The strategic value of flexibility in sequential decision making

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Abstract: This paper formalizes the notion of flexibility in sequential decision making and investigates conditions under which the use of flexibility as an additional criterion may be justified. The correlations between flexibility and value, and flexibility and risk, are studied under various assumptions of uncertainty and information. A number of approaches to constructing a multiple objective decision criterion are discussed. In particular, characteristics of a dual-objective value function, that accounts for both expected value and flexibility, are described. The usefulness of these results is illustrated by applying them to decision processes in discrete part manufacturing. Relationships between flexibility and manufacturing performance are shown and implications to part flow control are discussed.

1. Introduction

The issue of flexibility in sequential decision making arises when a decision maker has to choose between an irreversible position and one that is amendable in the future. Intuitively, a reversible, or flexible, position is preferred when the decision maker is uncertain about the future and/or expects to learn more about it with the passage of time. A flexible position gives decision makers the possibility to change their minds upon the receipt of new information. In this sense, flexibility limits the risks of an early commitment to an alternative when the value of this alternative is not known with certainty. A flexible position also allows greater responsiveness to future events by taking greater advantage of the learning that often takes place with the passage of time.

Traditionally, stochastic sequential decision problems are solved by first estimating the expected value or utility of each sequence of actions and then, selecting an action program, or policy, that maximizes this expected value. Implicit in the use of the expected value criterion is the assumption that the decision situation is well defined. That is, states, actions, returns, and future information are known and/or can be characterized by well defined distributions. When this is the case, an estimate of the expected value may indeed be an adequate and sufficient measure of the worth of an action program. However, the effectiveness of the expected value becomes dubious when these assumptions about the decision situation cannot be sustained. For instance, in a highly volatile and uncertain decision environment, the returns associated with each action program may not be easily anticipated as they are subject to fluctuation with a variety of factors. These factors themselves may not be easy to capture by a finite set of world states and an associated probability distribution.

Under such conditions, the initial estimate of the return of an action may not be as critical as its associated flexibility, which is the possibility it leaves to respond to changes and new information with adequate and timely subsequent actions. In fact, it could become

especially important to the decision maker not to commit early on to any irreversible position, regardless of how lucrative it may initially seem, but rather to leave as many future action options open as possible.

This paper formalizes these observations and investigates conditions under which the use of flexibility as an additional criterion may be justified. The correlations between flexibility and value, and flexibility and risk, are studied under various assumptions of uncertainty and information. A number of approaches to constructing a multiple objective decision criterion are discussed. In particular, axiomatics of a dual-objective value function, that accounts for both expected value and flexibility, are described and an example of such a function is presented. The usefulness of these results is illustrated by applying them to decision processes in discrete part manufacturing. Relationships between flexibility and manufacturing performance are shown and implications to decision making in production flow control are discussed.

The plan of the paper is as follows: section 2 provides a brief review of existing literature on the subject of flexibility in the decision sciences and in manufacturing. Section 3 formalizes the notions of uncertainty and information and introduces measures for each. Also, useful relationships between information, uncertainty and value are presented. Section 4 introduces the notion of flexibility in decision making and studies its relationship to value and risk under various assumptions. Section 5 discusses conditions under which flexibility should be explicitly accounted for in the decision criterion. Various approaches to constructing a dual objective criterion are suggested. In particular, the axiomatics of a value function are described. Finally, section 6 addresses the issue of flexibility in manufacturing and shows the implications of the results of sections 4 and 5 to flexible manufacturing systems design and control.

2. Flexibility in the literature

The importance of flexibility to decision making was recognized as early as 1921 with Lavington (Lavington 1921) drawing a connection between variability and the value of flexibility in considering "the risk arising from the immobility of invested resources". Later, it re-appeared in the context of the theory of the firm (Kalecki 1937) (Stigler 1939). Stigler, in particular, used flexibility to describe the insensitivity of an average production cost curve to changes in production volume. Hart (Hart 1940) proposed flexibility, defined as postponement of decisions until more information is gained, as an effective means of dealing with current uncertainty. This proposition was also taken up by Tintner (Tintner 1941) and Marshack (Marshack 1938, 1949).

However, an attempt to formalize the notion of flexibility and study its usefulness to general decision making was not made until the work of Marshack and Nelson in 1962 (Marshack and Nelson 1962). Marshack and Nelson suggested that flexibility be viewed as a mechanism which allows decision makers to take advantage of future information. They conjectured that the greater the current uncertainty and/or the greater the expected amount of future information the more important is flexibility. They also discussed some possible measures of this flexibility.

Rosenhead et al. (Rosenhead et al. 1972) further pursued this view of flexibility. In particular, they argued for the use of flexibility as an alternative decision criterion to expected value in uncertain environments. A similar argument was also put forth by Pye (Pye 1978). Mandelbaum (Mandelbaum 1978) discussed the importance of flexibility in the context of a two-period decision process and explored its effect on expected value under certain conditions of learning. Merkhofer (Merkhofer 1977) studied the effect of decision flexibility on the value of information and suggested that this value should appreciate with increases in flexibility. More recently, Jones and Ostroy (Jones and Ostroy 1984) proposed, in the context of a two-period decision process, a dual ranking of initial actions

based on variability of beliefs and action flexibility. They showed that, under certain conditions, an ordering based on variability of beliefs induces an order-preserving relation on flexibility.

The treatment of flexibility in existing literature still however suffers from several limitations. In particular, there is a lack of a unified framework for defining and evaluating flexibility. While it is generally agreed upon that greater future decision making flexibility is desirable in an uncertain environment, the relationship between flexibility, uncertainty, and value (or cost) remains ambiguous and poorly quantified. Necessary and/or sufficient conditions under which flexibility and value may be positively correlated still need to be clearly identified. Also, conditions, under which the use of flexibility, as part of the decision criterion, may be justified must be further investigated. Finally, a systematic methodology for incorporating flexibility considerations in the decision process need to be developed.

3.   Information, uncertainty and value

In this section, useful results from information theory and decision theory are reviewed. These results will be used in later sections to prove a number of propositions regarding flexibility and value.

3.1   Measuring uncertainty

Let $S = \{s_1, s_2, \ldots, s_n\}$[1] be the set of possible world (or system) states, and let $\Pi_t[S(t_i)] = (p_t(s_1, t_i), p_t(s_2, t_i), \ldots, p_t(s_n, t_i))$ be the probability distribution of the held beliefs at time t concerning the state of the world at a future time t_i, where $p_t(s_j, t_i)$ represents the believed probability at time t of the occurrence of state j at time t_i such that $\Sigma_j \, p_t(s_j, t_i) = 1$ and $t \leq t_i$. The time period t_i is referred to as the implementation time. If we

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[1] The set S is not necessarily finite nor discrete.

define uncertainty at time t to be the degree of ignorance of the future state of the world at t_i, then a measure, $U_t[S(t_i)]$, of this uncertainty should at least satisfy the following properties (Shannon and Weaver 1949):

(1) $U_t[S(t_i)]$ is minimum when $\|S\| = 1$,

(2) $U_t[S(t_i)]$ is maximum when $p_t(s_1, t_i) = p_t(s_2, t_i) = ... = p_t(s_n, t_i) = 1/n$,

(3) $U_t[S(t_i)]$ is minimum when $p_t(s_j, t_i) = 1$ for any $j = 1, 2, ..., n$,

(4) $U_t[S(t_i)]$ increases with an increase in n, $n = \|S\|$, given that all $p_t(s_j, t_i)$'s are kept equal, and

(5) $U_t[S(t_i)]$ does not change if an additional state is included in S, such that $\|S\| = n + 1$, but $p_t(s_{n+1}, t_i) = 0$.

