)^4 \xi_m^{\ 2} + 21.6061\left(\frac{\tau_m}{T_{m1}}\right)^3 \xi_m^{\ 3} - 24.1917\left(\frac{\tau_m}{T_{m1}}\right)^2 \xi_m^{\ 4} + 6.2798\frac{\tau_m}{T_{m1}}\xi_m^{\ 5}\right]$$

[4] $$K_c^{(104)} = \frac{1}{K_m}\left[0.1098 - 8.6290\frac{\tau_m}{T_{m1}} + 76.6760\frac{T_{m2}}{T_{m1}} - 3.3397\frac{\tau_m T_{m2}}{T_{m1}^{\ 2}} + 1.1863\left(\frac{\tau_m}{T_{m1}}\right)^{-0.8058}\right]$$

$$+\frac{1}{K_m}\left[23.1098\left(\frac{\tau_m}{T_{m1}}\right)^{0.6642} + 20.3519\left(\frac{\tau_m}{T_{m1}}\right)^{2.1482} - 52.0778\left(\frac{T_{m2}}{T_{m1}}\right)^{0.8405} - 12.1033\left(\frac{T_{m2}}{T_{m1}}\right)^{2.1123}\right]$$

$$+\frac{1}{K_m}\left[9.4709\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^{0.5306} + 13.6581\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^{-1.0781} - 19.4944\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^{-0.4500}\right]$$

$$+\frac{1}{K_m}\left[-28.2766\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^{1.1427} - 19.1463e^{\frac{\tau_m}{T_{m1}}} + 8.8420e^{\frac{T_{m2}}{T_{m1}}} + 7.4298e^{\frac{\tau_m T_{m2}}{T_{m1}^{\ 2}}} - 11.4753\frac{T_{m2}}{\tau_m}\right]$$

$$T_i^{(104)} = T_{m1}\left[-0.0145 + 2.0555\frac{\tau_m}{T_{m1}} + 0.7435\frac{T_{m2}}{T_{m1}} - 4.4805\left(\frac{\tau_m}{T_{m1}}\right)^2 + 1.2069\frac{\tau_m T_{m2}}{T_{m1}^{\ 2}} + 0.2584\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 7.7916\left(\frac{\tau_m}{T_{m1}}\right)^3\right]$$

$$+T_{m1}\left[-6.0330\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^2 + 3.9585\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^2 - 3.0626\left(\frac{T_{m2}}{T_{m1}}\right)^3 - 7.0263\left(\frac{\tau_m}{T_{m1}}\right)^4 + 7.0004\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^3\right]$$

$$+T_{m1}\left[-2.7755\left(\frac{T_{m2}}{T_{m1}}\right)^2\left(\frac{\tau_m}{T_{m1}}\right)^2 - 1.5769\left(\frac{\tau_m}{T_{m1}}\right)\left(\frac{T_{m2}}{T_{m1}}\right)^3 + 3.1663\left(\frac{T_{m2}}{T_{m1}}\right)^4 + 2.4311\left(\frac{\tau_m}{T_{m1}}\right)^5\right]$$

$$+T_{m1}\left[-0.9439\left(\frac{T_{m2}}{T_{m1}}\right)^5 - 2.4506\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^4 - 0.2227\left(\frac{T_{m2}}{T_{m1}}\right)^2\left(\frac{\tau_m}{T_{m1}}\right)^3 + 1.9228\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^3 - 0.5494\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^5\right]$$

$$T_d^{(104)} = T_{m1}\left[-0.0206 + 0.9385\frac{\tau_m}{T_{m1}} + 0.7759\frac{T_{m2}}{T_{m1}} - 2.3820\left(\frac{\tau_m}{T_{m1}}\right)^2 + 2.9230\frac{\tau_m T_{m2}}{T_{m1}^{\ 2}} - 3.2336\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 7.2774\left(\frac{\tau_m}{T_{m1}}\right)^3\right]$$

$$+T_{m1}\left[-9.9017\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^2 + 2.7095\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 6.1539\left(\frac{T_{m2}}{T_{m1}}\right)^3 - 11.1018\left(\frac{\tau_m}{T_{m1}}\right)^4\right]$$

$$+T_{m1}\left[10.6303\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^3 + 5.7105\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^2 - 7.9490\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^3 - 6.6597\left(\frac{T_{m2}}{T_{m1}}\right)^4\right]$$

$$+T_{m1}\left[8.0849\left(\frac{\tau_m}{T_{m1}}\right)^5 - 4.4897\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^4 - 7.6469\left(\frac{T_{m2}}{T_{m1}}\right)^2\left(\frac{\tau_m}{T_{m1}}\right)^3 + 2.1155\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^3\right]$$

$$+T_{m1}\left[5.0694\left(\frac{\tau_m}{T_{m1}}\right)\left(\frac{T_{m2}}{T_{m1}}\right)^4 + 4.1225\left(\frac{T_{m2}}{T_{m1}}\right)^5 - 2.274\left(\frac{\tau_m}{T_{m1}}\right)^6 + 0.519\frac{T_{m2}}{T_{m1}}\left(\frac{\tau_m}{T_{m1}}\right)^5 - 1.1295\left(\frac{T_{m2}}{T_{m1}}\right)^6\right]$$

$$+T_{m1}\left[2.2875\left(\frac{\tau_m}{T_{m1}}\right)^4\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 0.9524\left(\frac{\tau_m}{T_{m1}}\right)^3\left(\frac{T_{m2}}{T_{m1}}\right)^3 - 1.6307\left(\frac{\tau_m}{T_{m1}}\right)^2\left(\frac{T_{m2}}{T_{m1}}\right)^4 - 0.9321\frac{\tau_m}{T_{m1}}\left(\frac{T_{m2}}{T_{m1}}\right)^5\right]$$

Rule	K_c	T_i	T_d	Comment
Regulator - Min. IAE - Huang *et al.* [18] *Model: Method 1*	$K_c^{(105)}$ [5]	$T_i^{(105)}$	$T_d^{(105)}$ [N=10]	$0.4 \le \xi_m \le 1$; $0.05 \le \frac{\tau_m}{T_{m1}} \le 1$

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[5]
$$K_c^{(105)} = \frac{1}{K_m}\left[-35.7307 - 14.19\frac{\tau_m}{T_{m1}} + 1.4023\xi_m + 6.8618\xi_m\left(\frac{\tau_m}{T_{m1}}\right) - 0.9773\left(\frac{\tau_m}{T_{m1}}\right)^3 + 55.5898\xi_m\left(\frac{\tau_m}{T_m}\right)^{0.086}\right]$$

$$+\frac{1}{K_m}\left[-3.3093\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^2 + 53.8651\frac{\tau_m}{T_{m1}}\xi_m^{\ 2} + 11.4911\xi_m^{\ 3} + 0.8778\left(\frac{\tau_m}{T_{m1}}\right)^{-1.6624} - 29.8822\left(\frac{\tau_m}{T_{m1}}\right)^{-0.6951}\right]$$

$$+\frac{1}{K_m}\left[53.535\left(\frac{\tau_m}{T_{m1}}\right)^{-0.4762} - 16.9807\xi_m^{\ 1.1197} - 25.4293\xi_m^{\ 1.4622} - 0.1671\xi_m^{\ 58981} + 0.0034\xi_m\left(\frac{\tau_m}{T_m}\right)^{-2.1208}\right]$$

$$+\frac{1}{K_m}\left[-25.0355\frac{\tau_m}{T_{m1}}\xi_m^{\ 3.0836} - 54.9617\frac{\tau_m}{T_{m1}}\xi_m^{\ 1.2103} - 0.1398e^{\frac{\tau_m}{T_{m1}}} - 8.2721e^{\xi_m} + 6.3542e^{\frac{\tau_m\xi_m}{T_{m1}}} + 1.0479\frac{\xi_m T_{m1}}{\tau_m}\right]$$

$$T_i^{(105)} = T_{m1}\left[0.2563 + 11.8737\frac{\tau_m}{T_{m1}} - 1.6547\xi_m - 16.1913\left(\frac{\tau_m}{T_{m1}}\right)^2 - 9.7061\xi_m\frac{\tau_m}{T_{m1}} + 3.5927\xi_m^{\ 2} + 19.5201\left(\frac{\tau_m}{T_{m1}}\right)^3\right]$$

$$+T_{m1}\left[-14.5581\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^2 + 2.939\xi_m^{\ 2}\left(\frac{\tau_m}{T_{m1}}\right) - 0.4592\xi_m^{\ 3} - 34.6273\left(\frac{\tau_m}{T_{m1}}\right)^4\right]$$

$$+T_{m1}\left[50.5163\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^3 + 8.9259\xi_m^{\ 2}\left(\frac{\tau_m}{T_{m1}}\right)^2 + 8.6966\frac{\tau_m}{T_{m1}}\xi_m^{\ 3} - 6.9436\xi_m^{\ 4}\right]$$

$$+T_{m1}\left[27.2386\left(\frac{\tau_m}{T_{m1}}\right)^5 - 20.0697\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^4 - 42.2833\xi_m^{\ 2}\left(\frac{\tau_m}{T_{m1}}\right)^3 + 8.5019\left(\frac{\tau_m}{T_{m1}}\right)^2\xi_m^{\ 3}\right]$$

$$+T_{m1}\left[-12.2957\left(\frac{\tau_m}{T_{m1}}\right)\xi_m^{\ 4} + 8.0694\xi_m^{\ 5} - 7.7887\left(\frac{\tau_m}{T_{m1}}\right)^6 + 2.3012\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^5 - 2.7691\xi_m^{\ 6}\right]$$

$$+T_{m1}\left[8.8984\left(\frac{\tau_m}{T_{m1}}\right)^4\xi_m^{\ 2} + 10.2494\left(\frac{\tau_m}{T_{m1}}\right)^3\xi_m^{\ 3} - 5.4906\left(\frac{\tau_m}{T_{m1}}\right)^2\xi_m^{\ 4} + 4.6594\frac{\tau_m}{T_{m1}}\xi_m^{\ 5}\right]$$

$$T_d^{(105)} = T_{m1}\left[-0.021 + 3.3385\frac{\tau_m}{T_{m1}} + 0.185\xi_m - 0.5164\left(\frac{\tau_m}{T_{m1}}\right)^2 - 0.9643\xi_m\frac{\tau_m}{T_{m1}} - 0.8815\xi_m^{\ 2} + 0.584\left(\frac{\tau_m}{T_{m1}}\right)^3\right]$$

$$+T_{m1}\left[-12.513\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^2 + 1.3468\xi_m^{\ 2}\left(\frac{\tau_m}{T_{m1}}\right) + 2.3181\xi_m^{\ 3} - 5.2368\left(\frac{\tau_m}{T_{m1}}\right)^4\right]$$

$$+T_{m1}\left[15.3014\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^3 + 11.9607\xi_m^{\ 2}\left(\frac{\tau_m}{T_{m1}}\right)^2 - 2.0411\frac{\tau_m}{T_{m1}}\xi_m^{\ 3} - 3.1988\xi_m^{\ 4}\right]$$

$$+T_{m1}\left[3.4675\left(\frac{\tau_m}{T_{m1}}\right)^5 - 0.8219\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^4 - 15.0718\xi_m^{\ 2}\left(\frac{\tau_m}{T_{m1}}\right)^3 - 1.8859\left(\frac{\tau_m}{T_{m1}}\right)^2\xi_m^{\ 3}\right]$$

$$+T_{m1}\left[0.4841\left(\frac{\tau_m}{T_{m1}}\right)\xi_m^{\ 4} + 2.2821\xi_m^{\ 5} - 0.9315\left(\frac{\tau_m}{T_{m1}}\right)^6 + 0.529\xi_m\left(\frac{\tau_m}{T_{m1}}\right)^5 - 0.6772\xi_m^{\ 6}\right]$$

$$+T_{m1}\left[-1.4212\left(\frac{\tau_m}{T_{m1}}\right)^4 \xi_m{}^2 + 7.1176\left(\frac{\tau_m}{T_{m1}}\right)^3 \xi_m{}^3 - 2.3636\left(\frac{\tau_m}{T_{m1}}\right)^2 \xi_m{}^4 + 0.5497\frac{\tau_m}{T_{m1}}\xi_m{}^5\right]$$

Table 77: PID tuning rules - SOSPD model $\frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ or $\frac{K_m e^{-s\tau_m}}{T_{m1}{}^2 s^2 + 2\xi_m T_{m1} s + 1}$ - ideal controller with set-point weighting $U(s) = K_c\left(F_p R(s) - Y(s)\right) + \frac{K_c}{T_i s}\left[F_i R(s) - Y(s)\right] + K_c T_d s\left[F_d R(s) - Y(s)\right]$. *1 tuning rule*.

Rule	K_c	T_i	T_d	Comment
Ultimate cycle				
Oubrahim and Leonard [138] *Model: Method not specified*	$0.6K_u$ $F_p = 1.3\frac{16 - K_m K_u}{17 + K_m K_u}$	$0.5T_u$ $F_i = 1$	$0.125T_u$ $F_d = F_p{}^2$	Repeated pole $0.1 < \frac{\tau_m}{T_m} < 3$; 10% overshoot
	$0.6K_u$ $F_p = \frac{38}{29 + 35K_m K_u}$	$0.5T_u$ $F_i = 1$	$0.125T_u$ $F_d = F_p{}^2$	Repeated pole $0.1 < \frac{\tau_m}{T_m} < 3$; 20% overshoot

Table 78: PID tuning rules - SOSPD model $\frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ or $\frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1}s + 1}$ - non-interacting controller $U(s) = K_c\left(b + \frac{1}{T_i s}\right)[R(s) - Y(s)] - (c + T_d s)Y(s)$. *1 tuning rule*.