Other mathematically desirable properties are:

(6) $U_t[S(t_i)]$ is a continuous function of $p_t(s_1, t_i)$, $p_t(s_2, t_i)$, ..., $p_t(s_n, t_i)$,

(7) $U_t[S(t_i)]$ is differentiable with respect to $p_t(s_1, t_i)$, $p_t(s_2, t_i)$, ..., $p_t(s_n, t_i)$, and

(8) $U_t[S(t_i)]$ is a concave function of $p_t(s_1, t_i)$, $p_t(s_2, t_i)$, ..., $p_t(s_n, t_i)$ ensuring that a local maximum is also a global maximum.

The need for these properties is highly intuitive and follows directly from the definition of uncertainty. Property 1 states that uncertainty cannot exist without a multiplicity of states; property 2 formalizes the notion that uncertainty is the greatest when there is equal likelihood that any of the states could occur; property 3 is simply a reformulation of the definition and ensures that when there is certainty about the occurrence of a state, the associated uncertainty becomes zero; property 4 ensures that when the same belief distribution is maintained while the number of states is increased, uncertainty increases; finally, property 5 satisfies the requirement that when a state is added but is believed to never occur, overall uncertainty remains unaffected.

A measure that satisfies all these requirements is given by Shannon's measure of entropy (Shannon and Weaver 1949):

$$U_t[S(t_i)] = U_t[p(s_1, t_i), ..., p(s_n, t_i)] = -C \sum_{j=1}^{n} p_t(s_j, t_i)\log[p_t(s_j, t_i)] \qquad (3.1),$$

where C is an arbitrary positive constant, and the logarithm base is any number greater than 1 (following established convention, C will be set to 1 and the log base to 2 in the remainder of this discussion). This measure could also be extended to situations with continuous or infinite state spaces (see for instance (Ash 1965)).

3.2   Measuring information

Let $Y(t_o) = (y_1, y_2, ..., y_k)^2$ be a set signals that are observable at a time t_o. The signals are correlated to the state of the system at time t_i ($t \leq t_o \leq t_i$) by the following joint distribution:

$$\Pi_t[Y(t_o)] = (p_t(y_1, t_o), p_t(y_2, t_o), ..., p_t(y_k, t_o)),$$

$$\Pi_t[S(t_i)|Y(t_o)] = \begin{bmatrix} p_{1|1} & \cdots & p_{1|k} \\ \vdots & & \vdots \\ p_{n|1} & \cdots & p_{n|k} \end{bmatrix},$$

and

$$\Pi_t[S(t_i)]^T = \Pi_t[S(t_i)|Y(t_o)]\Pi_t[Y(t_o)]^T,$$

where $p_{i|j} = p(s_i, t_i|y_j\text{-}t_o)$ represents the probability of occurrence of state i at time t_i given that signal y_j is observed at time t_o, and $\Pi_t[Y(t_o)]$ is the a-priori probability distribution of the signals y_j's where $p_t(y_j, t_o)$ is the believed probability at time t of observing signal j at observation time t_o and $\Sigma_j\, p_t(y_j, t_o) = 1$.

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2 The signals y_i's are not necessarily scalar valued and may correspond to vectors.

Given a structure of the form $(\Pi_t[S(t_i)], \Pi_t[S(t_i)|Y(t_o)], \Pi_t[Y(t_o)])$, information can be defined as the expected amount of reduction in uncertainty that is due to observing the signals y_j's. Expected information can thus be measured as (Shannon and Weaver 1949):

$$I_t[S(t_i)|Y(t_o)] = U_t[S(t_i)] - U_t[S(t_i)|Y(t_o)] \qquad (3.2),$$

where

$$U_t[S(t_i)|Y(t_o)] = \sum_{j=1}^{k} p_t(y_j, t_o)U_t[S(t_i)|y_j] \qquad (3.3),$$

and

$$U_t[S(t_i)|y_j] = -\sum_{i=1}^{n} p_{i|j}\log(p_{i|j}) \qquad (3.4).$$

The actual amount of information that results from observing a particular signal y_j is given by

$$I_t[S(t_i)|y_j] = U_t[S(t_i)] - U_t[S(t_i)|y_j] \qquad (3.5).$$

If information is collected over several time periods, $t_1, t_2, \ldots, t_m$, then the total expected reduction in uncertainty is calculated as:

$$I_t[S(t_i)|Y(t_1), Y(t_2), \ldots, Y(t_m)] = \sum_{j=1}^{m} I_t[S(t_i)|Y(t_j)] \qquad (3.6),$$

where

$$I_t[S(t_i)|Y(t_j)] = U_t[S(t_i)|Y(t_{j-1})] - U_t[S(t_i)|Y(t_j)] \qquad (3.7)$$

and represents the information that is due to observation period t_j. The expression in 3.6 can also be written as:

$$I_t[S(t_i)|Y(t_1), Y(t_2), \ldots, Y(t_m)] = U_t[S(t_i)] - U_t[S(t_i)|Y(t_1), Y(t_2), \ldots, Y(t_m)] \qquad (3.8).$$

It can be shown (Shannon and Weaver 1949) that $I_t[S(t_i)|Y(t_o)] \geq 0$ (or more generally, $I_t[S(t_i)|Y(t_1), Y(t_2), \ldots, Y(t_m)]$) for any information structure $(\Pi_t[S(t_i)], \Pi_t[S(t_i)|Y(t_o)], \Pi_t[Y(t_o)])$, with the equality occurring only when $S(t_i)$ and $Y(t_o)$ are independent and/or when $U_t[S(t_i)] = 0$. This coincides with our intuitive expectation that the receipt of relevant information, in the presence of uncertainty, should always result in a

reduction of this uncertainty. On the other hand, if the information is irrelevant so that the observed signals are independent of future system states, then no reduction in the initial amount of uncertainty $U_t[S(t_i)]$ should be expected (i.e. $U_t[S(t_i)] = U_t[S(t_i)|Y(t_o)]$ and $I_t[S(t_i)|Y(t_o)] = 0$). Similarly, if future system states are known with certainty then any subsequent observations, regardless of their relevance, will carry no real informational value. Thus, if $U_t[S(t_i)] = 0$ then $I_t[S(t_i)|Y(t_o)] = 0$.

3.3 Measuring value in single stage decision making

For the sake of ease of exposition, the discussion in this section is restricted to single stage decision situations. The concepts introduced here will be extended in the next section to the more general case of multiple stages.

Let $A(t_i) = (a_1, a_2, ..., a_m)$ be the set of possible actions at time t_i for a given decision situation, and

$$R = \begin{bmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & & \vdots \\ r_{m1} & \cdots & r_{mn} \end{bmatrix}$$

be the return matrix, with r_{ij} being the return associated with action i given state of the world s_j. The expected value at time t ($t \leq t_i$) can then be calculated as:

$$E_t[r(t_i)] = \max_i \left(\sum_{j=1}^{n} p_t(s_j, t_i) \, r_{ij} \right) \qquad (3.9).$$

Note that the returns r_{ij} may not necessarily represent the actual returns but rather the utility of these returns to the decision maker.