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Hansen [150]	$K_c^{(105a)}$ [6]	$1.54T_i^{(105a)}$	$T_d^{(105a)}$	$b = 0.198$ $c = K_c^{(105a)} - 1/K_m$; zero overshoot
Model: Method 1	$K_c^{(105a)}$	$1.27T_i^{(105a)}$	$T_d^{(105a)}$	$b = 0.289$ $c = K_c^{(105a)} - 1/K_m$; minimum IAE
	$K_c^{(105a)}$	$1.75T_i^{(105a)}$	$T_d^{(105a)}$	$b = 0.143$ $c = K_c^{(105a)} - 1/K_m$; conservative tuning

[6] $K_c^{(105a)} = \frac{2\left[T_{m1}T_{m2} + (T_{m1}+T_{m2})\tau_m + 0.5\tau_m^2\right]^3}{9K_m\left[T_{m1}T_{m2}\tau_m + 0.5(T_{m1}+T_{m2})\tau_m^2 + 0.167\tau_m^3\right]^2}$, $T_i^{(105a)} = \frac{3\left[T_{m1}T_{m2}\tau_m + 0.5(T_{m1}+T_{m2})\tau_m^2 + 0.167\tau_m^3\right]}{\left[T_{m1}T_{m2} + (T_{m1}+T_{m2})\tau_m + 0.5\tau_m^2\right]}$,

$$T_d^{(105a)} = \frac{2\left[T_{m1}T_{m2} + (T_{m1}+T_{m2})\tau_m + 0.5\tau_m^2\right]^2}{3K_m\left[T_{m1}T_{m2}\tau_m + 0.5(T_{m1}+T_{m2})\tau_m^2 + 0.167\tau_m^3\right]} - \frac{T_{m1}+T_{m2}-\tau_m}{K_m}$$

Table 79: PID tuning rules - SOSPD model $\frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ or $\frac{K_m e^{-s\tau_m}}{T_{m1}{}^2 s^2 + 2\xi_m T_{m1} s + 1}$ - ideal controller with filtered derivative $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{sT_d}{N}}\right)$. *2 tuning rules*.

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Hang *et al*. [151] *Model: Method 1*	$\frac{\omega_p T_{m1}}{A_m K_m}$	$\frac{1}{2\omega_p - \frac{4\omega_p{}^2\tau_m}{\pi} + \frac{1}{T_{m1}}}$	T_{m2}	Sample A_m, ϕ_m provided. N = 20. $T_{m1} > T_{m2}$ $\omega_p = \frac{A_m\phi_m + 0.5\pi A_m(A_m - 1)}{(A_m{}^2 - 1)\tau_m}$
	$A_m = 2.0, \phi_m = 45^o$ $A_m = 3.0, \phi_m = 60^o$	$A_m = 4.0, \phi_m = 67.5^o$	$A_m = 5.0, \phi_m = 72^o$	
Robust				
Hang *et al*. [151] *Model: Method 1*	$\frac{T_{m1}}{K_m(\lambda + \tau_m)}$	T_{m1}	T_{m2}	N = 20. $T_{m1} > T_{m2}$

Table 80: PID tuning rules – SOSPD model $\frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$ - Two degree of freedom controller:

$$U(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N}s}\right)E(s) - K_c\left(\alpha + \frac{\beta T_d s}{1 + \frac{T_d}{N}s}\right)R(s)$$. *3 tuning rules*.

Rule	K_c	T_i	T_d	Comment
Servo/regulator tuning	**Minimum performance index**			
Taguchi and Araki [61a] *Model: ideal process*	$K_c^{(105b)}$ [7]	$T_i^{(105b)}$	$T_d^{(105b)}$	$\frac{\tau_m}{T_m} \le 1.0$; $\xi_m = 1.0$ Overshoot (servo step) $\le 20\%$; settling time $\le$ settling time of tuning rules of Chien *et al*. [10]
	$K_c^{(105c)}$	$T_i^{(105c)}$	$T_d^{(105c)}$	$\frac{\tau_m}{T_m} \le 1.0$; $\xi_m = 0.5$ Overshoot (servo step) $\le 20\%$; settling time $\le$ settling time of tuning rules of Chien *et al*. [10]
Minimum ITAE - Pecharroman and Pagola [134a] *Model: Method 15*	$0.7236\,K_u$	$0.5247\,T_u$	$0.1650\,T_u$	$\alpha = 0.5840$, $\beta = 1$, $N = 10$, $\phi_c = -139.65^0$ $K_m = 1$; $T_m = 1$; $\xi_m = 1$

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[7] $K_c^{(105b)} = \frac{1}{K_c}\left(1.389 + \frac{0.6978}{[\frac{\tau_m}{T_m} - 0.02295]^2}\right)$, $T_i^{(105b)} = T_m\left(0.02453 + 4.104\frac{\tau_m}{T_m} - 3.434\left[\frac{\tau_m}{T_m}\right]^2 + 1.231\left[\frac{\tau_m}{T_m}\right]^3\right)$

$T_d^{(105b)} = T_m\left(0.03459 + 1.852\frac{\tau_m}{T_m} - 2.741\left[\frac{\tau_m}{T_m}\right]^2 + 2.359\left[\frac{\tau_m}{T_m}\right]^3 - 0.7962\left[\frac{\tau_m}{T_m}\right]^4\right)$,

$\alpha = 0.6726 - 0.1285\frac{\tau_m}{T_m} - 0.1371\left[\frac{\tau_m}{T_m}\right]^2 + 0.07345\left[\frac{\tau_m}{T_m}\right]^3$, $\beta = 0.8665 - 0.2679\frac{\tau_m}{T_m} + 0.02724\left[\frac{\tau_m}{T_m}\right]^2$

$K_c^{(105c)} = \frac{1}{K_c}\left(0.3363 + \frac{0.5013}{[\frac{\tau_m}{T_m} - 0.01147]^2}\right)$, $T_i^{(105c)} = T_m\left(-0.02337 + 4.858\frac{\tau_m}{T_m} - 5.522\left[\frac{\tau_m}{T_m}\right]^2 + 2.054\left[\frac{\tau_m}{T_m}\right]^3\right)$

$T_d^{(105c)} = T_m\left(0.03392 + 2.023\frac{\tau_m}{T_m} - 1.161\left[\frac{\tau_m}{T_m}\right]^2 + 0.2826\left[\frac{\tau_m}{T_m}\right]^3\right)$,

$\alpha = 0.6678 - 0.05413\frac{\tau_m}{T_m} - 0.5680\left[\frac{\tau_m}{T_m}\right]^2 + 0.1699\left[\frac{\tau_m}{T_m}\right]^3$, $\beta = 0.8646 - 0.1205\frac{\tau_m}{T_m} - 0.1212\left[\frac{\tau_m}{T_m}\right]^2$

Rule	K_c	T_i	T_d	Comment
Minimum ITAE - Pecharroman and Pagola [134b] $\phi_c =$ phase corresponding to the crossover frequency; $K_m = 1$; $T_m = 1$; $0.1 < \tau_m < 10$ *Model: Method 15*	$0.803 K_u$	$0.509 T_u$	$0.167 T_u$	$\alpha = 0.585$, $\beta = 1$, $N = 10$, $\phi_c = -146^0$
	$0.727 K_u$	$0.524 T_u$	$0.165 T_u$	$\alpha = 0.584$, $\beta = 1$, $N = 10$, $\phi_c = -140^0$
	$0.672 K_u$	$0.532 T_u$	$0.161 T_u$	$\alpha = 0.577$, $\beta = 1$, $N = 10$, $\phi_c = -134^0$
	$0.669 K_u$	$0.486 T_u$	$0.170 T_u$	$\alpha = 0.550$, $\beta = 1$, $N = 10$, $\phi_c = -125^0$
	$0.600 K_u$	$0.498 T_u$	$0.157 T_u$	$\alpha = 0.543$, $\beta = 1$, $N = 10$, $\phi_c = -115^0$
	$0.578 K_u$	$0.481 T_u$	$0.154 T_u$	$\alpha = 0.528$, $\beta = 1$, $N = 10$, $\phi_c = -105^0$
	$0.557 K_u$	$0.467 T_u$	$0.149 T_u$	$\alpha = 0.504$, $\beta = 1$, $N = 10$, $\phi_c = -93^0$
	$0.544 K_u$	$0.466 T_u$	$0.141 T_u$	$\alpha = 0.495$, $\beta = 1$, $N = 10$, $\phi_c = -84^0$
	$0.537 K_u$	$0.444 T_u$	$0.144 T_u$	$\alpha = 0.484$, $\beta = 1$, $N = 10$, $\phi_c = -73^0$
	$0.527 K_u$	$0.450 T_u$	$0.131 T_u$	$\alpha = 0.477$, $\beta = 1$, $N = 10$, $\phi_c = -63^0$
	$0.521 K_u$	$0.440 T_u$	$0.129 T_u$	$\alpha = 0.454$, $\beta = 1$, $N = 10$, $\phi_c = -52^0$
	$0.515 K_u$	$0.429 T_u$	$0.126 T_u$	$\alpha = 0.445$, $\beta = 1$, $N = 10$, $\phi_c = -41^0$
	$0.509 K_u$	$0.399 T_u$	$0.132 T_u$	$\alpha = 0.433$, $\beta = 1$, $N = 10$, $\phi_c = -30^0$
	$0.496 K_u$	$0.374 T_u$	$0.123 T_u$	$\alpha = 0.385$, $\beta = 1$, $N = 10$, $\phi_c = -19^0$
	$0.480 K_u$	$0.315 T_u$	$0.112 T_u$	$\alpha = 0.286$, $\beta = 1$, $N = 10$, $\phi_c = -10^0$
	$0.430 K_u$	$0.242 T_u$	$0.084 T_u$	$\alpha = 0.158$, $\beta = 1$, $N = 10$, $\phi_c = -6^0$

Table 81: PID tuning rules - I^2PD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s^2}$ - controller $U(s) = K_c(1 + \frac{1}{T_i s})E(s) + K_c(b-1)R(s) - K_c T_d s Y(s)$. *1 tuning rule.*

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Hansen [91a] *Model: Method 1*	$3.75K_m\tau_m^{\ 2}$	$5.4\tau_m$	$2.5\tau_m$	b = 0.167

Table 82: PID tuning rules – SOSIPD model (repeated pole) $\dfrac{K_m e^{-s\tau_m}}{s(1+T_{m1}s)^2}$ - Two degree of freedom controller:

$$U(s) = K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+\frac{T_d}{N}s}\right)E(s) - K_c\left(\alpha+\frac{\beta T_d s}{1+\frac{T_d}{N}s}\right)R(s)$$. *2 tuning rules*.

Rule	K_c	T_i	T_d	Comment
Servo/regulator tuning	**Minimum performance index**			
Taguchi and Araki [61a] *Model: Method 2*	$K_c^{(105d)}$ [8]	$T_i^{(105d)}$	$T_d^{(105d)}$	$\frac{\tau_m}{T_m} \le 1.0$; Overshoot (servo step) $\le 20\%$; settling time $\le$ settling time of tuning rules of Chien *et al*. [10]
Minimum ITAE - Pecharroman and Pagola [134a] $\phi_c =$ phase corresponding to the crossover frequency; $K_m = 1$; $T_m = 1$; $0.1 < \tau_m < 10$ *Model: Method 1*	$1.672 K_u$	$0.366 T_u$	$0.136 T_u$	$\alpha = 0.601$, $\beta = 1$, $N = 10$, $\phi_c = -164^0$
	$1.236 K_u$	$0.427 T_u$	$0.149 T_u$	$\alpha = 0.607$, $\beta = 1$, $N = 10$, $\phi_c = -160^0$
	$0.994 K_u$	$0.486 T_u$	$0.155 T_u$	$\alpha = 0.610$, $\beta = 1$, $N = 10$, $\phi_c = -155^0$
	$0.842 K_u$	$0.538 T_u$	$0.154 T_u$	$\alpha = 0.616$, $\beta = 1$, $N = 10$, $\phi_c = -150^0$
	$0.752 K_u$	$0.567 T_u$	$0.157 T_u$	$\alpha = 0.605$, $\beta = 1$, $N = 10$, $\phi_c = -145^0$
	$0.679 K_u$	$0.610 T_u$	$0.149 T_u$	$\alpha = 0.610$, $\beta = 1$, $N = 10$, $\phi_c = -140^0$
	$0.635 K_u$	$0.637 T_u$	$0.142 T_u$	$\alpha = 0.612$, $\beta = 1$, $N = 10$, $\phi_c = -135^0$
	$0.590 K_u$	$0.669 T_u$	$0.133 T_u$	$\alpha = 0.610$, $\beta = 1$, $N = 10$, $\phi_c = -130^0$
	$0.551 K_u$	$0.690 T_u$	$0.114 T_u$	$\alpha = 0.616$, $\beta = 1$, $N = 10$, $\phi_c = -125^0$

[8] $K_c^{(105d)} = \dfrac{1}{K_c}\left(0.1778 + \dfrac{0.5667}{\frac{\tau_m}{T_m}+0.002325}\right)$,

$$T_i^{(105d)} = T_m\left(0.2011 + 11.16\frac{\tau_m}{T_m} - 14.98\left[\frac{\tau_m}{T_m}\right]^2 + 13.70\left[\frac{\tau_m}{T_m}\right]^3 - 4.835\left[\frac{\tau_m}{T_m}\right]^4\right)$$

$$T_d^{(105d)} = T_m\left(1.262 + 0.3620\frac{\tau_m}{T_m}\right),\ \alpha = 0.6666\ ,\ \beta = 0.8206 - 0.09750\frac{\tau_m}{T_m} + 0.03845\left[\frac{\tau_m}{T_m}\right]^2$$

Rule	K_c	T_i	T_d	Comment
Minimum ITAE - Pecharroman and Pagola [134a] – continued	$0.520 K_u$	$0.776 T_u$	$0.087 T_u$	$\alpha = 0.609$, $\beta = 1$, $N = 10$, $\phi_c = -120^0$
	$0.509 K_u$	$0.810 T_u$	$0.068 T_u$	$\alpha = 0.611$, $\beta = 1$, $N = 10$, $\phi_c = -118^0$

Table 83: PID tuning rules - SOSPD model with a negative zero $\frac{K_m(1-sT_{m3})e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ - controller with filtered derivative $G_c(s)=K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+\frac{T_d s}{N}}\right)$. *1 tuning rule*.

Rule	K_c	T_i	T_d	Comment
Robust				
Chien [50] *Model: Method 1*	$\frac{T_{m1}+T_{m2}-T_{m3}}{K_m(\lambda+\tau_m)}$	$T_{m1}+T_{m2}-T_{m3}$	$\frac{T_{m1}T_{m2}-(T_{m1}+T_{m2}-T_{m3})T_{m3}}{T_{m1}+T_{m2}-T_{m3}}$	$T_{m1}>T_{m2}>T_{m3}$ N=10; $\lambda=[T_{m1},\tau_m]$
	$\frac{2\xi T_{m2}-T_{m3}}{K_m(\lambda+\tau_m)}$	$2\xi T_{m1}-T_{m3}$	$\frac{T_{m1}^2-(2\xi T_{m1}-T_{m3})T_{m3}}{2\xi T_{m1}-T_{m3}}$	N=10; $\lambda=[T_{m1},\tau_m]$

Table 84: PID tuning rules - SOSPD model with a negative zero $\frac{K_m(1 - sT_{m3})e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})}$ - classical controller

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)\left(\frac{1 + T_d s}{1 + \frac{T_d s}{N}}\right)$$. *1 tuning rule*.