In order to alleviate the notation, the time indices will be heretofore eliminated. The expression in 3.9 can then be rewritten as:

$$E(r) = \max_i \left(\sum_{j=1}^{n} p(s_j) \, r_{ij} \right) \qquad (3.10).$$

If current beliefs are to be updated at a later time by an information structure ($\Pi(S)$, $\Pi(Y)$, $\Pi(S|Y)$) (more exactly, ($\Pi_t[S(t_i)]$, $\Pi_t[S(t_i)|Y(t_o)]$, $\Pi_t[Y(t_o)]$)), then the expected return becomes:

$$E(r|Y) = \sum_{l=1}^{k} p(y_l) \max_i \left(\sum_{j=1}^{n} p_{j|l} \, r_{ij} \right) \qquad (3.11).$$

The value of the information contained in Y can be measured as:

$$E[I(S|Y)] = E(r|Y) - E(r) \qquad (3.12).$$

The value of $E[I(S|Y)]$ quantifies the gain in value that is due to the receipt of the information $I(S|Y)$.

The quantity $E[I(S|Y)]$ can be shown (Degroot 1962) to be always non-negative, that is $E[I(S|Y)] \geq 0$, for any information structure ($\Pi(S)$, $\Pi(Y)$, $\Pi(S|Y)$), with $E[I(S|Y)] = 0$ only when $I(S|Y) = 0$. This result is highly intuitive. A reduction in uncertainty should always lead to better judgment of the returns associated which each action and thus to better action selection. The value of information should be zero if the information is irrelevant and/or the returns are known with certainty.

Also, it can be shown (Epstein 1980) (Jones and Ostroy 1984) that $E[I(S|Y)] \leq E[I(S|Y')]$ if $I(S|Y) \leq I(S|Y')$ which simply means that information which results in greater reduction of uncertainty should not result in degradation of value to the decision maker. These two results will, in later sections, prove to be of extreme importance to the study of the relationship between flexibility and value.

In the case where decisions are made in real time and opportunistically, the selection of an action is delayed until the implementation time so that the actual returns of all actions are known before a decision is made. The benefits of delaying decisions until implementation time are the benefits of perfect information. In fact, at the implementation time, $U_{t_i}[S(t_i)] = 0$. The resulting value increase is given by:

$$E[I(S|PI)] = E(r|PI) - E(r) \qquad (3.13),$$

where PI refers to perfect information and

$$E(r|PI) = \sum_{j=1}^{n} p(s_j)\max_i(r_{ij}) \qquad (3.14).$$

The quantity $E[I(S|PI)]$ can be shown to be always non negative and to be strictly positive when there exist at least two non-dominated actions.

3.4 Implications to multi-stage decision making

Multi-stage decision problems may be viewed as consisting of a sequence of single stage decision problems. However, the two problems differ in that in a multi-stage problem decisions that are taken at a given stage affect the nature of the decision situation at future stages (e.g. the number and type of future possible actions and their corresponding returns). The stages may also correspond to time periods. In this case, the resolution of the decision sequence is time-phased. This means that the implementation of actions is delayed until the corresponding time periods. Information in this context may be itself time phased. That is, at each stage partial information is gained as to the actual value of each sequence of actions. An example of such a problem is illustrated below.
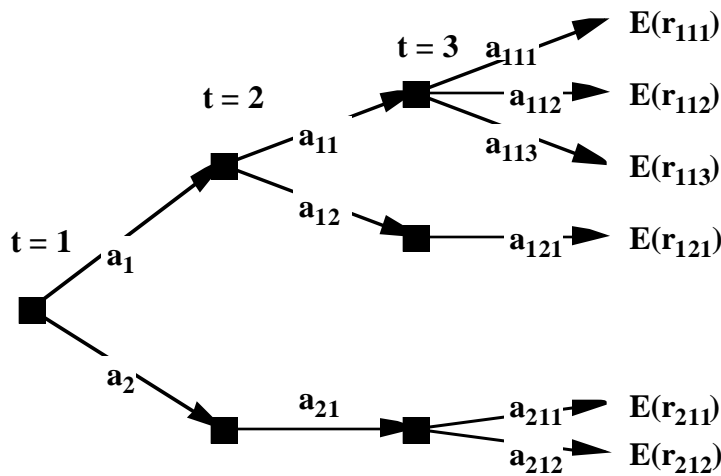

Figure 1   Example muti-stage decision problem

Without loss of generality, the multi-stage/multi-period decision problem can be collapsed, at least for the purpose of this discussion, into a two-stage/two-period problem that can be formalized using the following notation:

(1) $A = \{a_1, a_2, ..., a_m\}$ is the set of possible actions in period 1,

(2) $SS(a_i) = \{a_{i1}, a_{i2}, ..., a_{im_i}\}$ is the set of possible actions in period 2 once action $a_i \in A$ has been taken in period 1 ($SS(a_i)$ is called the set of successor actions to a_i), and

(3) r_{ijk} is the return associated with the action program (a_i, a_{ij}) given state s_k.

The expected value of return can then be expressed as

$$E(r) = \max_{a_i \in A}(\max_{a_{ij} \in SS(a_i)} (\sum_{k=1}^{n} p(s_k)r_{ijk})) \qquad (3.15),$$

or equivalently

$$E(r) = \max_{a_i \in A}(\max_{a_{ij} \in SS(a_i)} (E(r_{ij}))) \qquad (3.16).$$

In most time-phased decision situations, information as to the actual state of the world is gained with the passage of time. In the context of a two-period decision problem, the information gained after the initial action is taken is generally used to make a more informed decision in period 2. The receipt of information prior to taking the second period action can be modeled by an information structure $(\Pi(S), \Pi(Y), \Pi(S|Y))$ similar to those proposed for the single stage situation. The expected value of return, given information in period 2, is given by:

$$E(r|Y) = \max_{a_i \in A}[\sum_{l=1}^{k} p(y_l)\max_{a_{ij} \in SS(a_i)}(\sum_{k=1}^{n} p_{k|l}r_{ij})] \qquad (3.17).$$

As in the single stage situation, it can be shown that the value of information $E[I(S|Y)]$ is always non-negative, for any information structure $(\Pi(S), \Pi(Y), \Pi(S|Y))$, with $E[I(S|Y)] = 0$ only when $I(S|Y) = 0$. Also, it can be shown that $E[I(S|Y)] \leq E[I(S|Y')]$ if $I(S|Y) \leq I(S|Y')$.

4. Flexibility in multi-stage decision making

In this paper, flexibility is defined as the property of an action that describes the degree of future decision making freedom this action leaves once it is implemented. More precisely, the flexibility of an action is measured as a function of the number of actions that are possible at each of the subsequent stages. For example in a two-stage process, the flexibility of an action a_i, $F(a_i)$, is indicated by the size of its successor actions set $SS(a_i)$[3]. Thus, flexibility can be interpreted as the extent to which decision makers are allowed to change their minds once an initial action is taken. Intuitively, this is desirable in situations where the decision maker is uncertain about future outcomes and/or expects to gain more information about these outcomes with the passage of time. Flexibility may thus be viewed as a mechanism that enables decision makers to respond effectively to future changes by minimizing the degree of their future commitment.

The discussion below formalizes these intuitive observations and establishes several relationships between flexibility and value and flexibility and risk under various assumptions of uncertainty and information. The results obtained will be used in a later section to argue for the relevance of flexibility as an additional criterion for decision making under uncertainty. Note that although the discussion is restricted to two-stage decision processes, the results can be easily extended to multi-stage situations.

4.1 Flexibility and value

The following results identify conditions under which flexibility is correlated to value and establish fundamental relationships between flexibility, information and uncertainty. Flexibility of an action a_i, $F(a_i)$, will refer to the size of the action successor set. Flexibility will be measured as $F(a_i) = \|SS(a_i)\| - 1$ so that an action with only one successor has zero flexibility.