Rule	K_c	T_i	T_d	Comment
Robust				
Chien [50]	$\frac{T_{m2}}{K_m(\lambda + \tau_m)}$	T_{m2}	T_{m1}	$T_{m1} > T_{m2}$ N=10; $\lambda = [T_{m1}, \tau_m]$
Model: Method 1	$\frac{T_{m1}}{K_m(\lambda + \tau_m)}$	T_{m1}	T_{m2}	$T_{m1} > T_{m2}$ N=10; $\lambda = [T_{m1}, \tau_m]$

Table 85: PID tuning rules - SOSPD model with a negative zero $\frac{K_m(1 - sT_{m3})e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})}$ - series controller with filtered derivative $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)\left(1 + \frac{T_d s}{1 + \frac{T_d s}{N}}\right)$. *1 tuning rule*.

Rule	K_c	T_i	T_d	Comment
Robust				
Chien [50]	$\frac{T_{m2}}{K_m(\lambda + \tau_m)}$	T_{m2}	$T_{m1} - T_{m3}$	$T_{m1} > T_{m2} > T_{m3}$ N=10; $\lambda = [T_{m1}, \tau_m]$
Model: Method 1	$\frac{T_{m1}}{K_m(\lambda + \tau_m)}$	T_{m1}	$T_{m2} - T_{m3}$	$T_{m1} > T_{m2} > T_{m3}$ N=10; $\lambda = [T_{m1}, \tau_m]$

Table 86: PID tuning rules - SOSPD model with a positive zero $\frac{K_m(1+sT_{m3})e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ - ideal controller $G_c(s)=K_c\left(1+\frac{1}{T_i s}+T_d s\right)$. *1 tuning rule.*

Rule	K_c	T_i	T_d	Comment
	Minimum performance index			
Minimum IAE, ISE and ITAE – Wang *et al*. [97] *Model: Method 1*	$\rho\frac{(p_0q_1-p_1)}{{p_0}^2}$, $p_0=T_{m1}+T_{m2}+T_{m3}-\tau_m$ $q_1=T_{m1}+T_{m2}+0.5\tau_m$	$q_1-\frac{p_1}{p_0}$, $p_2=0.5T_{m1}T_{m2}\tau_m$ [1]	$\frac{p_0q_2-p_2}{p_0q_1-p_1}-\frac{p_1}{p_0}$, $p_1=T_{m1}T_{m2}+$ $0.5\tau_m(T_{m1}+T_{m2})-0.5T_{m3}\tau_m$ $q_2=T_{m1}T_{m2}$ $+0.5\tau_m(T_{m1}+T_{m2})$	$0.1\le\frac{\tau_m}{T_{m1}}\le 1$ $0.1\le\frac{T_{m2}}{T_{m1}}\le 1$ $0.1\le\frac{T_{m3}}{T_{m1}}\le 1$

1

$$\rho=3.5550-3.6167\frac{\tau_m}{T_{m1}}+2.1781\frac{T_{m2}}{T_{m1}}-5.5203\frac{T_{m3}}{T_{m1}}+\frac{\tau_m}{T_{m1}}\left[1.4704\frac{\tau_m}{T_{m1}}-0.4685\frac{T_{m2}}{T_{m1}}+1.4746\frac{T_{m3}}{T_{m1}}\right]$$
$$+\frac{T_{m2}}{T_{m1}}\left[-0.4685\frac{\tau_m}{T_{m1}}-0.4918\frac{T_{m2}}{T_{m1}}-0.3318\frac{T_{m3}}{T_{m1}}\right]+\frac{T_{m3}}{T_{m1}}\left[1.4746\frac{\tau_m}{T_{m1}}-0.3318\frac{T_{m2}}{T_{m1}}+2.5356\frac{T_{m3}}{T_{m1}}\right]\text{, minimum IAE}$$

$$\rho=3.9395-3.2164\frac{\tau_m}{T_{m1}}+1.6185\frac{T_{m2}}{T_{m1}}-5.8240\frac{T_{m3}}{T_{m1}}+\frac{\tau_m}{T_{m1}}\left[1.0933\frac{\tau_m}{T_{m1}}-0.2383\frac{T_{m2}}{T_{m1}}+1.3508\frac{T_{m3}}{T_{m1}}\right]$$
$$+\frac{T_{m2}}{T_{m1}}\left[-0.2383\frac{\tau_m}{T_{m1}}-0.6679\frac{T_{m2}}{T_{m1}}-0.0564\frac{T_{m3}}{T_{m1}}\right]+\frac{T_{m3}}{T_{m1}}\left[1.3508\frac{\tau_m}{T_{m1}}-0.0564\frac{T_{m2}}{T_{m1}}+2.5648\frac{T_{m3}}{T_{m1}}\right]\text{, minimum ISE}$$

$$\rho=3.2950-3.4779\frac{\tau_m}{T_{m1}}+2.5336\frac{T_{m2}}{T_{m1}}-5.5929\frac{T_{m3}}{T_{m1}}+\frac{\tau_m}{T_{m1}}\left[1.4407\frac{\tau_m}{T_{m1}}-0.5712\frac{T_{m2}}{T_{m1}}+1.5340\frac{T_{m3}}{T_{m1}}\right]$$
$$+\frac{T_{m2}}{T_{m1}}\left[-0.5712\frac{\tau_m}{T_{m1}}-0.3268\frac{T_{m2}}{T_{m1}}-0.5790\frac{T_{m3}}{T_{m1}}\right]+\frac{T_{m3}}{T_{m1}}\left[1.5340\frac{\tau_m}{T_{m1}}-0.5790\frac{T_{m2}}{T_{m1}}+2.7129\frac{T_{m3}}{T_{m1}}\right]\text{, minimum ITAE}$$

<u>Table 87</u>: PID tuning rules - SOSPD model with a positive zero $\frac{K_m(1+sT_{m3})e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ - ideal controller with filtered derivative $G_c(s)=K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+\frac{T_d s}{N}}\right)$. *1 tuning rule*.

Rule	K_c	T_i	T_d	Comment
Robust				
Chien [50] *Model: Method 1*	$\frac{T_{m1}+T_{m2}+\frac{T_{m3}\tau_m}{\lambda+T_{m3}+\tau_m}}{K_m(\lambda+T_{m3}+\tau_m)}$	$T_{m1}+T_{m2}+\frac{T_{m3}\tau_m}{\lambda+T_{m3}+\tau_m}$	$\frac{T_{m3}\tau_m}{\lambda+T_{m3}+\tau_m}+\frac{T_{m1}T_{m2}}{T_{m1}+T_{m2}+\frac{T_{m3}\tau_m}{\lambda+T_{m3}+\tau_m}}$	$T_{m1}>T_{m2}>T_{m3}$ N=10; $\lambda=[T_{m1},\tau_m]$
	$\frac{2\xi T_{m1}+\frac{T_{m3}\tau_m}{\lambda+T_{m3}+\tau_m}}{K_m(\lambda+T_{m3}+\tau_m)}$	$2\xi T_{m1}+\frac{T_{m3}\tau_m}{\lambda+T_{m3}+\tau_m}$	$\frac{T_{m3}\tau_m}{\lambda+T_{m3}+\tau_m}+\frac{T_{m3}^2}{2\xi T_{m1}+\frac{T_{m3}\tau_m}{\lambda+T_{m3}+\tau_m}}$	$T_{m1}>T_{m2}>T_{m3}$ N=10; $\lambda=[T_{m1},\tau_m]$

Table 88: PID tuning rules - SOSPD model with a positive zero $\frac{K_m(1+sT_{m3})e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ - classical controller

$$G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+T_d s}{1+\frac{T_d s}{N}}\right)$$. *1 tuning rule*.

Rule	K_c	T_i	T_d	Comment
Robust				
Chien [50]	$\frac{T_{m2}}{K_m(\lambda + T_{m3} + \tau_m)}$	T_{m2}	T_{m1}	$T_{m1} > T_{m2} > T_{m3}$ N=10; $\lambda = [T_{m1}, \tau_m]$
Model: Method 1	$\frac{T_{m1}}{K_m(\lambda + T_{m3} + \tau_m)}$	T_{m1}	T_{m2}	$T_{m1} > T_{m2} > T_{m3}$ N=10; $\lambda = [T_{m1}, \tau_m]$

Table 89: PID tuning rules - SOSPD model with a positive zero $\frac{K_m(1+sT_{m3})e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ - series controller with filtered derivative $G_c(s)=K_c\left(1+\frac{1}{T_i s}\right)\left(1+\frac{T_d s}{1+\frac{T_d s}{N}}\right)$. *1 tuning rule*.

Rule	K_c	T_i	T_d	Comment
Robust				
Chien [50]	$\frac{T_{m2}}{K_m(\lambda+T_{m3}+\tau_m)}$	T_{m2}	$T_{m1}+\frac{T_{m3}\tau_m}{\lambda+T_{m3}+\tau_m}$	N=10; $\lambda=[T,\tau_m]$, T = dominant time constant
Model: Method 1	$\frac{T_{m1}}{K_m(\lambda+T_{m3}+\tau_m)}$	T_{m1}	$T_{m2}+\frac{T_{m3}\tau_m}{\lambda+T_{m3}+\tau_m}$	N=10; $\lambda=[T,\tau_m]$, T = dominant time constant

Table 90: PID tuning rules - TOLPD model $\frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})(1+sT_{m3})}$ - ideal controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$. *1 tuning rule.*

Rule	K_c	T_i	T_d	Comment
	Minimum performance index			
Standard form optimisation - binomial - Polonyi [153] *Model: Method 1*	$\left(1 - 4\sqrt{\frac{T_{m2}}{6\tau_m}} + \frac{T_{m2}}{T_{m3}}\right)\frac{T_{m1}}{\tau_m}$	$4\sqrt{6T_{m2}\tau_m} + \tau_m$	τ_m	$T_{m1} > 10(T_{m2} + \tau_m)$
Standard form optimisation – minimum ITAE - Polonyi [153] *Model: Method 1*	$\left(1 - 2.1\sqrt{\frac{T_{m2}}{3.4\tau_m}} + \frac{T_{m2}}{T_{m3}}\right)\frac{T_{m1}}{\tau_m}$	$2.7\sqrt{3.4T_{m2}\tau_m} + \tau_m$	τ_m	$T_{m1} > 10(T_{m2} + \tau_m)$

Table 91: PID tuning rules - TOLPD model - $G_m(s)=\frac{K_m e^{-s\tau_m}}{(1+sT_m)^3}$ – Two degree of freedom controller:

$$U(s)=K_c\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+\frac{T_d}{N}s}\right)E(s)-K_c\left(\alpha+\frac{\beta T_d s}{1+\frac{T_d}{N}s}\right)R(s)$$. *1 tuning rule.*

Rule	K_c	T_i	T_d	Comment
Servo/regulator tuning	**Minimum performance index**			
Taguchi and Araki [61a] *Model: ideal process*	$K_c^{(105e)}$ [2]	$T_i^{(105e)}$	$T_d^{(105e)}$	$\frac{\tau_m}{T_m}\le 1.0$; Overshoot (servo step) $\le$ 20% ; settling time $\le$ settling time of tuning rules of Chien *et al*. [10]

[2] $$K_c^{(105e)}=\frac{1}{K_c}\left(0.4020+\frac{1.275}{\frac{\tau_m}{T_m}-0.003273}\right),$$

$$T_i^{(105e)}=T_m\left(0.3572+7.647\frac{\tau_m}{T_m}-12.86\left[\frac{\tau_m}{T_m}\right]^2+11.77\left[\frac{\tau_m}{T_m}\right]^3-4.146\left[\frac{\tau_m}{T_m}\right]^4\right)$$

$$T_d^{(105e)}=T_m\left(0.8335+0.2910\frac{\tau_m}{T_m}-0.04000\left[\frac{\tau_m}{T_m}\right]^2\right),\ \alpha=0.6661-0.2509\frac{\tau_m}{T_m}+0.04773\left[\frac{\tau_m}{T_m}\right]^2,$$

$$\beta=0.8131-0.2303\frac{\tau_m}{T_m}+0.03621\left[\frac{\tau_m}{T_m}\right]^2$$

Table 92: PID tuning rules - unstable FOLPD model $\frac{K_m e^{-s\tau_m}}{1 - sT_m}$ - ideal controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$.

3 tuning rules.

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
De Paor and O'Malley [86] *Model: Method 1*	$\frac{T_m}{K_m}\left[\cos\sqrt{(1-T_m\tau_m)T_m\tau_m}\right] +$ $\frac{T_m}{K_m}\left[\sqrt{\frac{1-T_m\tau_m}{T_m\tau_m}}\sin\sqrt{(1-T_m\tau_m)T_m\tau_m}\right]$	$\frac{1}{T_m\left[\sqrt{\frac{1-T_m\tau_m}{T_m\tau_m}}\right]\tan(0.75\phi_m)}$, $\phi_m = \tan^{-1}\sqrt{\frac{1-T_m\tau_m}{T_m\tau_m}} - \sqrt{T_m\tau_m(1-T_m\tau_m)}$	$\frac{1}{T_m{}^2}\left[\frac{T_m\tau_m}{1-T_m\tau_m}\right]\frac{1}{T_i}$	$\left\lvert\frac{\tau_m}{T_m}\right\rvert < 1$
Chidambaram [88] *Model: Method 1*	$\frac{1}{K_m}\left(1.3 + 0.3\frac{T_m}{\tau_m}\right)$	$T_m\left(25 - 27\frac{\tau_m}{T_m}\right)$	$0.46\tau_m$	$\frac{\tau_m}{T_m} < 0.6$
Valentine and Chidambaram [154] - dominant pole placement *Model: Method 1*	$\frac{1.165}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.245}$	$\frac{\left(0.176 + 0.36\frac{\tau_m}{T_m}\right)T_m}{0.179 - 0.324\frac{\tau_m}{T_m} + 0.161\left(\frac{\tau_m}{T_m}\right)^2}$	$\left(0.176 + 0.36\frac{\tau_m}{T_m}\right)T_m$	$\left\lvert\frac{\tau_m}{T_m}\right\rvert < 0.6$
	$\frac{1.165}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.245}$	$\left(0.176 + 0.36\frac{\tau_m}{T_m}\right)25T_m$	$\left(0.176 + 0.36\frac{\tau_m}{T_m}\right)T_m$	$0.6 \le \left\lvert\frac{\tau_m}{T_m}\right\rvert \le 0.8$
	$\frac{1.165}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.245}$	$\frac{0.176T_m + 0.36\tau_m}{0.12 - 0.1\frac{\tau_m}{T_m}}$	$\left(0.176 + 0.36\frac{\tau_m}{T_m}\right)T_m$	$0.8 < \left\lvert\frac{\tau_m}{T_m}\right\rvert \le 1$

<u>Table 93</u>: PID tuning rules - unstable FOLPD model $\frac{K_m e^{-s\tau_m}}{1 - sT_m}$ - non-interacting controller

$$U(s) = K_c\left(1 + \frac{1}{T_i s}\right)E(s) - \frac{K_c T_d s}{1 + \frac{sT_d}{N}} Y(s)$$. *2 tuning rules.*

Rule	K_c	T_i	T_d	Comment
Servo tuning	**Minimum performance index**			
Huang and Lin [155] - minimum IAE *Model: Method 2*	$-\frac{1}{K_m}\left(-0.433+0.2056\frac{\tau_m}{T_m}+0.3135\left[\frac{\tau_m}{T_m}\right]^{-1.0251}\right)$	$T_m\left(-0.0018+0.8193\frac{\tau_m}{T_m}+7.7853\left[\frac{\tau_m}{T_m}\right]^{6.6423}\right)$	$T_m\left(-0.0312+1.6333\frac{\tau_m}{T_m}+0.0399e^{\frac{7.6983\tau_m}{T_m}}\right)$	$0.01 \le \left\lvert\frac{\tau_m}{T_m}\right\rvert \le 0.8$; N=10
Regulator tuning	**Minimum performance index**			
Huang and Lin [155] - minimum IAE *Model: Method 2*	$-\frac{1}{K_m}\left(0.2675+01226\frac{\tau_m}{T_m}+0.8781\left[\frac{\tau_m}{T_m}\right]^{-1004}\right)$	$T_m\left(0.0005+2.4631\frac{\tau_m}{T_m}+9.5795\left[\frac{\tau_m}{T_m}\right]^{2.9123}\right)$	$T_m\left(0.0011+0.4759\frac{\tau_m}{T_m}\right)$	$0.01 \le \left\lvert\frac{\tau_m}{T_m}\right\rvert \le 0.8$; N=10

Table 94: PID tuning rules - unstable FOLPD model $\frac{K_m e^{-s\tau_m}}{1 - sT_m}$ - classical controller

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)\left(\frac{1 + T_d s}{1 + \frac{T_d s}{N}}\right)$$. *1 tuning rule.*

Rule	K_c	T_i	T_d	Comment
Regulator tuning	**Minimum performance index**			
Shinskey [16] - minimum IAE – *page 381.* *Method: Model 1*	$\frac{0.9091T_m}{K_m\tau_m}$	$1.70\tau_m$	$0.60\tau_m$	$\lvert\tau_m/T_m\rvert = 0.1$
	$\frac{T_m}{K_m\tau_m}$	$1.90\tau_m$	$0.60\tau_m$	$\lvert\tau_m/T_m\rvert = 0.2$
	$\frac{0.8929T_m}{K_m\tau_m}$	$2.00\tau_m$	$0.80\tau_m$	$\lvert\tau_m/T_m\rvert = 0.5$
	$\frac{0.8621T_m}{K_m\tau_m}$	$2.25\tau_m$	$0.90\tau_m$	$\lvert\tau_m/T_m\rvert = 0.67$
	$\frac{0.8333T_m}{K_m\tau_m}$	$2.40\tau_m$	$1.00\tau_m$	$\lvert\tau_m/T_m\rvert = 0.8$

Table 95: PID tuning rules - unstable SOSPD model $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{(1 - sT_{m1})(1 + sT_{m2})}$ - ideal controller

$G_c(s) = K_c\left(1 + \dfrac{1}{T_i s} + T_d s\right)$. *2 tuning rules.*

Rule	K_c	T_i	T_d	Comment
Ultimate cycle				
McMillan [58] *Model: Method not relevant*	$K_c^{(106)}$ [3]	$T_i^{(106)}$	$T_d^{(106)}$	Tuning rules developed from K_u, T_u
Robust				
Rotstein and Lewin [89] *Model: Method 1*	$\dfrac{T_{m1}\left[\lambda\left(\dfrac{\lambda}{T_{m1}} + 2\right) + T_{m2}\right]}{\lambda^2}$	$\lambda\left(\dfrac{\lambda}{T_{m1}} + 2\right) + T_{m2}$	$\dfrac{\lambda\left(\dfrac{\lambda}{T_{m1}} + 2\right)T_{m2}}{\lambda\left(\dfrac{\lambda}{T_{m1}} + 2\right) + T_{m2}}$	λ determined graphically – sample values provided
	K_m uncertainty = 50%	$\tau_m / T_m = 0.2$	$\lambda = [0.5T_m, 1.9T_m]$	
		$\tau_m / T_m = 0.4$	$\lambda = [1.3T_m, 1.9T_m]$	
	K_m uncertainty = 30%	$\tau_m / T_m = 0.2$	$\lambda = [0.4T_m, 4.3T_m]$	
		$\tau_m / T_m = 0.4$	$\lambda = [1.1T_m, 4.3T_m]$	
		$\tau_m / T_m = 0.6$	$\lambda = [2.2T_m, 4.3T_m]$	

[3] $K_c^{(106)} = \dfrac{1.111}{K_m}\dfrac{T_{m1}T_{m2}}{\tau_m^{\,2}}\left\{\dfrac{1}{1 + \left[\dfrac{(T_{m1} + T_{m2})T_{m1}T_{m2}}{(T_{m1} - T_{m2})(T_{m1} - \tau_m)\tau_m}\right]^{0.65}}\right\}^2$,

$T_i^{(106)} = 2\tau_m\left\{1 + \left[\dfrac{(T_{m1} + T_{m2})T_{m1}T_{m2}}{(T_{m1} - T_{m2})(T_{m1} - \tau_m)\tau_m}\right]^{0.65}\right\}$, $T_d^{(106)} = 0.5\tau_m\left\{1 + \left[\dfrac{(T_{m1} + T_{m2})T_{m1}T_{m2}}{(T_{m1} - T_{m2})(T_{m1} - \tau_m)\tau_m}\right]^{0.65}\right\}$

Table 96: PID tuning rules - unstable SOSPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{(1-sT_{m1})(1+sT_{m2})}$ - classical controller

$$G_c(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(\frac{1+T_d s}{1+\frac{T_d}{N}s}\right)$$. *2 tuning rules*.

Rule	K_c		T_i	T_d		Comment
Regulator tuning	**Minimum performance index**					
Minimum ITAE - Poulin and Pomerleau [82], [92] – *deduced from graph Model: Method 1*	$\frac{bT_{m1}\sqrt{1+\frac{aT_{m2}^2}{4(\tau_m+T_{m2})^2}}}{K_m\left(aT_{m1}-4[\tau_m+T_{m2}]\right)}$		$\frac{4T_{m1}(\tau_m+T_{m2})}{aT_{m1}-4(\tau_m+T_{m2})}$	T_{m2}		$0 \le \frac{\tau_m}{(T_d/N)} \le 2$; $0.1T_m \le \frac{T_d}{N} \le 0.33T_m$
	$(\tau_m+T_d/N)/T_{m1}$	a	b	$(\tau_m+T_d/N)/T_{m1}$	a	b
Output step load disturbance	0.05	0.9479	2.3546	0.30	1.6163	2.6612
	0.10	1.0799	2.4111	0.35	1.7650	2.7368
	0.15	1.2013	2.4646	0.40	1.9139	2.8161
	0.20	1.3485	2.5318	0.45	2.0658	2.9004
	0.25	1.4905	2.5992	0.50	2.2080	2.9826
Input step load disturbance	0.05	1.1075	2.4230	0.30	1.6943	2.7007
	0.10	1.2013	2.4646	0.35	1.8161	2.7637
	0.15	1.3132	2.5154	0.40	1.9658	2.8445
	0.20	1.4384	2.5742	0.45	2.1022	2.9210
	0.25	1.5698	2.6381	0.50	2.2379	3.0003

Table 97: PID tuning rules - unstable SOSPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{(1 - sT_{m1})(1 + sT_{m2})}$ - series controller

$G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)(1 + T_d s)$. *1 tuning rule.*

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Ho and Xu [90] *Model: Method 1*	$\frac{\omega_p T_{m1}}{A_m K_m}$	$1.57\omega_p - \omega_p{}^2\tau_m - \frac{1}{T_{m1}}$	T_{m2}	$\omega_p = \frac{A_m\phi_m + 1.57A_m(A_m - 1)}{(A_m{}^2 - 1)\tau_m}$

Table 98: PID tuning rules - unstable SOSPD model $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{(1-sT_{m1})(1+sT_{m2})}$ - non-interacting controller

$$U(s) = K_c\left(1+\frac{1}{T_i s}\right)E(s) - \frac{K_c T_d s}{1+\dfrac{sT_d}{N}}Y(s)$$. *2 tuning rules.*

Rule	K_c	T_i	T_d	Comment
Servo tuning	**Minimum performance index**			
Huang and Lin [155] - minimum IAE *Model: Method 2*	$K_c^{(107)}$ [4]	$T_i^{(107)}$	$T_d^{(107)}$	$T_{m2} \le T_{m1}$; $0.05 \le \dfrac{\tau_m}{T_{m1}} \le 0.4$

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4 $$K_c^{(107)} = -\frac{1}{K_m}\left[10.741 - 13.363\frac{\tau_m}{T_{m1}} + 0.099\left(\frac{\tau_m}{T_{m1}}\right)^{-1.344} + 727.914\frac{T_{m2}}{T_{m1}} - 708.481\left(\frac{T_{m2}}{T_{m1}}\right)^{0.995} + 9.915\frac{T_{m2}\tau_m}{T_{m1}^2}\right]$$
$$-\frac{1}{K_m}\left[84.273\left(\frac{T_{m1}}{\tau_m}\right)^{1.031}\left(\frac{T_{m2}}{T_{m1}}\right)^{0.997} - 90.959\frac{T_{m2}}{\tau_m} + 9.034e^{\frac{\tau_m}{T_{m1}}} - 2.386e^{\frac{T_{m2}}{T_{m1}}} - 16.304e^{\frac{T_{m2}\tau_m}{T_{m1}^2}}\right]$$

$$T_i^{(107)} = T_{m1}\left[-149.685 - 141.418\frac{\tau_m}{T_{m1}} - 88.717\left(\frac{\tau_m}{T_{m1}}\right)^{2.12} - 17.29\frac{T_{m2}}{T_{m1}} + 20.518\left(\frac{T_{m2}}{T_{m1}}\right)^{0.985} - 12.82\frac{T_{m2}\tau_m}{T_{m1}^2}\right]$$
$$+T_{m1}\left[3.611\left(\frac{\tau_m}{T_{m1}}\right)^{0.286}\left(\frac{T_{m2}}{T_{m1}}\right)^{1.988} + 0.000805\frac{T_{m2}}{\tau_m} + 141.702e^{\frac{\tau_m}{T_{m1}}} - 2.032e^{\frac{T_{m2}}{T_{m1}}} + 10.006e^{\frac{T_{m2}\tau_m}{T_{m1}^2}}\right]$$

$$T_d^{(107)} = T_{m1}\left[-0.4144 + 15.805\frac{\tau_m}{T_{m1}} - 142.327\left(\frac{\tau_m}{T_{m1}}\right)^2 + 0.7287\frac{T_{m2}}{T_{m1}} + 0.1123\left(\frac{T_{m2}}{T_{m1}}\right)^2 - 18.317\frac{T_{m2}\tau_m}{T_{m1}^2}\right]$$
$$+T_{m1}\left[486.95\left(\frac{\tau_m}{T_{m1}}\right)^3 - 10.542\left(\frac{T_{m2}}{T_{m1}}\right)^3 + 204.009\left(\frac{\tau_m^2 T_{m2}}{T_{m1}^3}\right) + 47.26\left(\frac{\tau_m T_{m2}^2}{T_{m1}^3}\right) + 396.349\left(\frac{\tau_m}{T_{m1}}\right)^4\right]$$
$$+T_{m1}\left[-138.038\left(\frac{\tau_m^3 T_{m2}}{T_{m1}^4}\right) + 52.155\left(\frac{\tau_m T_{m2}^3}{T_{m1}^4}\right) - 646.848\left(\frac{\tau_m^2 T_{m2}^2}{T_{m1}^4}\right) + 19.302\left(\frac{T_{m2}}{T_{m1}}\right)^4 - 4731.72\left(\frac{\tau_m}{T_{m1}}\right)^5\right]$$
$$+T_{m1}\left[425.789\left(\frac{\tau_m^4 T_{m2}}{T_{m1}^5}\right) - 289.746\left(\frac{\tau_m T_{m2}^4}{T_{m1}^5}\right) - 841.807\left(\frac{\tau_m^3 T_{m2}^2}{T_{m1}^5}\right) + 1313.72\left(\frac{\tau_m^2 T_{m2}^3}{T_{m1}^5}\right) - 3.7688\left(\frac{T_{m2}}{T_{m1}}\right)^5\right]$$
$$+T_{m1}\left[6264.79\left(\frac{\tau_m}{T_{m1}}\right)^6 - 161.469\left(\frac{\tau_m^5 T_{m2}}{T_{m1}^6}\right) + 204.689\left(\frac{\tau_m T_{m2}^5}{T_{m1}^6}\right) + 25.706\left(\frac{\tau_m^4 T_{m2}^2}{T_{m1}^6}\right)\right]$$
$$+T_{m1}\left[-791.857\left(\frac{\tau_m^2 T_{m2}^4}{T_{m1}^6}\right) + 648.217\left(\frac{\tau_m T_{m2}}{T_{m1}^2}\right)^3 - 5.71\left(\frac{T_{m2}}{T_{m1}}\right)^6\right]$$

Rule	K_c	T_i	T_d	Comment
Huang and Lin [155] - minimum IAE - continued *Model: Method 2*	$K_c^{(108)}$ [5]	$T_i^{(108)}$	$T_d^{(108)}$	$T_{m1} < T_{m2} \le 10T_{m1}$; $0.05 \le \frac{\tau_m}{T_{m1}} \le 0.25$