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[3] A more precise measure of decision flexibility can be found in (Benjaafar and Talavage 1992).

Theorem 1: *The expected return of an action program does not decrease with an increase in its flexibility.*

Proof: Given two actions a_i and a_j such that $SS(a_i)$ is a subset of $SS(a_j)$, it is easy to show that $E[r(a_j)] \geq E[r(a_i)]$, where $E[r(a_i)]$ and $E[r(a_j)]$ represent respectively the expected return that result from selecting a_i and a_j initially.

Although theorem 1 is a conceptually important result (it affirms that, under all conditions, increased flexibility may not result in a deterioration of value), a more useful result can be obtained for the case of perfect information, that is for information structures yielding $I(S|Y) = U(S)$.

Proposition 1: *Given perfect information, the expected return of an action strictly increases with increases in its flexibility under conditions of non-dominance and uncertainty.*

Proof: This proposition can be reinterpreted symbolically as: $E[r(a_j)|PI] > E[r(a_i)|PI]$ if:

(1) $I(S|Y) = U(S)$,

(2) $U(S) \neq 0$,

(3) the action set $SS(a_i)$ is a subset of $SS(a_j)$, and

(4) $SS(a_j)$ contains at least one action program non-contained in $SS(a_i)$ that is non-dominated[4].

Using the fact that there exists at least one state k for which the return of an action in $SS(a_j)$ is strictly smaller than that of any in $SS(a_i)$, it is straightforward to show that $E[r(a_j)|PI] > E[r(a_i)|PI]$.

Proposition 1 also means that $E[r(a_i)|PI]$ is a strictly monotonic function of $SS(a_i)$ when all actions in $SS(a_i)$ are non-dominated. That is, given perfect information, value should always be expected to strictly increase with increased flexibility.

Flexibility is also necessary for the realization of the value of information.

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[4] An action program i is dominated if for every state j there exists an action program k such that $c_{kj} \leq c_{ij}$.

Theorem 2: *The expected value of information is zero if flexibility is zero.*

Proof: This result can be restated as follows: For the expected value of an action to increase with the receipt of information, the size of its successor set must be strictly greater than 1. The expected value of an action program a_i is given by

$$E[r(a_i)] = \max_{a_{ij} \in SS(a_i)} \left(\sum_{k=1}^{n} p(s_k) r_{ijk} \right).$$

Given an information structure $(\Pi(S), \Pi(Y), \Pi(S|Y))$, this expected value becomes

$$E[r(a_i)|Y] = \sum_{l=1}^{k} p(y_l) \max_{a_{ij} \in SS(a_i)} \left(\sum_{k=1}^{n} p_{k|l} r_{ijk} \right).$$

It follows at once that for zero flexibility, i.e. $\|SS(a_i)\| = 1$, we have $E[r(a_i)] = E[r(a_i)|Y]$ while for $SS(a_i) > 1$, $E[r(a_i)] \leq E[r(a_i)|Y]$.

Theorem 2 is intuitive. Given no flexibility in period 2, the same future action will always be chosen regardless of state and regardless of information. The expected value of information is consequently equal to zero. This reiterates the fact that in order to take advantage of future information, some degree of flexibility is required. More importantly, it can be shown that given flexibility and under conditions of non-dominance, expected value in fact strictly increases with the receipt of perfect information.

Proposition 2: *The expected value of an action strictly increases with the receipt of perfect information if (1) its flexibility is greater than zero, (2) initial uncertainty is greater than zero, and (3) its set of successor actions contains at least two non-dominated actions.*

Proof: Given perfect information, the updated expected value can written as:

$$E[r(a_i)|PI] = \sum_{k=1}^{n} p(s_k)(r_{ijk} + d_{ijk})$$

or equivalently

$$E[r(a_i)|PI] = E[r(a_{ij})] + \sum_{k=1}^{n} p(s_k) d_{ijk}$$

- 15 -

where d_{ijk} represents the amount of regret in state k, which is the difference between the the maximum possible return in state k and the return r_{ijk} and $(d_{ijk} = \max_j(r_{ijk}) - r_{ijk})$. The expected value of information can then be calculated as

$$E[I(S|PI)] = \min_j(\sum_{k=1}^{n} p(s_k)d_{ijk})$$

which is strictly greater than zero if the action set $SS(a_i)$ contains at least two non-dominated actions. Consequently, $E[r(a_i)|PI] > E[r(a_i)]$.

These four results define situations under which cost reduction is positively correlated to flexibility. However, flexibility has, at times, no consequences on decision outcomes. The following two theorems describe such situations.

Theorem 3: *Flexibility is of no value when initial uncertainty is zero.*

Proof: If there is no initial uncertainty, then the value of each action can be deterministically evaluated. The action with the lowest return is then selected. Thus, the value does not depend on the size of the successor sets but rather on the returns of the individual actions in these sets.

Theorem 4: *Flexibility is of no value if no future information is expected.*

Proof: If no future information is expected, then the expected value of an action is calculated based on the initial estimate of the state space probabilities. This expected value is independent of the size of the successor set and depends solely on the returns associated with each of the programs in this set.

The above theorems and propositions are useful in identifying the fundamental relationships between flexibility, value and information. In particular, these include the following:

• Expected value is a non-decreasing function of flexibility. In fact, expected value strictly increases with increases in flexibility under certain conditions of non-dominance and/or given perfect information.

• Expected value is a non-decreasing function of information.

• Flexibility is of no value if no initial uncertainty exists and/or no future information is expected.

• For expected value to increase with the receipt of information, flexibility has to be greater than zero.

The following two examples further illustrate these results.

4.2   Example 1:   Dependent discrete return structures

Consider a decision situation with n possible states and m possible actions. Let u_{ik} represent the maximum return that is attainable in state k after an action a_i is taken in the first period.   Also, let $SS(a_i)$, the set of successor actions to a_i, contain m_i $(m_i \leq n)$ non-dominated actions where each action a_{ij} has a return equal to one of the u_{ik}'s in exactly one state. For the sake of simplifying the notation, let the return r_{ijk} of action a_{ij} be equal to u_{ij} when j = k (e.g. action a_{i2} has a return equal to u_{i2} in state s_2) while for all other states $j \neq k$, $r_{ijk} < u_{ik}$. The expected value of action a_i, given perfect information, can then be calculated as

$$E[r(a_i)|PI] = E[r(a_{ij})] + \sum_{k=1}^{n} p(s_k)d_{ijk} \qquad (4.1)$$

which can be rewritten as

$$E[r(a_i)|PI] = E[r(a_{ij})] + \sum_{k=1}^{m_i} p(s_k)[u_{ik} - r_{ijk}] + \sum_{k=m_i+1}^{n} p(s_k)[\max_j(r_{ijk}) - r_{ijk}] \quad (4.2).$$

From the above expression, the effect of increasing flexibility can be readily quantified. For instance, if the action set $SS(a_i)$ is increased from m_i to m_i +1, then the value of $E(r(a_i)|PI)$ increases by the amount

$$\Delta_{m_i+1} = p(s_k)[u_{i,m_i+1} - \max_{j \neq m_i+1} (r_{i,m_i+1,m_i+1})] \qquad (4.3).$$

This quantity can be thought as the expected value of flexibility. Note that under the required conditions of non-dominance, Δ_{m_i+1} is always strictly positive. In the limit case, that is when $m_i = n$, the expected value of action a_i becomes

$$E[r(a_i)|PI] = E[r(a_{ij})] + \sum_{k=1}^{n} p(s_k)[u_{ik} - r_{ijk}] \qquad (4.4)$$

which can be rewritten as

$$E[r(a_i)|PI] = E[r(a_{ij})] + \sum_{k=1}^{n} p(s_k)[u_{ik}] - E[r(a_{ij})] \qquad (4.5)$$

or simply

$$E[r(a_i)|PI] = E(u_i) \qquad (4.6).$$

This means that for maximum flexibility (or infinite flexibility if n is not finite) the expected value is also maximum. That is, when flexibility is maximized, expected value is also maximized.