[5] $$K_c^{(108)} = -\frac{1}{K_m}\left[-130.2 + 85.914\frac{\tau_m}{T_{m1}} + 34.82\left(\frac{\tau_m}{T_{m1}}\right)^{-0.3055} + 10.442\frac{T_{m2}}{T_{m1}} - 22.547\left(\frac{T_{m2}}{T_{m1}}\right)^{0.5174} - 14.698\frac{T_{m2}\tau_m}{T_{m2}^{\;2}}\right]$$
$$-\frac{1}{K_m}\left[52.408\left(\frac{T_{m1}}{\tau_m}\right)^{1.0077}\left(\frac{T_{m2}}{T_{m1}}\right)^{0.9879} - 51.47\left(\frac{T_{m2}}{\tau_m}\right) + 53.378e^{\frac{\tau_m}{T_{m1}}} - 0.000001e^{\frac{T_{m2}}{T_{m1}}} + 0.286e^{\frac{T_{m2}\tau_m}{T_{m1}^{\;2}}}\right]$$

$$T_i^{(108)} = T_{m1}\left[72.806 - 268.746\frac{\tau_m}{T_{m1}} - 4.9221\left(\frac{T_{m2}}{T_{m1}}\right) + 2468.19\left(\frac{\tau_m}{T_{m1}}\right)^2 + 0.6724\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 151.351\frac{T_{m2}\tau_m}{T_{m1}^{\;2}}\right]$$
$$+T_{m1}\left[-6914.46\left(\frac{\tau_m}{T_{m1}}\right)^3 - 0.0092\left(\frac{T_{m2}}{T_{m1}}\right)^3 - 795.465\left(\frac{\tau_m^{\;2}T_{m2}}{T_{m1}^{\;3}}\right) - 14.27\left(\frac{\tau_m T_{m2}^{\;2}}{T_{m1}^{\;3}}\right) + 5580.17\left(\frac{\tau_m}{T_{m1}}\right)^4\right]$$
$$+T_{m1}\left[1417.65\left(\frac{\tau_m^{\;3}T_{m2}}{T_{m1}^{\;4}}\right) + 0.4057\left(\frac{\tau_m T_{m2}^{\;3}}{T_{m1}^{\;4}}\right) + 55.536\left(\frac{\tau_m^{\;2}T_{m2}^{\;2}}{T_{m1}^{\;4}}\right) - 0.001119\left(\frac{T_{m2}}{T_{m1}}\right)^4\right]$$
$$+T_{m1}\left[-44.903e^{\frac{\tau_m}{T_{m1}}} + 0.000034e^{\frac{T_{m2}}{T_{m1}}} - 15.694e^{\frac{T_{m2}\tau_m}{T_{m1}^{\;2}}} + 678778\left(\frac{\tau_m}{T_{m1}}\right)^{19.056}\left(\frac{T_{m2}}{T_{m1}}\right)^{7.3464}\right]$$

$$T_d^{(108)} = T_{m1}\left[175.515 - 86.2\frac{\tau_m}{T_{m1}} + 348.727\left(\frac{\tau_m}{T_{m1}}\right)^{1.1798} - 0.008207\frac{T_{m2}}{T_{m1}} - 55.619\left(\frac{T_{m2}}{T_{m1}}\right)^{0.1064} + 0.0418\frac{T_{m2}\tau_m}{T_{m1}^{\;2}}\right]$$
$$+T_{m1}\left[78.959\left(\frac{\tau_m}{T_{m1}}\right)^{0.0355}\left(\frac{T_{m2}}{T_{m1}}\right)^{0.0827} + 0.005048\left(\frac{T_{m2}}{\tau_m}\right) - 187.01e^{\frac{\tau_m}{T_{m1}}} + 0.000001e^{\frac{T_{m2}}{T_{m1}}} - 0.0149e^{\frac{T_{m2}\tau_m}{T_{m1}^{\;2}}}\right]$$

Rule	K_c	T_i	T_d	Comment
Regulator tuning	**Minimum performance index**			
Huang and Lin [155] – minimum IAE *Model: Method 2*	$K_c^{(109)}$ [6]	$T_i^{(109)}$	$T_d^{(109)}$	$T_{m2} \le T_{m1}$; $0.05 \le \frac{\tau_m}{T_{m1}} \le 0.4$

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[6] $$K_c^{(109)} = -\frac{1}{K_m}\left[-174.167 - 31.364\frac{\tau_m}{T_{m1}} + 0.4642\left(\frac{\tau_m}{T_{m1}}\right)^{-1.164} - 103.069\frac{T_{m2}}{T_{m1}} - 83.916\left(\frac{T_{m2}}{T_{m1}}\right)^{2.54} - 66.962\frac{T_{m2}\tau_m}{T_{m1}^2}\right]$$
$$-\frac{1}{K_m}\left[59.496\left(\frac{T_{m1}}{\tau_m}\right)^{1.065}\left(\frac{T_{m2}}{T_{m1}}\right)^{1.014} - 70.79\frac{T_{m2}}{\tau_m} + 23.121e^{\frac{\tau_m}{T_{m1}}} + 126.924e^{\frac{T_{m2}}{T_{m1}}} + 26.944e^{\frac{T_{m2}\tau_m}{T_{m1}^2}}\right]$$

$$T_i^{(109)} = T_{m1}\left[0.008 + 2.0718\frac{\tau_m}{T_{m1}} + 6.431\left(\frac{\tau_m}{T_{m1}}\right)^2 + 0.4556\frac{T_{m2}}{T_{m1}} + 0.7503\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 2.4484\frac{T_{m2}\tau_m}{T_{m1}^2} - 18.686\left(\frac{\tau_m}{T_{m1}}\right)^3\right]$$
$$+T_{m1}\left[-2.9978\left(\frac{T_{m2}}{T_{m1}}\right)^3 - 21.135\left(\frac{T_{m2}\tau_m^2}{T_{m1}^3}\right) + 12.822\left(\frac{\tau_m T_{m2}^2}{T_{m1}^3}\right) + 39.001\left(\frac{\tau_m}{T_{m1}}\right)^4 + 22.848\left(\frac{T_{m2}\tau_m^3}{T_{m1}^4}\right)\right]$$
$$+T_{m1}\left[-4.754\left(\frac{T_{m2}^3\tau_m}{T_{m1}^4}\right) - 0.527\left(\frac{T_{m2}^2\tau_m^2}{T_{m1}^4}\right) + 1.64\left(\frac{T_{m2}}{T_{m1}}\right)^4\right]$$

$$T_d^{(109)} = T_{m1}\left[-0.0301 + 1.1766\frac{\tau_m}{T_{m1}} - 4.4623\left(\frac{\tau_m}{T_{m1}}\right)^2 + 0.5284\frac{T_{m2}}{T_{m1}} - 1.4281\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 4.6\frac{T_{m2}\tau_m}{T_{m1}^2} + 11.176\left(\frac{\tau_m}{T_{m1}}\right)^3\right]$$
$$+T_{m1}\left[1.0886\left(\frac{T_{m2}}{T_{m1}}\right)^3 - 5.0229\left(\frac{\tau_m^2 T_{m2}}{T_{m1}^3}\right) - 0.5039\left(\frac{\tau_m T_{m2}^2}{T_{m1}^3}\right) - 9.8564\left(\frac{\tau_m}{T_{m1}}\right)^4 - 7.528\left(\frac{T_{m2}\tau_m^3}{T_{m1}^4}\right)\right]$$
$$+T_{m1}\left[-2.3542\left(\frac{\tau_m T_{m2}^3}{T_{m1}^4}\right) + 9.3804\left(\frac{\tau_m^2 T_{m2}^2}{T_{m1}^4}\right) - 0.1457\left(\frac{T_{m2}}{T_{m1}}\right)^4\right]$$

Rule	K_c	T_i	T_d	Comment
Huang and Lin [155] - minimum IAE – continued *Model: Method 2*	$K_c^{(110)}$ [7]	$T_i^{(110)}$	$T_d^{(110)}$	$T_{m1} < T_{m2} \le 10T_{m1}$; $0.05 \le \frac{\tau_m}{T_{m1}} \le 0.25$

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[7] $$K_c^{(110)} = -\frac{1}{K_m}\left[1750.08 + 1637.76\frac{\tau_m}{T_{m1}} + 1533.91\left(\frac{\tau_m}{T_{m1}}\right)^{2.1984} - 7.917\frac{T_{m2}}{T_{m1}} + 6.187\left(\frac{T_{m2}}{T_{m1}}\right)^{0.791} - 6.451\frac{T_{m2}\tau_m}{T_{m1}^2}\right]$$
$$-\frac{1}{K_m}\left[0.002452\left(\frac{T_{m1}}{\tau_m}\right)^{3.2927}\left(\frac{T_{m2}}{T_{m1}}\right)^{1.0757} + 1.3729\frac{T_{m2}}{\tau_m} - 1739.77e^{\frac{\tau_m}{T_{m1}}} - 0.000296e^{\frac{T_{m2}}{T_{m1}}} + 2.311e^{\frac{T_{m2}\tau_m}{T_{m1}^2}}\right]$$

$$T_i^{(110)} = T_{m1}\left[51.678 - 57.043\frac{\tau_m}{T_{m1}} + 1337.29\left(\frac{\tau_m}{T_{m1}}\right)^2 + 0.1742\frac{T_{m2}}{T_{m1}} - 0.1524\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 7.7266\frac{T_{m2}\tau_m}{T_{m1}^2} - 6011.57\left(\frac{\tau_m}{T_{m1}}\right)^3\right]$$
$$+T_{m1}\left[0.0213\left(\frac{T_{m2}}{T_{m1}}\right)^3 - 65.283\left(\frac{T_{m2}\tau_m^2}{T_{m1}^3}\right) + 0.0645\left(\frac{\tau_m T_{m2}^2}{T_{m1}^3}\right) + 9135.52\left(\frac{\tau_m}{T_{m1}}\right)^4 + 274.851\left(\frac{T_{m2}\tau_m^3}{T_{m1}^4}\right)\right]$$
$$+T_{m1}\left[0.003926\left(\frac{T_{m2}^3\tau_m}{T_{m1}^4}\right) - 2.0997\left(\frac{T_{m2}^2\tau_m^2}{T_{m1}^4}\right) - 0.001077\left(\frac{T_{m2}}{T_{m1}}\right)^4 - 49.007e^{\frac{\tau_m}{T_{m1}}} + 0.000026e^{\frac{T_{m2}}{T_{m1}}}\right]$$
$$+T_{m1}\left[0.2977e^{\frac{T_{m2}\tau_m}{T_{m1}}}\right]$$

$$T_d^{(110)} = T_{m1}\left[-0.0605 + 4.6998\frac{\tau_m}{T_{m1}} - 29.478\left(\frac{\tau_m}{T_{m1}}\right)^2 + 0.0117\frac{T_{m2}}{T_{m1}} - 0.0129\left(\frac{T_{m2}}{T_{m1}}\right)^2 + 0.6874\frac{T_{m2}\tau_m}{T_{m1}^2} + 140.135\left(\frac{\tau_m}{T_{m1}}\right)^3\right]$$
$$+T_{m1}\left[0.002455\left(\frac{T_{m2}}{T_{m1}}\right)^3 - 1.4712\left(\frac{\tau_m^2 T_{m2}}{T_{m1}^3}\right) - 0.1289\left(\frac{\tau_m T_{m2}^2}{T_{m1}^3}\right) - 238.864\left(\frac{\tau_m}{T_{m1}}\right)^4 + 0.6884\left(\frac{T_{m2}\tau_m^3}{T_{m1}^4}\right)\right]$$
$$+T_{m1}\left[0.007725\left(\frac{\tau_m T_{m2}^3}{T_{m1}^4}\right) - 0.1222\left(\frac{\tau_m^2 T_{m2}^2}{T_{m1}^4}\right) - 0.000135\left(\frac{T_{m2}}{T_{m1}}\right)^4\right]$$

Table 99: PID tuning rules – general model with a repeated pole $G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_m)^n}$ - ideal controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$. *1 tuning rule.*

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Skoczowski and Tarasiejski [156] *Model: Method 1*	$\le \frac{\omega_g T_m}{K_m}\sqrt{\frac{\left(1+\omega_g{}^2 T_m{}^2\right)^{n-1}}{1+\omega_g{}^2 T_d{}^2}}$ [1]	T_m	$\frac{n-1}{n+2}T_m$, $n \ge 2$	

[1] $$\omega_g = \frac{-T_m\left[n^2 - 2n - 2 + \frac{2n+4}{\pi}\frac{\tau_m}{T_m} + \frac{4n+2}{\pi}\phi_m\right] \pm b}{2T_m{}^2\left[n^2 - 4n + 3 + \frac{4n+2}{\pi}\frac{\tau_m}{T_m} + \frac{2n-2}{\pi}\phi_m\right]}$$

with

$$b = T_m\sqrt{\left[n^2 - 2n - 2 + \frac{2n+4}{\pi}\frac{\tau_m}{T_m} + \frac{4n+2}{\pi}\phi_m{}^2\right]^2 + 4(n+2)\left(1 - \frac{2}{\pi}\phi_m\right)\left[n^2 - 4n + 3 + \frac{4n+2}{\pi}\frac{\tau_m}{T_m} + \frac{2n-2}{\pi}\phi_m\right]}$$

Table 100: PID tuning rules – general stable non-oscillating model with a time delay - ideal controller $G_c(s)=K_c\left(1+\frac{1}{T_i s}+T_d s\right)$. *1 tuning rule*.

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Gorez and Klan [147a] *Model: not specified*	$K_c^{(110a)}$ [2]	$T_i^{(110a)}$	$T_d^{(110a)}$	
	$K_c^{(110a)}$	$T_i^{(110a)}$	$T_i^{(110a)}\frac{\tau_m}{T_{ar}}\left(1-\frac{\tau_m}{T_{ar}}\right)$	
	$K_c^{(110a)}$	$T_i^{(110a)}$	$0.25T_i^{(110a)}$	

[2] $K_c^{(110a)}=\frac{T_i^{(110a)}}{T_i^{(110a)}+\tau_m}$, $T_i^{(110a)}=T_{ar}\frac{1+\sqrt{1+2\left(\frac{\tau_m}{T_{ar}}\right)^2}-2\frac{\tau_m}{T_{ar}}}{2}$,

$T_d^{(110a)}=\frac{T_{ar}}{T_i^{(110a)}}\left[\left(T_i^{(110a)}+T_{cr}\right)\frac{T_{cr}}{T_{ar}}+T_i^{(110a)}-T_{aa}-\frac{\tau_m^2}{2T_{ar}}\left(1-K_c^{(110a)}\left\{1+\frac{2\tau_m}{3T_i^{(110a)}}\right\}\right)\right]$

T_{ar} = average residence time of the process (which equals $T_m+\tau_m$ for a FOLPD process, for example); T_{aa} = additional apparent time constant; $T_{cr}=\left[1-K_c^{(110a)}\left(1+\frac{\tau_m}{2T_i^{(110a)}}\right)\right]\tau_m$

Table 101: PID tuning rules – fifth order model with delay $G_m(s) = \frac{K_m(1 + b_1 s + b_2 s^2 + b_3 s^3 + b_4 s^4 + b_5 s^5)e^{-s\tau_m}}{(1 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5)}$ – ideal controller $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$.

1 tuning rule.