In order to better understand the interaction between flexibility and value, the expression in 4.2 can be schematically rewritten as

$$E[r(a_i)|PI] = E[r(a_{ij})] + \phi(F(a_i))(E[u_i] - E[r(a_{ij})]) \qquad (4.7)$$

where $\phi(F(a_i))$, $0 \leq \phi(F(a_i)) \leq 1$ ($\phi(F(a_i)) = 0$ when $m_i = 1$ and $\phi(F(a_i)) = 1$ when $m_i = n$), and $\phi(F(a_i))$ is a strictly monotonic function of flexibility. It is clearer now that the updated expected value is a function of two factors: one that is dependent on the initial estimate of expected value $E[r(a_{ij})]$, and the second is dependent on and increases with flexibility $F(a_i)$. This relationship between flexibility and value is further illustrated in the next example.

4.3 Example 2: Independent continuous return structures

In a variety of decision situations, the previous action-state framework may be too restrictive. For instance the returns of each action may not share the dependence on the same states. In this case, the returns associated with each action are more appropriately modeled as independent distributions.

Let r_{i1}, r_{i2}, ..., r_{im_i} be independent random variables describing the returns associated with the actions a_{ij} in the successor set $SS(a_i)$. Then, given perfect information, the expected return of action a_i can be calculated as follows:

$$E[r(a_i)|PI] = E[\max(r_{i1}, r_{i2}, ..., r_{im_i})] \qquad (4.8)$$

Using the fact that the distribution function of the expected value is given by

$$F(y) = p((r(a_i) \leq y)|PI) = \prod_{j=1}^{m_i} p(r_{ij} \leq y) \qquad (4.9),$$

and assuming that the distribution r_{ij}'s are bounded and continuous, the expected value of return can be rewritten as

$$E[r(a_i)|PI] = \int_L^U y(dF(y))dy = U - \int_L^U \prod_{j=1}^{m_i} p(r_{ij} \leq y)dy \qquad (4.10).$$

where U and L are respectively the upper and lower bounds on the returns. It can clearly be seen that $E[r(a_i)|PI] \leq E[r(a_j)|PI]$ if $m_i \leq m_j$, where $m_i = \|SS(a_i)\|$ and $m_j = \|SS(a_j)\|$ and $SS(a_i)$ is a subset of $SS(a_j)$. This inequality is strict if there exists at least an action a_{jk} in $SS(a_j)$ that is not completely dominated by another action in $SS(a_i)$ (i.e. $p(r_{jk} \geq r_{il}) \neq 0$ for all actions l in $SS(a_i)$).

For the special case of identical return distributions, $E[r(a_i)|PI]$ can be expressed as:

$$E[r(a_i)|PI] = U - \int_L^U (p(r_{ij} \leq y))^{m_i} dy \qquad (4.11),$$

which makes $E[r(a_i)|PI] < E[r(a_j)|PI]$ for $m_i < m_j$ (that is, E is a strictly monotonic function of m_i). Note also that E approaches U, the upper bound on the distributions, as m_i approaches infinity.

These relationships can be seen even more readily by considering the following two special cases.

Case 1: Identical independent uniform returns

Let the random variables r_{ij} be identical, independent and uniformly distributed over an arbitrary finite range [L, U]. Using expression 4.11, it can easily be shown (see for instance Mandelbaum (Mandelbaum 1978)) that :

$$E[r(a_i)|PI] = U - \frac{(U - L)}{(m_i + 1)} \qquad (4.12)$$

which can be rewritten as

$$E[r(a_i)|PI] = \frac{U + L}{2} + (U - L)(\frac{1}{2} - \frac{1}{m_i + 1}) \qquad (4.13).$$

The above expression is clearly of the form suggested in section 4.2. That is, it is composed of two elements: (1) the mean, $(U + L)/2$, of the distributions which is also the expected return when flexibility is zero (i.e. $m_i = 1$), and (2) a factor which is an increasing (decreasing) function of flexibility (uncertainty). It is easy to verify that the value of $E[r(a_i)|PI]$ increases monotonically with increasing flexibility when initial uncertainty is greater than zero (i.e. $U \neq L$) and reaches a maximum equal to U when flexibility is infinite. At the same time, expected value, for a fixed level of flexibility, is clearly monotonically decreasing with increasing initial uncertainty (i.e. increasing $(U - L)$). Alternatively, it can be said that the value of flexibility appreciates with increasing initial uncertainty and has zero value when uncertainty is zero. Finally, it is interesting to note that much of the value increase that is due to flexibility is achieved at relatively low levels of flexibility. In fact, the plot of expected value versus flexibility follows a steep curve of diminishing returns (see also (Benjaafar and Talavage 1992)) since expected value is a linear function of $(m_i + 1)^{-1}$.

Case 2: Identical independent exponential Costs

Let the random variables r_{ij} represent costs and be independent and exponentially distributed with parameter λ_{ij}. Then it can be shown that $r(a_i)|PI$ is also exponentially distributed with parameter $\lambda_i = \Sigma_j \lambda_{ij}$ (see (Benjaafar and Talavage 1992)). Consequently,

$$E[r(a_i)|PI] = \cfrac{1}{\sum_{i=1}^{m_i} \lambda_{ij}} \qquad (4.14),$$

which is clearly a decreasing function of flexibility.

If the r_{ij}'s are identically distributed with parameter λ, the expected cost becomes

$$E[r(a_i)|PI] = \frac{1}{m_i\lambda} \qquad (4.15),$$

which can also be rewritten as

$$E[r(a_i)|PI] = \frac{1}{\lambda} - \frac{1}{\lambda}(1 - \frac{1}{m_i}) \qquad (4.16).$$

This is also of the form of equality 4.7. That is, it is composed of two factors with the first consisting of the initial cost estimate and the second being an increasing (decreasing) function of flexibility (uncertainty). Similarly to the uniform case, the plot of cost versus flexibility follows a steep curve of diminishing returns.

4.4   Flexibility and Risk

In the previous section, it was shown that under a variety of conditions, flexibility is positively correlated to expected value. In this section, it is shown that flexibility can in addition serve as a hedging mechanism against risk[5] .

In many decision situations, such as high stakes or one of a kind decisions, decision makers may be more concerned about minimizing risk than they are about maximizing expected value. This means that the decision maker may require a minimum level of return to be assured with a high degree of confidence. More exactly, the decision maker is interested in maximizing the value of the probability $p(r(a_i) \geq r_{min})$. A relationship between flexibility and risk is given by the following result:

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[5] Risk is defined here as the possibility of an undesirable outcome. It should be noted that this is different from the classical definition of risk in decision theory.

Theorem 5: *The expected risk of an action does not increase with an increase in the action flexibility.*

Proof: Given two actions a_i and a_j such that $SS(a_i)$ is a subset of $SS(a_j)$, it is easy to show that $p(r(a_j) \geq r_{min}) \geq p(r(a_i) \geq r_{min})$, where $p(r(a_i) \geq r_{min})$ is the probability that the return of action a_i is at least equal to r_{min}.