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Magnitude optimum - Vrancic *et al*. [159] *Model: Method 1*	[3] $K_c^{(112)}$	$T_i^{(112)}$	$T_d^{(112)}$	

3

$$K_c^{(112)} = \frac{a_1^3 - a_1^2 b_1 + a_1 b_2 - 2a_1 a_2 + a_2 b_1 + a_3 - b_3 + \tau_m\left(a_1^2 - a_1 b_1 - a_2 + b_2\right) + 0.5\left(a_1 - b_1\right)\tau_m^2 + 0.167\tau_m^3}{2K_m\left[-a_1^2 b_1 + a_1 a_2 + a_1 b_1^2 - a_3 - b_1 b_2 + b_3 + \left(a_1 - b_1\right)^2 \tau_m + \left(a_1 - b_1\right)\tau_m^2 + 0.333\tau_m^3 - \left(a_1 - b_1 + \tau_m\right)^2 T_d\right]}$$

$$T_i^{(112)} = \frac{a_1^3 - a_1^2 b_1 + a_1 b_2 - 2a_1 a_2 + a_2 b_1 + a_3 - b_3 + \tau_m\left(a_1^2 - a_1 b_1 - a_2 + b_2\right) + 0.5\left(a_1 - b_1\right)\tau_m^2 + 0.167\tau_m^3}{\left[a_1^2 - a_1 b_1 - a_2 + b_2 + \left(a_1 - b_1\right)\tau_m + 0.5\tau_m^2 - \left(a_1 - b_1 + \tau_m\right)T_d\right]}$$

$T_d^{(112)}$ … see attached sheet with $\delta = 0$.

Table 102: PID tuning rules – fifth order model with delay

$$G_m(s) = \frac{K_m(1 + b_1 s + b_2 s^2 + b_3 s^3 + b_4 s^4 + b_5 s^5)e^{-s\tau_m}}{\left(1 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5\right)}$$ – controller with filtered derivative

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d s}{N}}\right)$$. *1 tuning rule.*

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Magnitude optimum - Vrancic *et al*. [73] *Model: Method 1*	$K_c^{(113)}$ [4]	$T_i^{(113)}$	$T_d^{(113)}$	$8 \le N \le 20$

4

$$K_c^{(113)} = \frac{a_1^3 - a_1^2 b_1 + a_1 b_2 - 2a_1 a_2 + a_2 b_1 + a_3 - b_3 + \tau_m\left(a_1^2 - a_1 b_1 - a_2 + b_2\right) + 0.5(a_1 - b_1)\tau_m^2 + 0.167\tau_m^3}{2K_m\left[-a_1^2 b_1 + a_1 a_2 + a_1 b_1^2 - a_3 - b_1 b_2 + b_3 + (a_1 - b_1)^2 \tau_m + (a_1 - b_1)\tau_m^2 + 0.333\tau_m^3 - (a_1 - b_1 + \tau_m)^2 T_d - \frac{T_d^2}{N}(a_1 - b_1 + \tau_m)\right]}$$

$$T_i^{(113)} = \frac{a_1^3 - a_1^2 b_1 + a_1 b_2 - 2a_1 a_2 + a_2 b_1 + a_3 - b_3 + \tau_m\left(a_1^2 - a_1 b_1 - a_2 + b_2\right) + 0.5(a_1 - b_1)\tau_m^2 + 0.167\tau_m^3}{\left[a_1^2 - a_1 b_1 - a_2 + b_2 + (a_1 - b_1)\tau_m + 0.5\tau_m^2 - (a_1 - b_1 + \tau_m)T_d - \frac{T_d^2}{N}\right]}$$

$T_d^{(113)}$ = see attached sheet

4. Conclusions

The report has presented a comprehensive summary of the tuning rules for PI and PID controllers that have been developed to compensate SISO processes with time delay. Further work will concentrate on evaluating the applicability of these tuning rules to the compensation of processes with time delays, as the value of the time delay varies compared with the other dynamic variables.

Table 103: Tuning rules published by year and medium

Year	Journal articles	Conference papers/ correspondence	Books/ Ph.D. thesis	Total
1942	1			1
1950	1			1
1951	1			1
1952	1			1
1953	3			3
1954	1			1
1961			1	1
1964			1	1
1965	1			1
1967	1		1	2
1968	1			1
1969	2			2
1972	2			2
1973	1		1	2
1975	2			2
1979			1	1
1980	2			2
1982		1		1
1984	2	1		3
1985		1	1	2
1986	1			1
1987	1			1
1988	3	4	2	9
1989	4		3	7
1990	4		1	5
1991	5			5
1992	1	2		3
1993	5	4	1	10
1994	5	2	2	9
1995	14	2	1	17
1996	8	6	2	16
1997	11	4	1	16
1998	6	7		13
1999	12	1	1	14
2000	1	16		17
1941-1950	2	0	0	2

1951-1960	6	0	0	6
1961-1970	5	0	3	8
1971-1980	7	0	2	9
1981-1990	15	7	7	29
1991-2000	68	44	8	120
TOTAL	**103**	**51**	**20**	**174**

List of journals in which tuning rules were published and number of tuning rules published

Advances in Modelling and Analysis C, ASME Press	1
AIChE Journal	4
Automatica	12
British Chemical Engineering	1
Chemical Engineering Communications	5
Chemical Engineering Progress	1
Chemical Engineering Science	1
Control	1
Control and Computers	1
Control Engineering	7
Control Engineering Practice	4
European Journal of Control	1
Hydrocarbon Processing	2
Hungarian Journal of Industrial Chemistry	1
IEE Proceedings, Part D	9
(including IEE Proceedings - Control Theory and Applications, Proceedings of the IEE, Part 2)	
IEEE Control Systems Magazine	1
IEEE Transactions on Control Systems Technology	4
IEEE Transactions on Industrial Electronics and Control Instrumentation	1
IEEE Transactions on Industrial Electronics	1
IEEE Transactions on Industry Applications	1
Industrial and Engineering Chemistry Process Design and Development	2
Industrial Engineering Chemistry Research	12
International Journal of Adaptive Control and Signal Processing	1
International Journal of Control	3
International Journal of Electrical Engineering Education	1
International Journal of Systems Science	1
Instrumentation	1
Instrumentation Technology	2
Instruments and Control Systems	2
ISA Transactions	2
Journal of Chemical Engineering of Japan	2
Journal of the Chinese Institute of Chemical Engineers	3
Pulp and Paper Canada	1
Process Control and Quality	1
Thermal Engineering (Russia)	2
Transactions of the ASME	7
(including Transactions of the ASME. Journal of Dynamic Systems, Measurement and Control)	
Transactions of the Institute of Chemical Engineers	1

Classification of journals in which tuning rules were published and number of tuning rules published

Chemical Engineering Journals **32**

(AIChE Journal, British Chemical Engineering, Chemical Engineering Communications, Chemical Engineering Progress, Chemical Engineering Science, Transactions of the Institute of Chemical Engineers, Hungarian Journal of Industrial Chemistry, Industrial and Engineering Chemistry Process Design and Development, Industrial

Engineering Chemistry Research, Journal of Chemical Engineering of Japan, Journal of the Chinese Institute of Chemical Engineers)

Control Engineering Journals **53**

(Automatica, Control, Control and Computers, Control Engineering, Control Engineering Practice, European Journal of Control, IEE Proceedings, Part D, IEE Proceedings - Control Theory and Applications, Proceedings of the IEE, Part 2, IEEE Control Systems Magazine, IEEE Transactions on Control Systems Technology, International Journal of Adaptive Control and Signal Processing, International Journal of Control, International Journal of Systems Science, Instrumentation, Instrumentation Technology, Instruments and Control Systems, ISA Transactions, Process Control and Quality)

Mechanical Engineering Journals **8**

(Advances in Modelling and Analysis C, ASME Press, Transactions of the ASME, Transactions of the ASME. Journal of Dynamic Systems, Measurement and Control)

Electrical/Electronic Engineering Journals **4**

(EE Transactions on Industrial Electronics and Control Instrumentation, IEEE Transactions on Industrial Electronics, IEEE Transactions on Industry Applications, International Journal of Electrical Engineering Education)

Trade Journals **5**

(Hydrocarbon Processing, Pulp and Paper Canada, Thermal Engineering (Russia))

5. References

1. Koivo, H.N. and Tanttu, J.T., Tuning of PID Controllers: Survey of SISO and MIMO techniques. Proceedings of the IFAC Intelligent Tuning and Adaptive Control Symposium, Singapore, 1991, 75-80.
2. Hwang, S-H., Adaptive dominant pole design of PID controllers based on a single closed-loop test. Chemical Engineering Communications, 1993, 124, 131-152.
3. Astrom, K.J. and Hagglund, T., PID Controllers: Theory, Design and Tuning. Instrument Society of America, Research Triangle Park, North Carolina, 2nd Edition, 1995.
4. Bialkowski, W.L., Control of the pulp and paper making process. The Control Handbook. Editor: W.S. Levine, CRC/IEEE Press, Boca Raton, Florida, 1996, 1219-1242.
5. Isermann, R., Digital Control Systems Volume 1. Fundamentals, Deterministic Control. 2nd Revised Edition, Springer-Verlag, 1989.
6. Ender, D.B., Process control performance: not as good as you think. Control Engineering, 1993, September, 180-190.
7. Astrom, K.J. and Wittenmark, B., Computer controlled systems: theory and design. Prentice-Hall International Inc., 1984.
8. Ziegler, J.G. and Nichols, N.B., Optimum settings for automatic controllers. Transactions of the ASME, 1942, November, 759-768.

9. Hazebroek, P. and Van der Waerden, B.L., The optimum adjustment of regulators. Transactions of the ASME, 1950, April, 317-322.

10. Chien, K.-L., Hrones, J.A. and Reswick, J.B., On the automatic control of generalised passive systems. Transactions of the ASME, 1952, February, 175-185.

11. Cohen, G.H. and Coon, G.A., Theoritical considerations of retarded control. Transactions of the ASME, 1953, May, 827-834.

12. Wolfe, W.A., Controller settings for optimum control. Transactions of the ASME, 1951, May, 413-418.

13. Murrill, P.W., Automatic control of processes. International Textbook Co., 1967.

14. McMillan, G.K., Tuning and control loop performance - a practitioner's guide. Instrument Society of America, Research Triangle Park, North Carolina, 3rd Edition, 1994.

15. St. Clair, D.W., Controller tuning and control loop performance, Straight Line Control Co., Inc., 2nd Edition, 1997.

15a. Shinskey, F.G. (2000). PID-deadtime control of distributed processes, Preprints of the *Proceedings of PID '00: IFAC Workshop on digital control (Past, present and future of PID control)*, Terrassa, Spain, pp. 14-18.

16. Shinskey, F.G., Process Control Systems - Application, Design and Tuning. McGraw-Hill Inc., New York, 3rd Edition, 1988.

17. Shinskey, F.G., Process Control Systems - Application, Design and Tuning. McGraw-Hill Inc., New York, 4th Edition, 1996.

18. Huang, C.-T., Chou, C.-J. and Wang, J.-L., Tuning of PID controllers based on second-order model by calculation. Journal of the Chinese Institute of Chemical Engineers, 1996, 27(2), 106-120.

19. Yu, S. W., Optimal PI tuning for load disturbances. Journal of the Chinese Institute of Chemical Engineers, 1988, 19(6), 349-357.

20. Zhuang, M. and Atherton, D.P., Automatic tuning of optimum PID controllers. IEE Proceedings, Part D, 1993, 140(3), 216-224.

21. Rovira, A.A., Murrill, P.W. and Smith, C.L., Tuning controllers for setpoint changes. Instruments and Control Systems, 1969, 42, December, 67-69.

22. Khan, B.Z. and Lehman, B., Setpoint PI controllers for systems with large normalised dead time. IEEE Transactions on Control Systems Technology, 1996, 4(4), 459-466.

23. Haalman, A., Adjusting controllers for a deadtime process. Control Engineering, 1965, July, 71-73.

23a. Chen, C.-L. and Yang, S.-F., PI tuning based on peak amplitude ratio, Preprints of the *Proceedings of PID '00: IFAC Workshop on digital control (Past, present and future of PID control)*, Terrassa, Spain, 2000, pp. 195-198.

24. Pemberton, T.J., PID: The logical control algorithm. Control Engineering, 1972, 19(5), 66-67.

25. Smith, C.A. and Corripio, A.B., Principles and practice of automatic process control. John Wiley and Sons, New York, 2nd Edition, 1997.

26. Smith, C.L., Corripio, A.B. and Martin, J. (1975). Controller tuning from simple process models, *Instrumentation Technology*, **December**, 39-44.

27. Hang, C.C., Tan, C.H. and Chan, W.P., A performance study of control systems with dead time. IEEE Transactions on Industrial Electronics and Control Instrumentation, 1980, IECI-27(3), 234-241.

27a. Miluse, V., Vitecek, A. and Smutny, L. (2000a). Controller tuning for controlled plants with time delay, Preprints of the *Proceedings of PID '00: IFAC Workshop on digital control (Past, present and future of PID control)*, Terrassa, Spain, pp. 283-288.

27b. Miluse, V., Vitecek, A. and Smutny, L. (2000a). Controller tuning for controlled plants with time delay, Preprints of the *Proceedings of PID '00: IFAC Workshop on digital control (Past, present and future of PID control)*, Terrassa, Spain, pp. 289-294.

28. Gorecki, H., Fuska, S., Grabowski, P. and Korytowski, A., Analysis and synthesis of time delay systems, John Wiley and Sons, New York, 1989.

29. Chiu, K.C., Corripio, A.B. and Smith, C.L., Digital controller algorithms. Part III. Tuning PI and PID controllers. Instruments and Control Systems, 1973, December, 41-43.

30. Astrom, K.J., Hagglund, T., Hang, C.C. and Ho., W.K., Automatic tuning and adaptation for PID controllers - a survey. Control Engineering Practice, 1993, 1(4), 699-714.

31. Davydov, N.I., Idzon, O.M. and Simonova, O.V., Determining the parameters of PID-controller settings using the transient response of the controlled plant. Thermal Engineering, 1995, 42(10), 801-807.

32. Schneider, D.M., Control of processes with time delay. IEEE Transactions on Industry Applications, 1988, 24 (2), 186-191.

33. McAnany, D.E., A pole placement technique for optimum PID control parameters. Proceedings of the ISA/93 Advances in Instrumentation and Control Conference, McCormick Place, Chicago, Illinois, USA, 1993, 48, 1775-1782.

34. Leva, A., Maffezzoni, C. and Scattolini, R., Self-tuning PI-PID regulators for stable systems with varying delay. Automatica, 1994, 30(7), 1171-1183.