Theorem 5 simply states that increased flexibility can only result in reducing risk. More insight into the relationship between risk and flexibility can be gained by considering specific cases. For instance let r_{i1}, r_{i2}, ..., r_{im} be independent, non-dominated, and continuous random variables representing the returns of the actions in the successor set $SS(a_i)$, then given perfect information, the following result holds:

Proposition 3: *Given perfect information, the expected risk of an action strictly decreases with an increase in its flexibility under the condition that actions in its successor set have independent, non-dominated, and continuously distributed returns.*

Proof: Since $r(a_i)|PI = max(r_{i1}, r_{i2}, ..., r_{im})$ and

$$p(r(a_i) \geq r_{min}|PI) = 1 - p(r(a_i) \leq r_{min}|PI)$$

then

$$p(r(a_i) \geq y|PI) = 1 - \prod_{j=1}^{m} p(r(a_{ij}) \leq y),$$

from which the desired result directly follows.

Note also that, since the relation is exponential, the probability appreciates significantly even with small increases in m. This means that in almost all risky situations some degree of flexibility is justified.

5. Flexibility versus expected value

It should be evident from the results and examples of section 4 that the value of an action in a stochastic multi-stage decision process is a function of four factors: (1) initial

uncertainty, (2) initial estimate of expected value, (3) expected future information, and (4) expected future flexibility. These dependencies can be described symbolically as:

$$E[r(a_i)|Y] = f[E(a_i), F(a_i), U, I] \qquad (5.1),$$

where $E(a_i)$, $F(a_i)$, U, and I refer respectively to initial estimate of expected value, expected future flexibility, initial uncertainty, and expected future information. An explicit expression for $f[E(a_i), F(a_i), U, I]$ can be obtained, as we have done so far, when the decision situation is well defined. That is, states, actions, returns, and future information are known and/or can be characterized by well defined distributions. However, this becomes difficult when these assumptions regarding the decision situation cannot be sustained. For instance, in a highly volatile and uncertain decision environment, the returns associated with each action may not be easy to anticipate as they are subject to fluctuation with a variety of factors. These factors themselves may not be easy to capture by a finite set of world states and an associated probability distribution.

Under these and other conditions, the use of the criterion $E[r(a_i)|Y]$ becomes impractical. Consequently, a more useful criterion, or criteria, needs to be constructed which, while being simpler to compute, should preserve, or approximate, the same preference structure as that produced by $E[r(a_i)|Y]$.

In current decision making practices, the initial estimate of expected value $E[r(a_i)]$ is generally used in the absence of an exact expression of $E[r(a_i)|Y]$ (see for instance (Winterfeldt and Edwards 1986)). Such practices however ignore information about future flexibility which is generally available. This information can be particularly useful in situations where there is a great deal of initial uncertainty that is expected to be completely or partially elucidated with the passage of time. In such situations, there is much to be gained by taking into account the degree of future flexibility associated with each current action. Thus, a more useful criterion would be based on both the initial estimate of expected value

and future flexibility. The decision problem can then be viewed and solved as one with dual objectives: expected value and flexibility.

A variety of approaches may be used to solve decision problems of this kind. For example, an efficient set of expected value-flexibility vectors may be generated. This would reduce the set of alternatives that the decision maker has to consider to only those that are not dominated. Once an efficient set is obtained, decisions can be based simply on a satisficing criterion, where first the value of one attribute is constrained by a minimal satisficing value and then the action with the highest value for the remaining attribute is selected. Alternatively, the problem may be reduced to that of a single attribute by assigning a value or utility to every attribute vector. This latter option is pursued in this paper. In particular, key requirements for a dual-attribute value function are identified.   An example of such a function is then proposed. This function could be used either directly as a decision criterion or as an intermediary step for deriving the decision maker's utility function.

5.1   Axiomatics of a value function

Based on the results of section 4, a value function, $v(\bullet)$, that ranks initial actions based on both their expected value and their future flexibility (i.e. $E(a_i)$ and $F(a_i)$) and that preserves the same preference structure as $E[r(a_i)|Y]$ should at least satisfy the following basic requirements:

(1) $v[E(a_i), F(a_i)] \approx^6 E(a_i)$ when $F(a_i) = 0$,

(2) $v[E(a_i), F(a_i)] \approx E(a_i)$ when $U = 0$,

(3) $v[E(a_i), F(a_i)] \approx E(a_i)$ when $I = 0$,

(4) $v[E(a_i), F(a_i)]$ does not decrease with an increase in $F(a_i)$ for fixed $E(a_i)$, U, and I,

(5) $v[E(a_i), F(a_i)]$ does not decrease with an increase in I for fixed $E(a_i)$, U, and $F(a_i)$,

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[6] The symbol $\approx$ is used to denote that. the value function is strategically equivalent to expected value and results in the same ranking of initial actions.

(6) $v[E(a_i), F(a_i)]$ does not decrease with an increase in $E(a_i)$ for fixed U, I, and $F(a_i)$,

(7) $v[E(a_i), F(a_i)] > E(a_i)$ when $I = U$, $U \neq 0$, and $F(a_i)$ is due to at least two non-dominated actions, and

(8) $v[E(a_i), F(a_i)]$ strictly increase with $F(a_i)$, when $I = U$, $U \neq 0$, and F is due to a set of non-dominated actions.

Axioms 1-3 indicate that flexibility may not be part of the decision criterion if either flexibility in period 2 is zero, initial uncertainty is zero, and/or no information is expected prior to taking the second period action. Axioms 4-8 describe monotonicity relationships between the value function and each of the attributes under various decision making conditions. Depending on the decision situation, other properties may be derived. In the next section an example value function that satisfies axioms 1-8 is presented.

5.2 Example value function:

A function of the following form can be shown to satisfy axioms 1-8:

$$v[E(a_i), F(a_i)] = E(a_i) + [E_{max}(a_i) - E(a_i)]\iota(I)\phi(F(a_i)) \qquad (5.2)$$

where $0 \leq \iota(I) \leq 1$ and $\iota(I)$ is a monotonically increasing function of I ($\iota(I) = 0$ when $I = 0$ and $\iota(I) = 1$ when there is perfect information), $0 \leq \phi(F(a_i)) \leq 1$ and $\phi(F(a_i))$ is a monotonically increasing function of flexibility ($\phi(F(a_i)) = 0$ when $F(a_i) = 0$ and $\phi(F(a_i)) = 1$ when $F(a_i)$ is maximum or infinite), $E(a_i)$ is the initial estimate of the action expected value, and $E_{max}(a_i)$ is the maximum expected value achievable once the action is taken in period 1. Note this function is of the form suggested in 4.7.

An example of the function $\iota(I)$ is given by the normalized measure of information

$$\iota(I) = 1 - \frac{U(S|Y)}{U(S)} \qquad (5.3),$$

while an example of the function $\phi(F(a_i))$ is given by the normalized measure of flexibility

$$\phi(F(a_i)) = 1 - \frac{1}{\|SS(a_i)\|} \qquad (5.4).$$

The value of $\iota(I)$ need not be exact and may simply be a rough estimate of the expected amount of future information.

It can be seen that for either zero information or zero flexibility, the value function is equivalent to the expected value. The function $v[E(a_i), F(a_i)]$ strictly increases with either an increase in flexibility or in information. In the limit case, when information is perfect and flexibility is maximum, the value function is equal to $E_{max}(a_i)$. Note that with increases in the amount of expected information (or initial uncertainty), actions with the greater flexibility become more attractive. Alternatively, the greater is the expected information (or the initial uncertainty), the more attractive become those actions with the higher flexibility. This means that decision makers are made to seek the most flexible positions either when initial uncertainty is high and/or a great deal of future information is expected.
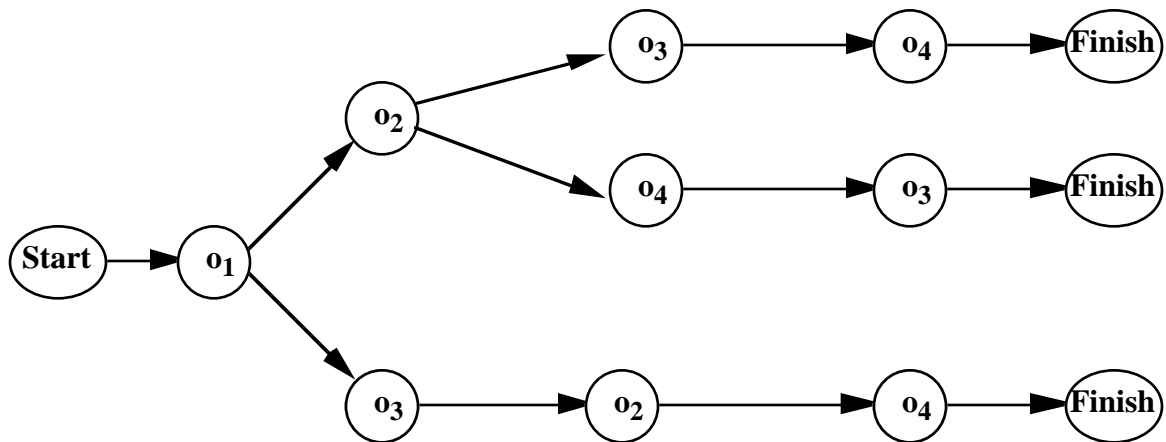
6. Example application: Flexibility in manufacturing

In this section, the usefulness of the results of this paper is illustrated by applying them to decision processes in discrete part manufacturing. In a previous study, it was shown that the flow of parts in a flexible manufacturing system (FMS) can be modeled as a multi-stage decision process (Benjaafar 1992). Flexibility was shown to arise at each stage of a part production from the availability of alternative processing and routing options from which a part can choose. For instance, a part, upon completing its current operation, may have a choice between more than one machine for carrying out its next operation. These alternative machines may not necessarily be identical and may have different production times or costs (e.g. a choice between a dedicated milling machine and a machining center). The ordering of all the part remaining operations may itself be flexible, with some of the operations having to be performed in sequence while others not being sequentially constrained. Thus at each stage, a part is faced with choices as to what should be its next operation and to which machine it should be assigned. Each of these choices is usually

associated with a different cost (or value) and results in a different portfolio of future options (i.e. future flexibility). For instance, in the example part of figure 2, choosing to perform operation 1 on machine 2 results in a different cost from that of performing it on machine 4 or 5. Similarly, choosing to carry out operation 2, once operation 1 is completed, produces a different set of future routing and sequencing options from that produced by choosing operation 3.
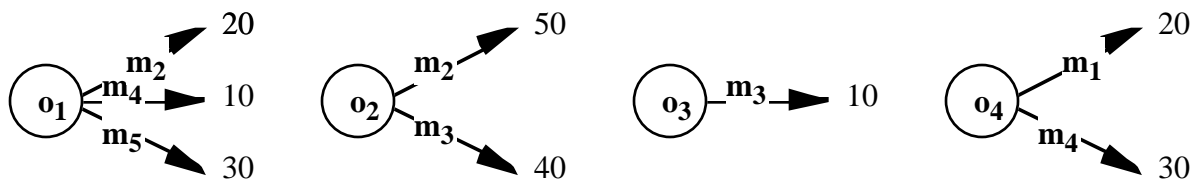
(a) Routing flexibility and processing costs

(b) Operation sequencing flexibility

Figure 2   Process planning flexibilities for an example part

6.2   Planning versus opportunism in part flow control

With the introduction of flexibility in part manufacturing specification, it becomes necessary to make a choice between the various possible manufacturing alternatives. It becomes also necessary to adopt a mechanism for making and implementing decisions. Because of the time-phased nature of the manufacturing process, this entails (1) the selection of a planning horizon determining the degree of lagging between action selection and action implementation, and (2) the adoption of a decision criterion based on which alternative actions are evaluated.

There have generally been two approaches to the timing of flow control decisions in manufacturing: the first is planning-based while the second is real time-based (Benjaafar 1992). Under a planning regime, a unique process plan is selected for each part at a pre-production stage. This process plan would then restrict the manufacturing possibilities for a workpiece to a single and fixed sequence of operations on a set of fixed machines. This sequence is usually chosen so that it optimizes a certain global performance measure(s). From a decision theoretic point of view, this consists of selecting a minimum cost action program.

Such an approach may be justified in a deterministic environment where the costs/returns associated with each manufacturing option are exactly known. However, this is often not the case. In fact, in most of today's systems, production demand is uncertain and production lead times are subject to a great deal of variability. This results in unpredictable part mixes, system loads, machine utilizations, etc., which in turn cause stochasticity in the costs, or desirability, of the available manufacturing alternatives (e.g. queue sizes and levels of inventory in front of each machine). The modeling of this stochasticity is often either difficult or computationally costly (Benjaafar and Talavage 1992).

The alternative to planning is to make decisions in real time. That is to let decision making be contemporaneous with action implementation. In the manufacturing context, this means allowing for flexible process plans and delaying operation selection and operation assignment until production time. This enables decisions to be made based on the present state of the system and minimizes the pre-commitment to actions whose desirability is uncertain. Such an approach to decision making is also called opportunistic.

In view of the results of previous sections, it should be clear that, under conditions of uncertainty, opportunism is superior to planning. This is particularly the case when initial uncertainty is important enough to warrant the delaying of action selection until more information is collected. The following example illustrates such a situation. A detailed discussion can be found in [Benjaafar 1992] and [Benjaafar and Talavage 1992].

Example: A flexible flow line

Consider a part that has to go through an ordered sequence of n operations (o_1, o_2, ..., o_n) as shown in figure 3. Each of these operations is associated with a set of machines on which it can be performed with each of the machines possibly having a different processing cost and processing time. Let the objective of the decision maker be to minimize average total part flow time, where flow time, t_f, is defined as the total time a part spends in process, or

$$t_f = \sum_{i=1}^{n} t_w(i) + \sum_{i=1}^{n} t_p(i) = \sum_{i=1}^{n} t_s(i) \qquad (6.1)$$

where $t_w(i)$ is the waiting time for the selected machine in stage i, $t_p(i)$ is the processing time on the selected machine of operation i, and $t_s(i)$ is the sum $t_w(i) + t_p(i)$ and referred to as the sojourn time. Finally, let the sojourn times be subject to variability with system states. Then, given a planning decision maker, expected flow time can be calculated as:

$$E(t_f|\text{Planning}) = \sum_{i=1}^{n} \min_j(E(t_s(o_i, m_j))) \qquad (6.2)$$

where $t_s(o_j, m_j)$ is the sojourn time if operation o_i is assigned to machine m_j. Note that the underlying assumption is that a planning decision maker would select the sequence of assignments that minimizes the a-priori estimate of expected flow time.