35. Hang, C.C., Ho, W.K. and Cao, L.S., A comparison of two design methods for PID controllers. Proceedings of the ISA/93 Advances in Instrumentation and Control Conference, McCormick Place, Chicago, Illinois, USA, 1993, 48, 959-967.

36. Hang, C.C., Lee, T.H. and Ho, W.K., Adaptive Control. Instrument Society of America, Reseaech Triangle Park, North Carolina, 1993.

37. Ho, W.K., Hang, C.C. and Zhou, J.H., Tuning of PID controllers based on gain and phase margin specifications. Automatica, 1995, 31(3), 497-502.

38. Kookos, I.K., Lygeros, A.I. and Arvanitis, K.G., On-line PI controller tuning for integrator/dead time processes. European Journal of Control, 1999, 5, 19-31.

39. Tan, K.K., Lee, T.H. and Wang, Q.G., Enhanced automatic tuning procedure for process control of PI/PID controllers. AIChE Journal, 1996, 42(9), 2555-2562.

40. Voda, A. and Landau, I.D., The autocalibration of PI controllers based on two frequency measurements. International Journal of Adaptive Control and Signal Processing, 1995, 9, 395-421.

41. Friman, M. and Waller, K.V., A two channel relay for autotuning. Industrial Engineering Chemistry Research, 1997, 36(7), 2662-2671.

42. Smith, L., A modified Smith predictor for extruded diameter control. InstMC Mini Symposium - Algorithms and Architectures for Industrial Controllers (in UKACC International Conference on Control '98), Swansea, Wales, Lecture 5, 1998.

43. Cox, C.S., Daniel, P.R. and Lowdon, A., Quicktune: a reliable automatic strategy for determining PI and PPI controller parameters using a FOLPD model. Control Engineering Practice, 1997, 5(10), 1463-1472.

44. Cluett, W.R. and Wang, L., New tuning rules for PID control. Pulp and Paper Canada, 1997, 3(6), 52-55.

45. Abbas, A., A new set of controller tuning relations. ISA Transactions, 1997, 36(3), 183-187.

46. Bi, Q., Cai, W.-J., Lee, E.-L., Wang, Q.-G., Hang, C.-C. and Zhang, Y., Robust identification of first-order plus dead-time model from step response, Control Engineering Practice, 1999, 7(1), 71-77.

47. Wang, Y.-G. and Shao, H.-H. (2000). Optimal tuning for PI controller, *Automatica*, **36**, 147-152.

48. Brambilla, A., Chen, S. and Scali, C., Robust tuning of conventional controllers. Hydrocarbon Processing, 1990, November, 53-58.

49. Rivera, D.E., Morari, M. and Skogestad, S., Internal Model Control. 4. PID controller design. Industrial and Engineering Chemistry Process Design and Development, 1986, 25(1), 252-265.

50. Chien, I-L., IMC-PID controller design - an extension. Proceedings of the IFAC Adaptive Control of Chemical Processes Conference, Copenhagen, Denmark, 1988, 147-152.

51. Thomasson, F.Y., Tuning guide for basic control loops. Proceedings of the 1997 process control, electrical and information conference (TAAPI), 1997, 137-148.

52. Fruehauf, P.S., Chien, I.-L. and Lauritsen, M.D., Simplified IMC-PID tuning rules. Proceedings of the ISA/93 Advances in Instrumentation and Control Conference, McCormick Place, Chicago, Illinois, USA, 1993, 48, 1745-1766.

53. Chen, C.-L., Huang, H.-P. and Hsieh, C.-T. (1999). Tuning of PI/PID controllers based on specification of maximum closed-loop amplitude ratio, *Journal of Chemical Engineering of Japan*, **32**, 6, 783-788.

54. Ogawa, S., PI controller tuning for robust performance. Proceedings of the IEEE Conference on Control Applications, 1995, 101-106.

55. Lee, Y., Park, S., Lee, M. and Brosilow, C., PID controller tuning for desired closed-loop responses for SI/SO systems. AIChE Journal, 1998, 44(1), 106-115.

56. Isaksson, A.J. and Graebe, S.F., Analytical PID parameter expressions for higher order systems, Automatica, 1999, 35, 1121-1130.

57. Chun, D., Choi, J.Y. and Lee, J., Parallel compensation with a secondary measurement, Industrial Engineering Chemistry Research, 1999, 38(4), 1575-1579.

58. McMillan, G.K., Control loop performance. Proceedings of the ISA/84 International Conference and Exhibition. Advances in Instrumentation, Houston, Texas, USA, 1984, 39, 1, 589-603.

59. Shinskey, F.G., Feedback controllers for the process industries. McGraw-Hill Inc., New York, 1994.

60. Hwang, S.-H., Closed-loop automatic tuning of single-input-single-output systems. Industrial Engineering Chemistry Research, 1995, 34(7), 2406-2417.

61. Hwang, S.-H. and Fang, S.-M., Closed-loop tuning method based on dominant pole placement. Chemical Engineering Communications, 1995, 136, 45-66.

61a. Taguchi, H. and Araki, M., Two-degree-of-freedom PID controllers – their functions and optimal tuning, Preprints of the *Proceedings of PID '00: IFAC Workshop on digital control (Past, present and future of PID control)*, Terrassa, Spain, 2000, pp. 95-100.

62. Hwang, S.-H. and Chang, H.-C., A theoretical examination of closed-loop properties and tuning methods of single-loop PI controllers. Chemical Engineering Science, 1987, 42(10), 2395-2415.

63. Pessen, D.W., A new look at PID-controller tuning. Transactions of the ASME. Journal of Dynamic Systems, Measurement and Control, 1994, 116, 553-557.

64. Parr, E.A., Industrial Control Handbook, Vol. 3. BSP Professional Books, 1989.

65. Hang, C.C., Astrom, K.J. and Ho, W.K., Refinements of the Ziegler-Nichols tuning formula. IEE Proceedings, Part D, 1991, 138(2), 111-118.

66. Hagglund, T. and Astrom, K.J., Industrial adaptive controllers based on frequency response techniques. Automatica, 1991, 27(4), 599-609.

67. Leva, A., PID autotuning algorithm based on relay feedback. IEE Proceedings, Part D, 1993, 140(5), 328-338.

68. Astrom, K.J., Ziegler-Nichols auto-tuners. Report TFRT-3167, Department of Automatic Control, Lund Institute of Technology, Lund, Sweden, 1982.

69. Calcev, G. and Gorez, R., Iterative techniques for PID controller tuning. Proceedings of the 34th Conference on Decision and Control, New Orleans, LA ., USA, 1995, 3209-3210.

70. Cox, C.S., Arden, W.J.B. and Doonan, A.F., CAD Software facilities tuning of traditional and predictive control strategies. Proceedings of the ISA/94 International Conference, Exhibition and Training Program. Advances in Instrumentation and Control, Anaheim, CA., U.S.A., 1994, 49, 2, 241-250.

71. Vrancic, D., Peng, Y., Strmcnik, S. and Hanus, R., A new tuning method for PI controllers based on a process step response. Proceedings of the CESA '96 IMACS Multiconference Symposium on Control, Optimisation and Supervision, Lille, France, 1996, 2, 790-794.

72. Vrancic, D., Design of anti-windup and bumpless transfer protection. Part II: PID controller tuning by multiple integration method. PhD thesis, University of Ljubljana, J. Stefan Institute, Ljubljana, Slovenia, 1996.

73. Vrancic, D., Peng, Y., Strmcnik, S. and Juricic, D., A multiple integration tuning method for filtered PID controller. Proceedings of the IFAC 1999 14th World Congress, Beijing, China, 1999, Preprints, Paper 3b-02-3.

74. Chien, I.-L., Huang, H.-P. and Yang, J.-C., A simple multiloop tuning method for PID controllers with no proportional kick, Industrial Engineering Chemistry Research, 1999, 38(4), 1456-1468.

75. Tyreus, B.D. and Luyben, W.L., Tuning PI controllers for integrator/dead time processes. Industrial Engineering Chemistry Research, 1992, 31(11), 2625-2628.

76. Wang, L. and Cluett, W.R., Tuning PID controllers for integrating processes. IEE Proceedings - Control Theory and Applications, 1997, 144(5), 385-392.

77. Rotach, V. Ya. (1995). Automatic tuning of PID-controllers – expert and formal methods, *Thermal Engineering*, **42**, 10, 794-800.

78. Poulin, E. and Pomerleau, A.. PI settings for integrating processes based on ultimate cycle information. IEEE Transactions on Control Systems Technology, 1999, 7(4), 509-511.

79. Penner, A., Tuning rules for a PI controller. Proceedings of the ISA/88 International Conference and Exhibition, Houston, Texas, USA, 1988, 1037-1051.

80. Srividya, R. and Chidambaram, M., On-line controllers tuning for integrator plus delay systems. Process Control and Quality, 1997, 9, 59-66.

81. Tan, W., Liu, K. and Tam, P.K.S., PID tuning based on loop-shaping H_∞ control. IEE Proceedings - Control Theory and Applications, 1998, 145(6), 485-490.

82. Poulin, E. and Pomerleau. A., PID tuning for integrating and unstable processes. IEE Proceedings - Control Theory and Applications, 1996, 143(5), 429-435.

83. McAvoy, T.J. and Johnson, E.F. (1967). Quality of control problem for dead-time plants, *Industrial and Engineering Chemistry Process Design and Development*, **6**, 4, 440-446.

84. Lopez, A.M., Smith, C.L. and Murrill, P.W. (1969). An advanced tuning method, *British Chemical Engineering*, **14**, 11, 1553-1555.

85. Hougen, J.O., Measurement and Control Applications. Instrument Society of America, Research Triangle Park, North Carolina, 1979.

86. De Paor, A.M. and O'Malley, M., Controllers of Ziegler-Nichols type for unstable processes with time delay. International Journal of Control, 1989, 49(4), 1273-1284.

87. Venkatashankar, V. and Chidambaram, M., Design of P and PI controllers for unstable first order plus time delay systems. International Journal of Control, 1994, 60(1), 137-144.

88. Chidambaram, M., Design of PI and PID controllers for an unstable first-order plus time delay system. Hungarian Journal of Industrial Chemistry, 1995, 23, 123-127.

89. Rotstein, G.E. and Lewin, D.E., Simple PI and PID tuning for open-loop unstable systems. Industrial Engineering Chemistry Research, 1991, 30(8), 1864-1869.

90. Ho, W.K. and Xu, W., PID tuning for unstable processes based on gain and phase-margin specifications. IEE Proceedings - Control Theory and Applications, 1998, 145(5), 392-396.

91. Luyben, W.L., Tuning temperature controllers on openloop unstable reactors, Industrial Engineering Chemistry Research, 1998, 37, 4322-4331.

91a. Hansen, P.D. (2000). Robust adaptive PID controller tuning for unmeasured load rejection, Preprints of the *Proceedings of PID '00: IFAC Workshop on digital control (Past, present and future of PID control)*, Terrassa, Spain, pp. 487-494.

92. Poulin, E. and Pomerleau. A., Unified PID design method based on a maximum peak resonance specification. IEE Proceedings - Control Theory and Applications, 1997, 144(6), 566-574.

93. Astrom, K.J. and Hagglund, T., Automatic tuning of PID Controllers, Instrument Society of America, Research Triangle Park, North Carolina, 1988.

94. Sain, S.G. and Ozgen, C., Identification and tuning of processes with large deadtime. Control and Computers, 1992, 20(3), 73-78.

95. Cheng, G.S. and Hung, J.C., A Least-Squares Based Self-Tuning of PID Controller. Proceedings of the IEEE South East Conference, Raleigh, North Carolina, USA, 1985, 325-332.

96. Gerry, J.P., How to control processes with large dead times, http://www.expertune.com/artdt.html, 1998.

97. Wang, F.-S., Juang, W.-S. and Chan, C.-T., Optimal tuning of PID controllers for single and cascade control loops. Chemical Engineering Communications, 1995, 132, 15-34.

97a. Pi-Mira, J., Mateo, E., Sarrate-Estruch, R. and Quevedo-Casin, J., LS-3000 digital PID controller, Preprints of the *Proceedings of PID '00: IFAC Workshop on digital control (Past, present and future of PID control)*, Terrassa, Spain, 2000, pp. 465-471.

98. Astrom, K.J. and Hagglund, T., Automatic tuning of simple regulators with specifications on phase and amplitude margins. Automatica, 1984, 20(5), 645- 651.

99. Li, Z., Su., X. and Lin, P., A practical algorithm for PID auto-tuning. Advances in Modelling and Analysis C, ASME Press, 1994, 40(2), 17-27.

100. Suyama, K., A simple design method for sampled-data PID control systems with adequate step responses. Proceedings of the International Conference on Industrial Electronics, Control, Instrumentation and Automation, 1992, 1117-1122.

101. Juang, W.-S. and Wang, F.-S., Design of PID controller by concept of Dahlin's Law. Journal of the Chinese Institute of Chemical Engineers, 1995, 26(2), 133-136.

102. Camacho, O.E., Smith, C. and Chacon, E., Toward an implementation of sliding mode control to chemical processes, Proceedings of the IEEE International Symposium on Industrial Electronics, 1997, 3, 1101-1105.

103. Ho, W.K., Lim, K.W. and Xu, W., Optimal gain and phase margin tuning for PID controllers. Automatica, 1998, 34(8), 1009-1014.

104. Ho, W.K., Lim, K.W., Hang, C.C. and Ni, L.Y., Getting more phase margin and performance out of PID controllers, Automatica, 1999, 35, 1579-1585.

104a. Morilla, F., Gonzalez, A. and Duro, N., Auto-tuning PID controllers in terms of relative damping, Preprints of the *Proceedings of PID '00: IFAC Workshop on digital control (Past, present and future of PID control)*, Terrassa, Spain, 2000, pp. 161-166.

105. Morari, M. and Zafiriou, E., Robust process control. Prentice-Hall Inc., Englewood Cliffs, New Jersey, 1989.

106. Horn, I.G., Arulandu, J.R., Gombas, C.J., VanAntwerp, J.G. and Braatz, R.D., Improved filter design in internal model control. Industrial Engineering Chemistry Research, 1996, 35(10), 3437-3441.

106a. Normey-Rico, J.E., Alcala, I., Gomez-Ortega, J. and Camacho, E.F. (2000). Robust PID tuning application to a mobile robot path tracking problem, Preprints of the *Proceedings of PID '00: IFAC Workshop on digital control (Past, present and future of PID control)*, Terrassa, Spain, pp. 648-653.

107. Witt, S.D. and Waggoner, R.C., Tuning parameters for non-PID three-mode controllers. Hydrocarbon Processing, 1990, June, 74-78.

108. Kaya, A. and Scheib, T.J., Tuning of PID controls of different structures. Control Engineering, 1988, July, 62-65.

109. Tsang, K.M. and Rad, A. B., A new approach to auto-tuning of PID controllers. International Journal of Systems Science, 1995, 26(3), 639-658.

110. Tsang, K.M., Rad, A.B. and To, F.W., Online tuning of PID controllers using delayed state variable filters. Proceedings of the IEEE Region 10 Conference on Computer, Communication, Control and Power Engineering, 1993, 4, 415-419.

111. Hang, C.C. and Astrom, K.J., Refinements of the Ziegler-Nichols tuning formuale for PID auto-tuners. Proceedings of the ISA/88 International Conference and Exhibition. Advances in Instrumentation, 1988, 43, 3, 1021-1030.

112. Hang, C.-C. and Cao, L., Improvement of transient response by means of variable set-point weighting. IEEE Transactions on Industrial Electronics, 1996, 43(4), 477-484.

113. Gong, X., Gao, J. and Zhou, C., Extension of IMC tuning to improve controller performance. Proceedings of the IEEE International Conference on Systems, Man and Cybernetics, 1996, 1770-1775.

114. VanDoren, V.J., Ziegler-Nichols methods facilitate loop tuning. Control Engineering, 1998, December.

114a. Argelaguet, R., Pons, M., Quevedo, J. and Aguilar, J., A new tuning of PID controllers based on LQR optimization, Preprints of the *Proceedings of PID '00: IFAC Workshop on digital control (Past, present and future of PID control),* Terrassa, Spain, 2000, pp. 303-308.

115. Blickley, G.J., Modern control started with Ziegler-Nichols tuning. Control Engineering, 1990, 2 October, 11-17.

116. De Paor, A.M., A fiftieth anniversary celebration of the Ziegler-Nichols PID controller. International Journal of Electrical Engineering Education, 1993, 30, 303-316.

117. Corripio, A.B., Tuning of industrial control systems. Instrument Society of America, Research Triangle Park, North Carolina, 1990.

118. Mantz, R.J. and Tacconi, E.J., Complementary rules to Ziegler and Nichols' rules for a regulating and tracking controller. International Journal of Control, 1989, 49(5), 1465-1471.

119. Atkinson, P. and Davey, R.L., A theoretical approach to the tuning of pneumatic three-term controllers. Control, 1968, March, 238-242.

120. Perry, R.H. and Chilton, C.H., Chemical engineers handbook, McGraw-Hill, New York, 5th edition, 1973.

121. Luo, K-N., Kuo, C.-Y. and Sheu, L.-T. (1996). A novel method for fuzzy self-tuning PID controllers, *Proceedings of the Asian Fuzzy Systems Symposium*, 194-199.

122. Yu, C.-C. (1999). *Autotuning of PID controllers*, Advances in Industrial Control Series, Springer-Verlag London Ltd.

123. Karaboga, D. and Kalinli, A., Tuning PID controller parameters using Tabu search algorithm. Proceedings of the IEEE International Conference on Systems, Man and Cybernetics, 1996, 134-136.

124. Hang, C.C. and Astrom, K.J., Practical aspects of PID auto-tuners based on relay feedback. Proceedings of the IFAC Adaptive control of Chemical Processes Conference, Copenhagen, Denmark, 1988, 153-158.

125. Shin, C.-H., Yoon, M.-H. and Park, I.-S., Automatic tuning algorithm of the PID controller using two Nyquist points identification. Proceedings of the Society of Instrument and Control Engineers annual conference, Tokyo, Japan, 1997, 1225-1228.

126. Harriott, P., Process Control, McGraw-Hill, New York, 1964.

127. Zhang, G., Shao, C. and Chai, T., A new method for independently tuning PID parameters. Proceedings of the 35th Conference on Decision and Control, Kobe, Japan, 1996, 2527-2532.

127a. Garcia, R.F. and Castelo, F.J.P., A complement to autotuning methods on PID controllers, Preprints of the *Proceedings of PID '00: IFAC Workshop on digital control (Past, present and future of PID control)*, Terrassa, Spain, 2000, pp. 101-104.

128. Fu, M., Olbrot, A.W. and Polis, M.P., Comments on 'Optimal gain for proportional-integral-derivative feedback'. IEEE Control Systems Magazine, 1989, January, 100-101.

129. Pessen, D.W., How to tune in a three-mode controller. Instrumentation, 1954, 7(3), 29-32.

130. Grabbe, E.M., Ramo, S. and Woolridge, D.E. (Editors), Handbook of automation, computation and control. Vol 3: Systems and Components, John Wiley and Sons, New York, 1961.

131. Pessen, D.W., Optimum three-mode controller settings for automatic start-up. Transactions of the ASME, 1953, July, 843-849.

132. Ford, R.L., The determination of the optimum process-controller settings and their confirmation by means of an electronic simulator. Proceedings of the IEE, Part 2, 1953, 101(80), April, 141-155, 173-177.

133. Luyben, W.L., Tuning proportional-integral-derivative controllers for integrator/deadtime processes. Industrial Engineering Chemistry Research, 1996, 35(10), 3480-3483.

134. Belanger, P.W. and Luyben, W.L., Design of low-frequency compensators for improvement of plantwide regulatory performances. Industrial Engineering Chemistry Research, 1997, 36(12), 5339-5347.

134a. Pecharroman, R.R. and Pagola, F.L., Control design for PID controllers auto-tuning based on improved identification, Preprints of the *Proceedings of PID '00: IFAC Workshop on digital control (Past, present and future of PID control)*, Terrassa, Spain, 2000, pp. 89-94.

134b. Pecharroman, R.R., *Private communication*, 9 May 2000.

135. Zhang, W., Xu, X. and Sun, Y., Quantitative performance design for integrating processes with time delay, Automatica, 1999, 35, pp. 719-723.

136. Tan, W., Liu, J. and Sun, W., PID tuning for integrating processes. Proceedings of the 1998 IEEE International Conference on Control Applications, Trieste, Italy, 1998, 2, 873-876.

137. Chien, I.-L. and Fruehauf, P.S., Consider IMC tuning to improve controller performance. Chemical Engineering Progress, 1990, October, 33-41.

138. Oubrahim, R. and Leonard, F., PID tuning by a composed structure. Proceedings of the UKACC International Conference on Control '98, Swansea, Wales, 1998, 2, 1333-1338.

139. Sung, S.W., O, J., Lee, I.-B., Lee, J. and Yi, S.-H., Automatic tuning of PID controller using second-order plus time delay model. Journal of Chemical Engineering of Japan, 1996, 29(6), 991-999.

140. Ho, W.K., Hang, C.C. and Cao, L.S., Tuning of PID controllers based on gain and phase margin specifications. Automatica, 1995, 31(3), 497-502.

141. Ho, W.K., Hang, C.C. and Zhou, J., Self-tuning PID control of a plant with under-damped response with specifications on gain and phase margins. IEEE Transactions on Control Systems Technology, 1997, 5(4), 446-452.

142. Ho, W.K., Hang, C.C., Zhou, J.H. and Yip, C.K., Adaptive PID control of a process with underdamped response. Proceedings of the Asian Control Conference, Tokyo, Japan, 1994, 335-338.

143. Wang, Q.-G., Lee, T.-H., Fung, H.-W., Bi, Q. and Zhang, Y., PID tuning for improved performance. IEEE Transactions on Control Systems Technology, 1999, 7(4), 457-465.

144. Wang, Y.-G. and Shao, H.-H. (1999). PID autotuner based on gain- and phase-margin specification, *Industrial Engineering Chemistry Research*, **38**, 3007-3012.

145. Pemberton, T.J., PID: The logical control algorithm II. Control Engineering, 1972, 19(7), 61-63.

146. Smith, C.L., Corripio, A.B. and Martin, J., Controller tuning from simple process models. Instrumentation Technology, 1975, December, 39-44.

147. Wang, T.-S. and Clements, W.C., Adaptive multivariable PID control of a distillation column with unknown and varying dead time. Chemical Engineering Communications, 1995, 132, 1-13.

147a. Gorez, R. and Klan, P., Nonmodel-based explicit design relations for PID controllers, Preprints of the *Proceedings of PID '00: IFAC Workshop on digital control (Past, present and future of PID control)*, Terrassa, Spain, 2000, pp. 141-148.

147b. Seki, H., Ogawa, M. and Ohshima, M. (2000). Retuning PID temperature controller for an unstable gas-phase polyolefin reactor, Preprints of the *Proceedings of PID '00: IFAC Workshop on digital control (Past, present and future of PID control)*, Terrassa, Spain, pp. 473-478.

148. Landau, I.D. and Voda, A., An analytical method for the auto-calibration of PID controllers. Proceedings of the 31st Conference on Decision and Control, Tucson, Arizona, USA, 1992, 3237-3242.

149. Jahanmiri, A. and Fallahi, H.R., New methods for process identification and design of feedback controller. Transactions of the Institute of Chemical Engineers, 1997, 75(A), July, 519-522.

150. Hansen, P.D., Controller structure and tuning for unmeasured load disturbance. Proceedings of the American Control Conference, Philadelphia, Pennsylvania, 1998, 1, 131-136.

151. Hang, C.C., Ho, W.H. and Cao, L.S., A comparison of two design methods for PID controllers. ISA Transactions, 1994, 33, 147-151.

152. Polonyi, M.J.G., PID controller tuning using standard form optimisation. Control Engineering, 1989, March, 102-106.

153. Valentine, C.C. and Chidambaram, M., PID control of unstable time delay systems. Chemical Engineering Communications, 1997, 162, 63-74.

154. Huang, C.-T. and Lin, Y.-S., Tuning PID controller for open-loop unstable processes with time delay. Chemical Engineering Communications, 1995, 133, 11-30.

155. Skoczowski, S. and Tarasiejski, L., Tuning of PID controllers based on gain and phase margin specifications using Strejc's process model with time delay. Proceedings of the Third International Symposium on Methods and Models in Automation and Robotics (MMAR '96), Miedzyzdroje, Poland, 1996, 765-770.

156. Lennartson, B. and Kristiansson, B., Pass band and high frequency robustness for PID control. Proceedings of the 36th IEEE Conference on Decision and Control, San Diego, California, U.S.A., 1997, 2666-2671.

157. Kristiansson, B. and Lennartson, B., Robust design of PID controllers including auto-tuning rules. Proceedings of the American Control Conference, 1998, 5, 3131-3132.

158. Kristiansson, B. and Lennartson, B., Optimal PID controllers for unstable and resonant plants. Proceedings of the Conference on Decision and Control, 1998.

158a. Kristiansson, B. and Lennartson, B., Near optimal tuning rules for PI and PID controllers, Preprints of the *Proceedings of PID '00: IFAC Workshop on digital control (Past, present and future of PID control)*, Terrassa, Spain, 2000, pp. 369-374.

159. Vrancic, D., Peng, Y. and Strmcnik, S., A new PID controller tuning method based on multiple integrations. Control Engineering Practice, 1999, 7(5), 623-633.

160. Hwang, S.-H. (1995). Closed-loop automatic tuning of single-input-single-output systems, *Industrial Engineering Chemistry Research*, **34**, 2406-2417.

161. Ferretti, G., Maffezzoni, C. and Scattolini, R. (1991). Recursive estimation of time delay in sampled systems, *Automatica*, **27**, 653-661.

162. Nishikawa, Y., Sannomiya, N., Ohta, T. and Tanaka, H. (1984). A method for auto-tuning of PID control parameters, *Automatica*, **20**, 321-332.

163. Lee, J. and Sung, S.W. (1993). Comparison of two identification methods for PID controller tuning, *AIChE Journal*, **39**, 695-697.

164. Deshpande, P.B. (1980). Process identification of open-loop unstable systems, *AIChE Journal*, **26**, 305-308.

165. Pecharroman, R.R. and Pagola, F.L. (1999). Improved identification for PID controllers auto-tuning, Proceedings of the *5th European Control Conference (ECC '99)*, Karlsruhe, Germany, Paper F453, BA-12.

Appendix 1: List of symbols used (more than once) in the paper.

a_1, a_2 = PID filter parameters

A_m = gain margin

b = setpoint weighting factor

b_1 = PID filter parameter

c = derivative term weighting factor

E(s) = Desired variable, R(s), minus controlled variable, Y(s)

FOLPD model = First Order Lag Plus time Delay model

FOLIPD model = First Order Lag plus Integral Plus time Delay model

$G_c(s)$ = PID controller transfer function

$G_p(j\omega)$ = process transfer function at frequency ω

$|G_p(j\omega)|$ = magnitude of $G_p(j\omega)$, $\angle G_p(j\omega)$ = phase of $G_p(j\omega)$

IAE = integral of absolute error

IMC = internal model controller

IPD model = Integral Plus time Delay model

ISE = integral of squared error

ISTES = integral of squared time multiplied by error, all to be squared

ISTSE = integral of squared time multiplied by squared error

ITAE = integral of time multiplied by absolute error

k_i = integral gain of the parallel PID controller

k_p = proportional gain of the parallel PID controller

k_d = derivative gain of the parallel PID controller

K_c = Proportional gain of the controller

K_m = Gain of the process model

K_u = Ultimate gain

$K_{25\%}$ = proportional gain required to achieve a quarter decay ratio

m = multiplication parameter in the lead controller

M_s = closed loop sensitivity

N = determination of the amount of filtering on the derivative term

PP = perturbance peak = peak of system output when unit step disturbance is added

R(s) = Desired variable

RT= recovery time = time for perturbed system output (when a unit step disturbance is added) to come to its final value

SOSPD model = Second Order System Plus time Delay model

T_d = Derivative time of the controller

T_{CL} = desired closed loop system time constant

T_f = Time constant of the filter in series with the PID controller

T_i = Integral time of the controller

T_m = Time constant of the FOLPD process model

T_{m1}, T_{m2}, T_{m3} = Time constants of the higher order process models

T_u = Ultimate time constant

$T_{25\%}$ = period of the quarter decay ratio waveform

TOLPD model = Third Order Lag Plus time Delay model

TS = settling time

U(s) = manipulated variable

Y(s) = controlled variable

λ = Parameter that determines robustness of compensated system.

ξ_m = damping factor of an underdamped process model, ξ = damping factor of the compensated system

$\kappa = 1/K_m K_u$

ϕ = phase lag, ϕ_m = phase margin, ϕ_ω = phase lag at an angular frequency of ω

$\phi_c =$ phase corresponding to the crossover frequency

τ_m = time delay of the process model, $\tau = \tau_m / (\tau_m + T_m)$

ω = angular frequency, ω_u = ultimate frequency, ω_ϕ = angular frequency at a phase lag of ϕ