In contrast, an opportunistic decision maker, would make decisions at each stage based on the *actual* values of sojourn time. Thus, a part will be assigned to the machine with the actual shortest sojourn time (e.g. shortest queue). Consequently the expected value of flow time becomes

$$E(t_f|\text{Opportunism}) = \sum_{i=1}^{n} E(\min(t_s(o_i, m_1), t_s(o_i, m_2), ..., t_s(o_i, m_{m_i}))) \quad (6.3).$$

Using the results of section 4, it can easily be shown that $E(t_f|\text{Opportunism}) \leq E(t_f|\text{Planning})$, with the inequality being strict under conditions of non-dominance. It can also be shown that, under conditions of non-dominance, the value of $E(t_f|\text{Opportunism})$ strictly decreases with increases in flexibility (i.e. the number of machines available for each operation). Increased flexibility has no direct effect on expected sojourn time if machine assignments are made at a pre-production stage. This means that in order to realize the benefit of flexibility, an opportunistic part flow controller is necessary. On the other hand, it can be shown that the need for flexibility diminishes as uncertainty is decreased or eliminated (Benjaafar 1992). In fact, flexibility has an effect on performance only in the presence of variability in the system operating conditions. Also, in the absence of variability, both planning and opportunism become equivalent control mechanisms and result in similar performance. A more detailed discussion of the relationships between flexibility, uncertainty, and performance can be found in (Benjaafar 1992) and (Benjaafar and Talavage 1992).

These relationships can be seen more clearly by considering a single stage where all sojourn times are independent and identically and uniformly distributed with upper and

lower bound U and L respectively. The expected sojourn time for a planner is then, as shown in section 4, equal to

$$E(t_s|\text{Planning}) = \frac{U + L}{2} \qquad (6.5),$$

while for an opportunist it is equal to

$$E(t_s|\text{Planning}) = L + \frac{U - L}{m_i + 1} = \frac{U + L}{2} - (U - L)(\frac{1}{2} - \frac{1}{m_i + 1}) \qquad (6.5).$$

Clearly, for flexibility greater than zero (i.e. $m_i > 1$), $E(t_s|\text{Opportunism}) < E(t_s|\text{Planning})$. It is also easy to see that $E(t_s|\text{Opportunism})$ is a strictly decreasing function of m_i, with $\lim(E(t_s|\text{Opportunism})) = L$ when $m_i \to \infty$. Flexibility has no effect on sojourn time when uncertainty is zero (i.e. $U = L$), and both opportunism and planning become then equivalent (i.e. $E(t_s|\text{Opportunism}) = E(t_s|\text{Planning})$). However, the need for flexibility increases as uncertainty increases (uncertainty being measured by the difference $(U - L)$).
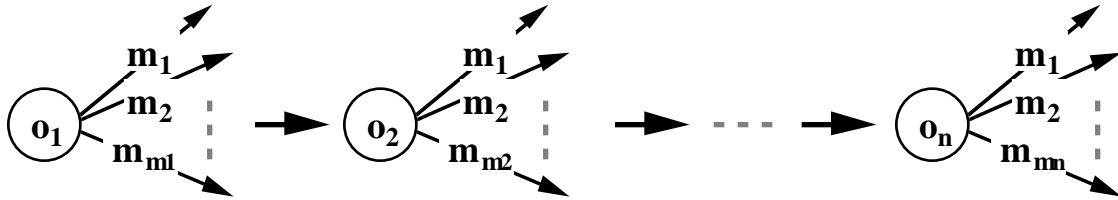

Figure 3   A flexible flow line

6.2   Flexibility in part flow control

In the above example, a single fixed sequence of operations is assumed. Within this sequence, the choice of a machine at a given stage was of no consequence on future stages. This is certainly not always the case. At each stage, the decision maker has often a choice between alternative operations and machines with each choice determining the number and type of operations and machines at subsequent stages (see for instance figure 2). Consequently, a decision criterion should account for both the immediate cost of an action and for its impact on future choices.

In a planned environment, decision makers would generally select the sequence of operation-to-machine assignments that maximizes their expected value. However this was shown to lead, under conditions of uncertainty, to less than an optimal performance. Based on insights from sections 4 and 5, a dual value function of expected value and flexibility could be constructed. For instance, using sojourn time as a measure of the worth of an operation assignment, a value function would account both for the the initial estimate of the resulting flow time and for future flexibility. Thus, each time a decision is made (e.g. an operation is selected or a machine is assigned), its effect on part flow time and on future flexibility would both be evaluated. Such a value function could be of the form :

$$v[E(t_f), F] = E(t_f) - [E(t_f) - E(t_{f_{min}})]\phi(F) \qquad (6.6)$$

where $E(t_f)$ represents an initial estimate of the the expected flow time resulting from the action in consideration, $E(t_{f_{min}})$ represents the minimum possible flow time, and $\phi(F)$ is a normalized measure of future flexibility (i.e. a function of the number of all possible future sequences of operation-to-machine assignments).

At times, it may be difficult to even generate rough estimates of flow times, thus making value functions of the above form is difficult to implement. In this case, decisions could be based solely on estimates of future flexibility. In fact, such a strategy has been advocated and tested by Yao and Pei (Yao and Pei 1990) and (Benjaafar 1992) who proposed the "principle of least reduction in flexibility" as the main criterion for ordering parts, selecting operations and assigning machines. They showed, using simulation, that this criterion results in significant reductions in flow times when compared to conventional scheduling and dispatching rules such as "shortest processing time" or "earliest due date".

An alternative to this approach is to combine estimates of future flexibility with whatever local information on value is available. For instance, the immediate sojourn time for a given operation is usually known with certainty. Thus, an operation could be assigned to a machine based both on the associated sojourn time in the current stage and on future

flexibility. Machines for each operation can then be ranked based on a value function of the following form:

$$v(m_i) = t_s(m_j)(1 + \phi(F(m_i))) \qquad (6.7)$$

where $t_s(m_j)$ is the immediate sojourn time resulting from assigning the operation to machine m_j and $\phi(F(m_i))$ is a normalized measure of the flexibility that remains once machine m_j is selected.

Finally, decision criteria may be devised, along similar lines, to deal with situations where several parts of possibly different types are simultaneously competing for the same resources. An aggregate measure of total system flexibility could be constructed and parts could be selected partially based on the effect of the assignment on future system flexibility.

7.   Conclusion

In this paper, the notion of flexibility in sequential decision making under uncertainty was formalized. Several fundamental relationships regarding flexibility and value were identified. These include the following:

(1) Flexibility is of no value if no initial uncertainty exists and/or no future information is expected.

(2) Expected value should not decrease with an increase in flexibility. In fact, it should be expected to strictly increase under certain conditions of non-dominance and/or given perfect information.

(3) For expected value to increase with the receipt of information, flexibility has to be greater than zero.

(4) In order to take better advantage of either flexibility or information decisions have to be made opportunistically.

It was also shown that in situations where either uncertainty or information are difficult to model, flexibility ought to be explicitly accounted for in the decision criterion.

Various approaches for this purpose were suggested. In particular, key characteristics of a dual value function were discussed.

These results were then applied to decision processes in manufacturing. For example, it was shown that opportunism, when coupled with flexible process plans, is a superior decision making mechanism to planning, in environments with a high degree of variability. The effectiveness of opportunism was illustrated by studying its effect on part flow time when queuing and processing times are stochastic. Also, it was suggested that, in situations where future system states cannot be easily predicted, control strategies should attempt to maximize future system flexibility. This flexibility would then allow the system to deal more effectively with unexpected changes.

The results of this paper have however wider implications to many areas of sequential decision making. The need to account for flexibility arises whenever current decision have future outcomes, and the nature of these outcomes is uncertain. Examples include investment decisions, marketing strategy, engineering design, and product development. Future research will investigate the relationship between flexibility and performance in several of these areas, and will develop systematic strategies for incorporating flexibility in the associated decision making processes.

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